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COLLECTIVE PHENOMENA IN THE INTERACTION
OF RADIATION WITH MATTER

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Коллективные явления при взаимодействии излучения с веществом

Цель этого сообщения — представить в сжатом виде основные идеи развитого автором метода описания сильнонеравновесных коллективных явлений, характерных для взаимодействия излучения с веществом, а также дать краткую иллюстрацию применения этого метода к нескольким проблемам. Развитый подход называется методом разделения масштабов, так как он базируется на возможности выделения различных пространственно-временных масштабов, характерных для неравновесной статистической системы. Этот общий подход может быть применен к разнообразным физическим проблемам, некоторые из которых рассмотрены в данном сообщении. Это сверхизлучение ядерных спинов, нитеобразование в резонансных средах, полуконфайнмент нейтральных атомов и отрицательный электрический ток.

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Collective Phenomena in the Interaction of Radiation with Matter

The aim of this communication is to present in a concentrated form the main ideas of a method, developed by the author, for treating strongly nonequilibrium collective phenomena typical of the interaction of radiation with matter, as well as to give a survey of several applications of the method. The latter is called the Scale Separation Approach since its basic techniques rely on the possibility of separating different space-time scales in statistical systems. This approach is rather general and can be applied to diverse physical problems, several of which are discussed here. These problems are: Superradiance of nuclear spins, filamentation in resonant media, semiconfinement of neutral atoms, and negative electric current.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

1 Introduction

Strongly nonequilibrium processes that occur in statistical systems and involve their interaction with radiation are usually described by complicated nonlinear differential and integro-differential equations. For treating these difficult problems, an approach has been developed [1,2] called the *scale separation approach* since its main idea is to formulate the evolution equations in such a form where it could be possible to separate several characteristic space-time scales. In many cases, different scales appear rather naturally being directly related to the physical properties of the considered system. A part of the approach is the *method of self-similar approximants* [3,4] that permits one to sum asymptotic series, if necessary, as well as to define typical scales of arising spatial structures.

The scale separation approach has been employed for solving several interesting physical problems related to strongly nonequilibrium processes occurring under the interaction of radiation with matter. As an illustration, the following phenomena are selected for this report: *Superradiance of Nuclear Spins* [1,2,5], *Filamentation in Resonant Media* [6,7], *Semiconfinement of Neutral Atoms* [8,9], and *Negative Electric Current* [10].

Since the scale separation approach makes the mathematical foundation for the following applications, its general scheme is described in Section 2 which is mainly based on Ref. [2] and partially, on Refs. [6,7,10] and [3,4]. In Sections 3 to 6 concrete physical effects are briefly reviewed and the most important results are summarized. Section 7 concludes the presentation.

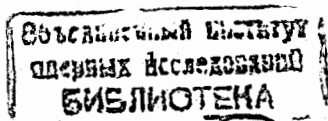
2 Scale Separation Approach

Because of the pivotal role of this approach for treating different physical problems, its general scheme will be presented here in an explicit way. It is possible to separate the following main steps, or parts, of the approach.

2.1 Stochastic quantization of short-range correlations

When considering nonequilibrium processes in statistical systems, one needs to write evolution equations for some averages $\langle A_i \rangle$ of operators $A_i(t)$ where t is time and $i = 1, 2, \dots, N$ enumerates particles composing the considered system. For simplicity, a discrete index i is used, although everywhere below one could mean an operator $A(\vec{r}_i, t)$ depending on a continuous space variable \vec{r}_i .

There is the well known problem in statistical mechanics consisting in the fact that writing an evolution equation for $\langle A_i \rangle$ one does not get a closed system of equations but a hierarchical chain of equations connecting correlation functions of higher orders. Thus, an equation for $\langle A_i \rangle$ contains the terms as $\sum_j \langle A_i B_j \rangle$ with double correlators $\langle A_i B_j \rangle$, and the evolution equations for the latter involve the terms with tripple correlators, and so on. The simplest way for making the system of equations closed



is the mean-field type decoupling $\langle A_i B_j \rangle \rightarrow \langle A_i \rangle \langle B_j \rangle$. When considering radiation processes, this decoupling is called the semiclassical approximation. Then the term $\sum_j \langle A_i B_j \rangle$ reduces to $\langle A_i \rangle \sum_j \langle B_j \rangle$, so that one can say that $\langle A_i \rangle$ is subject to the action of the mean field $\sum_j \langle B_j \rangle$. The semiclassical approximation describes well coherent processes, when long-range correlations between atoms govern the evolution of the system, while short-range correlations, due to quantum fluctuations, are not important. However, the latter may become of great importance for some periods of time, for example, at the beginning of a nonequilibrium process when long-range correlations have had yet no time to develop. Then neglecting short-range correlations can lead to principally wrong results.

To include the influence of short-range correlations, the semiclassical approximation can be modified as follows:

$$\sum_j \langle A_i B_j \rangle = \langle A_i \rangle \left(\sum_j \langle B_j \rangle + \xi \right), \quad (1)$$

where ξ is a random variable describing local short-range correlations. It is natural to treat ξ as a Gaussian stochastic variable with the stochastic averages

$$\langle \xi \rangle = 0, \quad \langle |\xi|^2 \rangle = \sum_j |\langle B_j \rangle|^2, \quad (2)$$

where the second moment is defined so that to take into account incoherent local fluctuations. Since short-range correlations are often due to quantum fluctuations, the manner of taking them into account by introducing a stochastic variable ξ can be called the stochastic quantization. Then the decoupling (1) may be termed the *stochastic semiclassical approximation*. This kind of approximation has been used for taking into account quantum spontaneous emission of atoms in the problem of atomic superradiance.

2.2 Separation of solutions onto fast and slow

The usage of the stochastic semiclassical approximation makes it possible to write down a closed set of stochastic differential equations. The next step is to find such a change of variables which results in the possibility of separating the functional variables onto fast and slow, so that one comes to the set of equations having the form

$$\frac{du}{dt} = f(\varepsilon, u, s, \xi, t), \quad \frac{ds}{dt} = \varepsilon g(\varepsilon, u, s, \xi, t), \quad (3)$$

where $\varepsilon \ll 1$ is a small parameter, such that

$$\lim_{\varepsilon \rightarrow 0} f \neq 0, \quad \lim_{\varepsilon \rightarrow 0} \varepsilon g = 0. \quad (4)$$

As is evident, dealing with only two functions, u and s , and one small parameter ε is done just for simplicity. All procedure is straightforwardly applicable to the case of many functions and several small parameters.

From Eqs. (3) and (4) it follows that

$$\lim_{\varepsilon \rightarrow 0} \frac{du}{dt} \neq 0, \quad \lim_{\varepsilon \rightarrow 0} \frac{ds}{dt} = 0, \quad (5)$$

which permits one to classify the solution u as fast, compared to the slow solution s . In turn, the slow solution s is a *quasi-invariant* with respect to the fast solution u .

The above classification of solutions onto fast and slow concerns time variations. In the case of partial differential equations, one has, in addition to time, a space variable \vec{r} . Then the notion of fast or slow functions can be generalized as follows [10]. Let $\vec{r} \in V$, with $\text{mes} V \equiv V$, and $t \in [0, T]$, where T can be infinite. Assume that

$$\lim_{\varepsilon \rightarrow 0} \left\langle \left\langle \frac{1}{V} \int_V \frac{\partial u}{\partial t} d\vec{r} \right\rangle \right\rangle \neq 0, \quad \lim_{\varepsilon \rightarrow 0} \left\langle \left\langle \frac{1}{T} \int_0^T \vec{\nabla} u dt \right\rangle \right\rangle \neq 0, \quad (6)$$

while

$$\lim_{\varepsilon \rightarrow 0} \left\langle \left\langle \frac{1}{V} \int_V \frac{\partial s}{\partial t} d\vec{r} \right\rangle \right\rangle = 0, \quad \lim_{\varepsilon \rightarrow 0} \left\langle \left\langle \frac{1}{T} \int_0^T \vec{\nabla} s dt \right\rangle \right\rangle = 0. \quad (7)$$

Then the solution u is called *fast on average*, with respect to both space and time, as compared to s that is *slow on average*. In such a case s is again a quasi-invariant as compared to u . In general, it may, of course, happen that one solution is fast with respect to time but slow in space, or vice versa, when compared to another function. The notion of quasi-invariants with respect to time is known in the Hamiltonian mechanics where they are also called adiabatic invariants. Here this notion is generalized to the case of both space and time variables [10].

2.3 Averaging method for multifrequency systems

After classifying in Eqs. (3) the function u as fast and s as slow, one can resort to the Krylov-Bogolubov averaging technique [11] extended to the case of multifrequency systems. This is done as follows.

Since the slow variable s is a quasi-invariant for the fast variable u , one considers the equation for the fast function u , with the slow one kept fixed,

$$\frac{\partial X}{\partial t} = f(\varepsilon, X, z, \xi, t). \quad (8)$$

Here z is treated as a fixed parameter. The solution to Eq. (8), that is

$$X = X(\varepsilon, z, \xi, t), \quad z = \text{const}, \quad (9)$$

has to be substituted into the right-hand side of the equation for the slow function, and for this right-hand side one defines the average

$$\bar{g}(\varepsilon, z) \equiv \left\langle \left\langle \frac{1}{T} \int_0^T g(\varepsilon, X(\varepsilon, z, \xi, t), z, \xi, t) dt \right\rangle \right\rangle, \quad (10)$$

in which τ is the characteristic oscillation time of the fast function. In many cases, it is possible to take $\tau \rightarrow \infty$, especially when the period of fast oscillations is not well defined [2]. Then one comes to the equation

$$\frac{dz}{dt} = \varepsilon \bar{g}(\varepsilon, z) \quad (11)$$

defining a solution

$$z = z(\varepsilon, t). \quad (12)$$

Substituting the latter into X , one gets

$$y(\varepsilon, \xi, t) = X(\varepsilon, z(\varepsilon, t), \xi, t). \quad (13)$$

The pair of solutions (9) are called the *generating solutions* since these are the first crude approximations one starts with. More elaborate solutions are given by Eqs. (12) and (13) which are termed *guiding centers*.

Notice two points that difference the case considered from the usual averaging techniques. The first point is that in Eq. (8) the small parameter ε is not set zero. And the second difference is in the occurrence of the stochastic average in Eq. (10). Leaving ε in Eq. (8) makes it possible to correctly take into account attenuation effects, as will be shown in applications.

2.4 Generalized expansion about guiding centers

Higher-order corrections to solutions may be obtained by presenting the latter as asymptotic expansions about the guiding centers (12) and (13). To this end, k -order approximations are written as

$$u_k = y(\varepsilon, \xi, t) + \sum_{n=1}^k y_n(\varepsilon, \xi, t) \varepsilon^n, \\ s_k = z(\varepsilon, t) + \sum_{n=1}^k z_n(\varepsilon, \xi, t) \varepsilon^n. \quad (14)$$

Such series are called *generalized asymptotic expansions* [12] since the expansion coefficients depend themselves on the parameter ε . The right-hand sides of Eqs. (3) are to be expanded similarly to Eq. (14) yielding

$$f(\varepsilon, u_k, s_k, \xi, t) \simeq f(\varepsilon, y, z, \xi, t) + \sum_{n=1}^k f_n(\varepsilon, \xi, t) \varepsilon^n \quad (15)$$

and an equivalent expansion for g . These expansions are to be substituted into Eqs. (3) with equating the like terms with respect to the powers of ε . In the first order, this gives

$$\frac{dy_1}{dt} = f_1(\varepsilon, \xi, t) - \bar{g}(\varepsilon, z) X_1(\varepsilon, \xi, t), \quad \frac{dz_1}{dt} = g(\varepsilon, y, z, \xi, t) - \bar{g}(\varepsilon, z), \quad (16)$$

where

$$X_1(\varepsilon, \xi, t) \equiv \frac{\partial}{\partial z} X(\varepsilon, z, \xi, t), \quad z = z(\varepsilon, t).$$

For the approximations of order $n \geq 2$, one gets

$$\frac{dy_n}{dt} = f_n(\varepsilon, \xi, t), \quad \frac{dz_n}{dt} = g_n(\varepsilon, \xi, t). \quad (17)$$

The functions f_n and g_n depend on y_1, y_2, \dots, y_n and on z_1, z_2, \dots, z_n (see for details [2]). But it is important that the dependence on y_n and z_n is linear. Therefore all equations (16) and (17) are linear and can be easily integrated. Thus, the approximants (14) are defined. Each k -order approximation can also be improved by invoking the self-similar summation of asymptotic series [3,4].

2.5 Selection of scales for space structures

The solutions of differential or integro-differential equations in partial derivatives are often nonuniform in space exhibiting the formation of different spatial structures. Also, it often happens that a given set of equations possesses several solutions corresponding to different spatial patterns or to different scales of such patterns [13]. Examples of these spatial structures could be photon filaments in laser media [6,7]. When one has a set of solutions describing different possible patterns, the question arises which of these solutions, and respectively patterns, to prefer? This problem of pattern selection is a general and very important problem constantly arising in considering spatial structures. In some cases this problem can be solved as follows.

Assume that the obtained solutions describe spatial structures that can be parametrized by a multiparameter b , so that the k -order approximations $u_k(b, t)$ and $s_k(b, t)$ include the dependence on b whose value is however yet undefined. To define b , and respectively the related pattern, one may proceed in the spirit of the self-similar approximation theory [3,4], by treating b as a control function, or a set of control functions if b is a multiparameter. According to the theory [3,4], control functions are to be defined from fixed-point conditions for an approximation cascade, which is to be constructed for an observed quantity. For the latter, one may take the energy which is a functional $E[u, s]$ of the solutions. In experiments, one usually measures an average energy whose k -order approximation writes

$$E_k(b) \equiv \ll \frac{1}{\tau} \int_0^\tau E[u_k(b, t), s_k(b, t)] dt \gg, \quad (18)$$

where τ is a period of fast oscillations. For the sequence of approximations, $\{E_k(b)\}$, it is possible to construct an approximation cascade [3,4] and to show that its fixed point can be given by the condition

$$\frac{\partial}{\partial b} E_k(b) = 0, \quad (19)$$

from which one gets the control function $b = b_k$ defining the corresponding pattern. According to optimal control theory, control functions are defined so that to minimize

a cost functional. In this case, it is natural to take for the latter the average energy (18). Therefore, if the fixed-point equation (19) has several solutions, one may select of them that one which minimizes the cost functional (18),

$$E_k(b_k) = \text{abs min}_b E_k(b). \quad (20)$$

Equations (19) and (20) have a simple physical interpretation as the minimum conditions for the average energy (18). However, one should keep in mind that there is no in general such a principle of minimal energy for nonequilibrium systems [13]. Therefore the usage of the ideas from the self-similar approximation theory [3,4] provides a justification for employing conditions (19) and (20) for nonequilibrium processes.

In the following sections a brief survey is given of several physical examples the scale separation approach has been applied to, and the main results are formulated.

3 Superradiance of Nuclear Spins

A system of neutral spins in an external magnetic field, prepared in a strongly nonequilibrium state and coupled with a resonance electric circuit, displays rather nontrivial relaxation behaviour somewhat similar to that of an inverted system of atoms. This is why the optical terminology, such as superradiance, has been used for describing collective relaxation processes in nonequilibrium nuclear magnets [1,2,5].

For a system of nuclear spins interacting through dipole forces the evolution equations can be derived [2] for the averages

$$u \equiv \frac{1}{N} \sum_{i=1}^N \langle S_i^- \rangle, \quad s \equiv \frac{1}{N} \sum_{i=1}^N \langle S_i^z \rangle, \quad (21)$$

in which N is the number of spins, angle brackets mean statistical averaging, S_i^- is a lowering spin operator, and S_i^z is the z -component of a spin operator. Following the ideology of the scale separation approach, local fluctuating fields are presented by stochastic variables ξ_0 and ξ . In this way, one comes to the evolution equations for the transverse spin variable

$$\frac{du}{dt} = i(\omega_0 - \xi_0 + i\gamma_2)u - i(\gamma_3 h + \xi)s \quad (22)$$

and the longitudinal average spin

$$\frac{ds}{dt} = \frac{i}{2}(\gamma_3 h + \xi)u^* - \frac{i}{2}(\gamma_3 h + \xi^*)u - \gamma_1(s - \zeta). \quad (23)$$

It is also convenient to consider the equation

$$\frac{d}{dt}|u|^2 = -2\gamma_2|u|^2 - i(\gamma_3 h + \xi)su^* + i(\gamma_3 h + \xi^*)su. \quad (24)$$

In equations (22)–(24) dimensionless units are used for the resonator magnetic field h satisfying the Kirchoff equation

$$\frac{dh}{dt} + 2\gamma_2 h + \omega^2 \int_0^t h(t') dt' = -2\alpha_0 \frac{d}{dt}(u^* + u) + \gamma_3 f. \quad (25)$$

Here ω_0 is the Zeeman frequency of spins in an external uniform magnetic field, ω is the resonator natural frequency, γ_1 and γ_2 are the spin-lattice and spin-spin relaxation parameters, respectively, γ_3 is the resonator ringing width, ζ is a stationary spin polarization, α_0 is the coupling between spins and the resonator, and f is an electromotive force. The random local fields are defined as Gaussian stochastic variables with the stochastic averages

$$\langle\langle \xi_0^2 \rangle\rangle = \langle\langle |\xi|^2 \rangle\rangle = \gamma_2^*, \quad (26)$$

where γ_2^* is the inhomogeneous dipole broadening.

There are the following small parameters in the system:

$$\frac{\gamma_1}{\omega_0} \ll 1, \quad \frac{\gamma_2}{\omega_0} \ll 1, \quad \frac{\gamma_2^*}{\omega_0} \ll 1, \quad \frac{\gamma_3}{\omega} \ll 1, \quad \frac{\Delta}{\omega_0} \ll 1, \quad (\Delta \equiv \omega - \omega_0). \quad (27)$$

This makes it admissible to classify the functions u and h as fast, while s and $|u|^2$ as slow, and to apply the method of Section 1. The behaviour of solutions to Eqs. (22)–(25) depends on initial conditions for $u(0)$, and $s(0)$, on the existence of an electromotive driving force $f(t)$, on the pumping related to the parameter ζ , and on the value of the effective coupling parameter

$$g = \pi^2 \eta \frac{\rho \mu_n^2 \omega_0}{\hbar \gamma_2 \omega}, \quad (28)$$

in which η is a filling factor; ρ , spin density; and μ_n is a nuclear magnetic moment.

The first interesting result is that the electromotive force does not influence much macroscopic samples [2] since the corresponding correlation time is proportional to N , that is, the effective, interaction strength of an electromotive force with the spin system is proportional to N^{-1} . This shows, in particular, that the role of the thermal Nyquist noise for starting the relaxation process is negligible. The main cause triggering the motion of spins leading to coherent self-organization is the presence of *nonsecular dipole interactions* [1,2]. The latter result gives an answer to the problem, posed by Bloembergen and Pound [14]: What is the origin of self-organized coherent relaxation in spin systems?

All possible regimes of nonlinear spin dynamics have been analysed [1,2,5]. When the nonresonant external pumping is absent, that is $\zeta > 0$, there are seven qualitatively different transient relaxation regimes: *free induction*, *collective induction*, *free relaxation*, *collective relaxation*, *weak superradiance*, *pure superradiance*, and *triggered superradiance* [2]. In the presence of pumping, realized e.g. by means of dynamical

nuclear polarization directing nuclear spins against an external constant magnetic field, one has $\zeta \leq 0$. Then three dynamical regimes can be observed, depending on the value of ζ with respect to the pumping thresholds

$$\zeta_1 = -\frac{1}{g}, \quad \zeta_2 = -\frac{1}{g} \left(1 + \frac{\gamma_1^*}{2\gamma_2} \right), \quad (29)$$

where γ_1^* is an effective pumping rate.

Two stationary points can exist for the slow solutions s and w , where

$$w \equiv |u|^2 - \left(\frac{\gamma_2^*}{\omega_0} \right)^2 s^2.$$

These fixed points are

$$\begin{aligned} s_1^* &= \zeta, & w_1^* &= 0, \\ s_2^* &= -\frac{1}{g}, & w_2^* &= -\frac{\gamma_1^*(1+g\zeta)}{g^2\gamma_2}. \end{aligned} \quad (30)$$

When $\zeta_1 < \zeta \leq 0$, then the first fixed point is a stable node and the second one is a saddle point. For $\zeta = \zeta_1$, both stationary points merge together, being neutrally stable. After the bifurcation at the value $\zeta = \zeta_1$, in the region $\zeta_2 \leq \zeta < \zeta_1$, the first fixed point loses its stability becoming a saddle while the second fixed point becomes a stable node. Finally, when $\zeta < \zeta_2$, the second fixed point transforms to a stable focus, and the first one is, as earlier, a saddle point.

In this way, there are three qualitatively different lasting relaxation regimes induced by the pumping. The first one is a monotonic relaxation to the first stationary solution with practically no coherence, $w_1^* = 0$. The second regime is a monotonic relaxation to the second stationary solution with a nonzero coherence, $w_2^* \neq 0$. And the third regime is that of pulsing relaxation to the coherent stationary point. Note that the pumping rate γ_1^* can be larger than γ_2 , so that w_2^* can reach the order of unity. The three lasting relaxation regimes occurring in the presence of pumping can be called, respectively, *incoherent monotone attenuation*, *coherent monotone relaxation*, and *coherent pulsing relaxation*.

4 Filamentation in Resonant Media

In optical resonant media there appear space structures when the radiation wavelength is much less than the characteristic sizes of the laser system. Such structures are described by nonlinear differential equations in partial derivatives. The general problem in dealing with these equations is the nonuniqueness of their solutions each of which corresponds to a particular spatial structure [13]. The related problem of pattern selection can be treated by the method of subsection 2.5. Here this is illustrated by the theory of filamentation in optical resonant media [6,7].

The Hamiltonian for a system of resonant atoms can be written as

$$\hat{H} = \frac{1}{2} \sum_i \omega_0 (1 + \sigma_i^2) - \frac{1}{2} \sum_i \left(\vec{d}^{\dagger} \cdot \vec{E}_{0i}^{\dagger} \sigma_i^- + \sigma_i^{\dagger} \vec{d} \cdot \vec{E}_{0i}^- \right) -$$

$$-\frac{1}{2} \sum_{i \neq j} \left(\vec{d}^{\dagger} \cdot \vec{E}_{ij}^{\dagger} \sigma_i^- + \sigma_i^{\dagger} \vec{d} \cdot \vec{E}_{ij}^- \right), \quad (31)$$

where the standard notation is used for the Pauli matrices σ_i^{\pm} and \vec{d} is a transition dipole for a transition with the frequency ω_0 ; the electric fields

$$\vec{E}_{0i}^- = \vec{E}_0 e^{i(kz_i - \omega t)} \quad \left(k \equiv \frac{\omega}{c} \right),$$

$$\vec{E}_{ij}^- = \frac{k_0^2}{r_{ij}} \vec{n}_{ij} \times \left(\vec{d} \times \vec{n}_{ij} \right) e^{i k_0 r_{ij}} \sigma_j^- \quad \left(k_0 \equiv \frac{\omega_0}{c} \right) \quad (32)$$

correspond to the laser mode and to the reradiated field, respectively, and

$$r_{ij} \equiv |\vec{r}_{ij}|, \quad \vec{n}_{ij} \equiv \frac{\vec{r}_{ij}}{r_{ij}}, \quad \vec{r}_{ij} \equiv \vec{r}_i - \vec{r}_j.$$

The resonant medium has cylindrical shape of radius R and length L . The transition wavelength λ is such that

$$\frac{\lambda}{R} \ll 1, \quad \frac{R}{L} \ll 1. \quad (33)$$

It is convenient to pass to a continuous space variable \vec{r} by transforming the sums in integrals according to the rule

$$\sum_{i=1}^N f_i = \rho \int f(\vec{r}) d\vec{r} \quad \left(\rho \equiv \frac{N}{V} \right).$$

Then the evolution equations for the statistical averages

$$u(\vec{r}, t) \equiv \langle \sigma^-(\vec{r}, t) \rangle, \quad s(\vec{r}, t) \equiv \langle \sigma^2(\vec{r}, t) \rangle \quad (34)$$

satisfy partial integro-differential equations. Because of the inequalities

$$\frac{\gamma_2}{\omega_0} \ll 1, \quad \frac{|\Delta|}{\omega_0} \ll 1, \quad (35)$$

where $\Delta \equiv \omega - \omega_0$ is the detuning parameter, the function u is fast in time as compared to the slow function s .

The solutions to the evolution equations describe a bunch of filaments aligned along the z axis which is the axis of the sample. Each filament has a radius R_f and is centered at a point $\{x_n, y_n\}$ in the transverse cross-section. The location of the filament centers is distributed chaotically. Thus, the solutions may be presented as expansions over filaments in the form

$$\begin{aligned} u(\vec{r}, t) &= \sum_{n=1}^{N_f} u_n(t) e^{ikz} \Theta \left(R_f - \sqrt{(x - x_n)^2 + (y - y_n)^2} \right), \\ s(\vec{r}, t) &= \sum_{n=1}^{N_f} s_n(t) \Theta \left(R_f - \sqrt{(x - x_n)^2 + (y - y_n)^2} \right), \end{aligned} \quad (36)$$

where N_f is the number of filaments and $\Theta(\cdot)$ is a unit-step function. The number of filaments is related to the filament radius R_f and the pumping characteristic

$$\zeta(t) = \frac{1}{V} \int s(\vec{r}, t) d\vec{r}, \quad (37)$$

which yields

$$N_f = \frac{1}{2} (1 + \zeta) \left(\frac{R}{R_f} \right)^2. \quad (38)$$

The filament radius R_f can be defined according to the procedure of subsection 2.5. To this end, R_f is considered as a control function parametrizing the filamentary space structure. It is possible to construct the average energy (18) corresponding to the Hamiltonian (31). Minimizing this energy functional with respect to R_f gives

$$R_f = 0.22 \sqrt{\lambda L}. \quad (39)$$

Then the number of filaments (38) can be written as

$$N_f = 3.3 (1 + \zeta) F \left(F \equiv \frac{\pi R^2}{\lambda L} \right), \quad (40)$$

where F is a Fresnel number.

The predictions of the theory [6,7] have been found to be in a good agreement with measurements, as has been confirmed in a series of experiments [15–19] with different lasers.

5 Semiconfinement of Neutral Atoms

Dynamics of neutral atoms in nonuniform magnetic fields concerns problems of current experimental and theoretical interest, especially with regard to atoms in magnetic traps, where the atoms can be cooled down to experience the Bose–Einstein condensation [20]. The motion of confined atoms is usually described by means of the adiabatic approximation assuming that the atom spins are permanently aligned along the local magnetic field and, thus, adiabatically follow its direction. To study the more general case, when atoms are permitted to escape from a trap, one has to invoke a more refined approximation, such as the scale separation approach described in Section 2.

The motion of neutral atoms in magnetic fields can be presented by the semiclassical equations for the quantum–mechanical average of the real–space coordinate, $\vec{R} = \{R_\alpha\}$, and for the average $\vec{S} = \{S_\alpha\}$ of the spin operator, with $\alpha = x, y, z$. The first equation writes

$$\frac{d^2 R_\alpha}{dt^2} = \frac{\mu_0}{m} \vec{S} \cdot \frac{\partial \vec{B}}{\partial R_\alpha} + \gamma \xi_\alpha, \quad (41)$$

where μ_0 is magnetic moment, m is mass of an atom, \vec{B} is a magnetic field, and $\gamma \xi_\alpha$ is a collision term. The equation for the average spin is

$$\frac{d\vec{S}}{dt} = \frac{\mu_0}{m} \vec{S} \times \vec{B}. \quad (42)$$

The total magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$ consists of two terms. One is the quadrupole field

$$\vec{B}_1 = B_1' (R_x \vec{e}_x + R_y \vec{e}_y + \lambda R_z \vec{e}_z), \quad (43)$$

in which λ is the anisotropy parameter. The second term is a transverse field

$$\vec{B}_2 = B_2 (\vec{e}_x \cos \omega t + \vec{e}_y \sin \omega t). \quad (44)$$

The characteristic frequencies

$$\omega_1 \equiv \left(\frac{\mu_0 B_1'}{m R_0} \right)^{1/2}, \quad \omega_2 \equiv \frac{\mu_0 B_2}{\hbar}, \quad (45)$$

where $R_0 \equiv B_2/B_1'$, satisfy the inequalities

$$\frac{\omega_1}{\omega_2} \ll 1, \quad \frac{\omega}{\omega_2} \ll 1. \quad (46)$$

Because of the latter, the spin variable \vec{S} has to be classified as fast, compared to the slow atomic variable \vec{R} .

The collision term in Eq. (41) contains a collision rate γ and a random collision variable $\xi_\alpha(t)$ defined by the stochastic averages

$$\langle \xi_\alpha(t) \rangle = 0, \quad \langle \xi_\alpha(t) \xi_\beta(t') \rangle = 2D_\alpha \delta_{\alpha\beta} \delta(t - t'), \quad (47)$$

where D_α is a diffusion rate.

The semiconfining regime of motion can be realized by preparing for the spin variable nonadiabatic initial conditions

$$S_x^0 = S_y^0 = 0, \quad S_z^0 = S \neq 0, \quad (48)$$

which can be done e.g. by means of an external pulse at $t = 0$. Then it is possible to show [8,9] that the motion of atoms becomes axially restricted by the value

$$z_m R_0 = \begin{cases} \min_t R_z(t) & (\lambda S > 0) \\ \max_t R_z(t) & (\lambda S < 0), \end{cases}$$

such that

$$z_m^3 = z_0^3 - \frac{3z_0^2}{2\lambda^3 S \omega_1^2}. \quad (49)$$

Atomic collisions do not disturb the semiconfined motion provided that temperature T is sufficiently low satisfying the condition

$$\frac{k_B T \hbar \rho^2 a_0^2}{m^2 \omega_1^3} \ll 1, \quad (50)$$

where ρ is the density of particles and a_0 is a scattering length.

The semiconfining regime of motion makes it possible to form well-collimated beams of neutral particles by means of only magnetic fields. This regime can be employed for creating coherent beams of Bose atoms from atom lasers.

6 Negative Electric Current

Semiconductors with nonuniform distribution of charge carriers can exhibit rather unusual transport properties. For example, in a sample biased with an external constant voltage, the transient effect of negative electric current can happen [10].

Transport properties of semiconductors are usually described by the semiclassical drift-diffusion equations. In what follows a plane device is considered and all quantities are expressed in dimensionless form, so that the space variable is $x \in [0, 1]$. The continuity equation writes

$$\frac{\partial \rho_i}{\partial t} + \mu_i \frac{\partial}{\partial x} (\rho_i E) - D_i \frac{\partial^2 \rho_i}{\partial x^2} + \frac{\rho_i}{\tau_i} = 0, \quad (51)$$

where $\rho(x, t)$ is a charge density; $E(x, t)$ is the electric current; μ_i , D_i , and τ_i are mobility, diffusion coefficient, and relaxation time, respectively, for the carriers of type i . The Poisson equation is

$$\frac{\partial E}{\partial t} = 4\pi \sum_i \rho_i. \quad (52)$$

At the initial time, the distribution of charge carriers is nonuniform, given by

$$\rho_i(x, 0) = f_i(x). \quad (53)$$

The sample is biased with an external constant voltage, which means that

$$\int_0^1 E(x, t) dx = 1. \quad (54)$$

The total electric current through the semiconductor sample is

$$J(t) \equiv \int_0^1 j(x, t) dx, \quad (55)$$

where the density of current

$$j = \sum_i \left(\mu_i E - D_i \frac{\partial}{\partial x} \right) \rho_i + \frac{1}{4\pi} \frac{\partial E}{\partial t}.$$

Owing to the voltage integral (54), one has

$$\int_0^1 \frac{\partial}{\partial t} E(x, t) dx = 0. \quad (56)$$

It is also possible to show that

$$\lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau \frac{\partial}{\partial x} E(x, t) dt = 0. \quad (57)$$

This means that the function E can be considered as slow on average in time and space. Then, treating E as a quasi-invariant, one may find the solutions to Eqs. (51) and (52) in order to analyse their general space-time behaviour.

Negative electric current can appear only when the initial charge distribution is essentially nonuniform. If this initial distribution forms a narrow layer located at the point $x = a$, then the current (55) becomes negative for some short time close to $t = 0$, if one of the following conditions holds true:

$$\begin{aligned} a < \frac{1}{2} - \frac{1}{4\pi Q} & \quad \left(Q > \frac{1}{2\pi} \right), \\ a > \frac{1}{2} + \frac{1}{4\pi|Q|} & \quad \left(Q < -\frac{1}{2\pi} \right), \end{aligned} \quad (58)$$

where

$$Q \equiv \sum_i Q_i, \quad Q_i \equiv \int_0^1 \rho_i(x, 0) dx. \quad (59)$$

The effect of the negative electric current can be employed for various purposes, as is discussed in Ref. [10]. For instance, when the initial charge layer is formed by an ion beam irradiating the semiconductor sample, then the location a corresponds to the mean free path of these ions. In this case, by measuring the negative current $J(0)$, one can define this mean free path

$$a = \frac{1}{2} - \frac{1}{4\pi Q} \left[1 - \frac{J(0)}{\sum_i \mu_i Q_i} \right]. \quad (60)$$

This formula is valid for both positive and negative values of Q .

7 Conclusion

A general method has been developed for treating strongly nonequilibrium processes in statistical systems. This method, called the scale separation approach, is essentially useful for describing collective phenomena in the interaction of radiation with matter. To emphasize the generality of the approach, it is illustrated here by several different physical examples. The common feature of all considered systems is that their evolution is described by nonlinear differential or integro-differential equations. Such equations, as is known, are difficult to solve. The scale separation approach makes it possible to find accurate approximate solutions. The accuracy of the latter has been confirmed by numerical calculations and by comparison with experiment, when available. Using this approach several interesting physical problems have been solved and new effects are predicted.

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