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TRAPPED BOSE-CONDENSATE IN GRAVITY FIELD

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Among the fundamental physical phenomena planned by NASA for researching in space [1], the possibility of influence a gravity field on the critical BEC temperature of ^{4}He was discussed [2]. The experiments on BEC in trapped atomic gases in space are unlikely in nearest future, but it is interesting to consider theoretically the contribution of gravity field constant g to the critical BEC temperature in the trap.

Let us consider a three dimensional isotropic harmonic trap with the frequency ω and the height h in direction of the gravity field z. The potential of the trap may be written as

$$U_0 + mgz, \quad \frac{m\omega^2}{2}(r - r_0)^2 + mgz = U(r) + mgz, \quad z \geq 0,$$

where m is the mass of an atom. The first expression describes the field out of the trap with the potential barrier U_0, and the second one describes the field inside of the trap with the center in point r_0. The new variable $Z = z - z_0$ gives the shifts of the field mgz_0, so that the potential of the trap $U(z)$ in z-direction takes the form (see Fig. 1)

$$U(z) = \begin{cases} U_0 + mgZ, & Z < -h/2, \quad Z > h/2, \\ U_g + (m\omega^2/2)(Z + \Delta)^2, & -h/2 < Z < h/2, \quad \Delta = g/\omega^2. \end{cases} \tag{1}$$

The gravity field shifts the minimum of potential to the right from its center $Z = 0$ by the value Δ and down by the value $U_g = -mg^2/2\omega^2$. The difference between new (shifted) potential barriers U_+ (on the right) and U_- (on the left)

$$U_\pm = \frac{m\omega^2}{2}\left(\Delta \pm \frac{h}{2}\right)^2 + U_g, \quad U_0 - U_- = U_+ - U_0 = \frac{1}{2}mgh$$

is equal mgh (difference in energy of atom between two potential barriers in z-direction). The full numbers of atoms N in volume of the trap will be conserved if we neglect its loss owing to laser cooling. The diminishing value of potential barrier U_- leads to decreasing of the number of trapped atoms and trapping volume and increasing of the number of atoms out of the potential. For any ω and h the corresponding one-dimensional "volumes" are equal $-h/2 < Z < h_g < h$ and $h_g < Z < h/2$. The numerical values on Fig. 1 for the atom number ~ 50

$$h = 4mm, \ \Delta \sim 1mm, \ U_0 \sim mgh \sim 10^{-8}eV, \ U_g \sim 10^{-9}eV, \ \hbar\omega \sim 10^{-13}eV$$

correspond approximately to the experimental results on the whole [3,4],but takes the difference from latter ones with a purpose of reconciliations the upper and lower part of the trap in scale of presented figure. It should be kept in mind that the parabolic trap approximation is rough near the walls of the trap.

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It follows from our estimations that the condition of macroscopic stability of the trap in gravity field $g < \omega^2 h$ is valid for the experiments [3,4] (and analogous) and this inequality will be satisfied for the frequencies more than 100 Hz. For all that the quantum dynamics of atoms undergoes nonperturbate disturbance because of the great shift of the trap potential ($U_g \gg \hbar\omega$) in comparison with its frequency. Note, that this disturbance appears in view of a finite size of the trap (mesoscopic phenomena). It wouldn't lead to any observing phenomena if the potential of the trap was defined on the whole axis $-\infty < Z < \infty$.

The critical temperature T_c is connected with the condensate density $\rho(T)$. To calculate the density we use the Hartree–Fock–Bogoliubov's method has been developed in [5] for the path integral. In according to [5] the condensate density and quasiclassical chemical potential ν are defined from the variational equations

$$\delta S_{ef}(\rho,\nu) = 0 \tag{2}$$

for the effective action of condensate. These equations are obtained after the integration over the trajectories μ of noncondensate bosons in formula for the partition function Q

$$Q = Sp\,\exp(-\beta H) = \int d\rho d\nu \int D\mu \exp S(0,\beta) = \int d\rho d\nu\ \exp S_{ef}(0,\beta).$$

The equation (2) describes a semiclassical dynamics of atoms due to the condition $\hbar\omega\beta \ll 1$, that is valid for the critical BEC temperature $\sim 10^{-6}K$ in [2,3]. The effective action S_{ef} is evaluated for Bogoliubov's Hamiltonian generalized in [6,7] for the case of the system with broken translation symmetry (1) in volume V

$$S_{ef}(\rho,\nu) = -\beta\rho^2\gamma_0 V + \beta\nu(\rho - R)V - \beta\rho w_{00} +$$

$$\frac{1}{2}\sum_{nn'\neq 0}\left[\beta(W_{nn'} - \nu\delta_{nn'}) - 4 ln\, sh\frac{\beta E_{nn'}}{4} + \beta A_{nn'}\right], \quad A_{nn'} = -\frac{V\phi_n^* M_{nn'} m \phi_{n'}}{E_{nn'}^2},$$

$$\rho = |b_0|^2, \quad m = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M = \begin{pmatrix} W_{nn'} - \nu\delta_{nn'} & 2\gamma_{nn'}\rho \\ -2\gamma_{nn'}\rho & -W_{nn'} + \nu\delta_{nn'} \end{pmatrix},$$

$$W_{nn'} = w_{nn'}/V + 2\rho\gamma_{nn'}, \quad E_{nn'}^2 = (M|_{11})^2 - (M|_{12})^2,$$

$$\phi_n^* = (b_0^*, b_0)(\rho\gamma_{0n} + w_{0n}/V).$$

One–particle matrix elements

$$w_{nn'} = \frac{1}{2m}\int \nabla u_n \nabla u_{n'} dr + \int U(r) u_n u_{n'} dr$$

for the potential (1) satisfy the condition $w_{nn'} = \delta_{nn'} w_{nn}$. Matrix elements of pair interaction $G(r,r')$ between the atoms in δ–approximation $G(r,r') \to G\cdot\delta(r - r')$

$$\gamma_0 = \frac{G}{2}\int dr u_0^4, \quad \gamma_{0n} = \frac{G}{2}\int dr u_0^3 u_n, \quad \gamma_{nn'} = \frac{G}{2}\int dr\, u_0^2 u_n u_{n'}$$

are calculated in basis u_n ($n = n_x, n_y, n_z$) for the functions of harmonic oscillator in x, y direction with the potential barrier U_0 at the endpoints of segment (-h/2,h/2) and in Z direction with the potential barrier $(U_- - U_g) < U_0$ at the left endpoint of the trap. Equations (2) for the chemical potential ν and condensate density $\rho(T)$ for the trap with the constant density of atoms $R = N/V$ may be evaluated approximately by dividing the sum over full number of trap levels into two terms with $n \ll n_0$ and $n \gg n_0$. Parameter $n_0 \simeq 100$ is defined under the condition

$$n_0\hbar\omega \simeq \rho\gamma_{n_0n_0} \simeq GN_0, \quad N_0 \sim N,$$

following from the inequality

$$10^{-13}eV \simeq \hbar\omega \ll GN_0 \simeq 10^{-11}eV, \quad G = \frac{4\pi\hbar^2 a}{m},$$

where the energy of interaction between atoms is defined from the estimation [3] for the scattering length $a = 4.9nm$ with density of atoms $R \sim 10^{13}cm^{-3}$, N_0 - is the number of condensed particles.

Full number of the trap levels $n_{max} = U_0/\hbar\omega \sim 10^4$. Thus, the density of condensate is written in the form $\rho = \rho^< + \rho^>$, where $\rho^< = \rho|_{n\ll n_0}$, $\rho^> = \rho|_{n\gg n_0}$. Then, taking into account the diagonal form of w_{nn}, we get the corresponding solutions of (2) in the form

$$\rho^< = R + \left[\frac{w_{00}}{V} - \sum_{n\neq 0}^{n_0}\left(T\frac{\gamma_{nn}}{2w_{nn}} + D_{nn}^<\right)\right]\left(2\gamma_{00} + \sum_{n,n'\neq 0}^{n_0} 2\gamma_{nn'}\right)^{-1},$$

$$D_{nn}^< = \frac{3{\gamma_{0n}}^2 w_{nn}}{2V\gamma_{nn}^2} \tag{3}$$

$$\rho^> = R\left(\gamma_{00} + \sum_{n,n'=n_0}^{n_{max}}\gamma_{nn'}\right)\left[\sum_{n=n_0}^{n_{max}}(\gamma_{nn} + TD_{nn}^>)\right]^{-1},$$

$$D_{nn}^> = \frac{4V\gamma_{nn}^2}{(w_{nn} - w_{00})^2} \tag{4}$$

for the chemical potential $\nu^< = 2\rho\gamma_{00}$ and $\nu^> = w_{00}(V)^{-1}$ correspondingly. The solution (3) is valid for repulsive interaction $G > 0$ between atoms, as the solution (4) is valid for any sign of G. The densities of condensate (3,4) decrease as far as the temperature increases. Note, that the density $\rho^<$ is turn to 0 at finite temperature, while $\rho^>$ decreases uniformly to zero with unrestricted increasing of the temperature. The latter kind of the density behavior occurs for the metastable condition of condensate in the trap, which has been considered theoretically in [8].

The solution (3,4) of equations (2) will be estimated below for the temperature interval $0,1T_c < T < 0,6T_c$, where the upper boundary is the temperature limit for application of HFB's method [9] and under this condition we may neglect the term $D^>_{nn}$, while under the lower boundary of this condition we may neglect the terms $w_{00}(V)^{-1}$ and D_{nn} in formulae (3,4). Solutions of the equations (2) for the ideal Bose–gas have the form

$$\nu = w_{00}(V)^{-1}, \quad \rho + \frac{1}{V}\sum_{n\neq 0}^{n_{max}} \frac{1}{\exp\left[\beta(w_{nn} - w_{00})(V)^{-1}\right] - 1} = R. \qquad (5)$$

The latter formula specifies the approximation for $\rho^>$ in absence of interaction and gives the trivial result for the system with translation invariance ($w_{00} = 0$).

The broken translation symmetry and the gravity field manifest themselves both in diminishing of a potential barrier $U_0 \to (U_- - U_g) < U_0$ and in the appearance of matrix elements w_{00}, $\gamma_{n\neq n'}$ in (3,4). We neglect the contribution of w_{00} and D_{nn} at given temperature interval, but the cut off the upper sum limit in (3,4)are taken into consideration.

Due to the degeneracy of states of the three-dimensional oscillator the matrix elements of boson interaction $\gamma_{nn'}$ are the sums of the matrix elements [1]

$$\int_{-\infty}^{\infty} e^{-2Z^2} H_n H_{n'} dZ = (-1)^{(n-n')/2} 2^{(n+n'-1)/2} \Gamma\left(\frac{n+n'+1}{2}\right). \qquad (6)$$

Here H – Hermite polinoms; the use of the whole axes Z instead of the segment $(-h/2, h/2)$ in (6) differs the calculation in the order of tunneling effects contribution that are negligible: the out-of-trap amplitude decreases as $\exp(-\alpha Z)$, $\alpha = 10^3$ for the most shallow trap with potential barrier $U_0 = \hbar\omega$.

[1] The power of the first term in a right hand side of (6) on the page 503 of [10], shown as $(n+n')/2$, is incorrect. The correct formula (6) follows from the more general formula page 502 of the same book.

Peculiarity of summation over n and n, n' in (3,4)

$$\rho^< \sim R - TS^<, \quad S^< = \sum_{n\neq 0}^{n_0} \frac{\gamma_{nn}}{w_{nn}} \left(\sum_{n,n'\neq 0}^{n_0} \gamma_{nn'}\right)^{-1},$$

$$\rho^> \sim R \sum_{n,n'=n_0}^{n_{max}} \gamma_{nn'} \cdot \left(\sum_{n=n_0}^{n_{max}} \gamma_{nn}\right)^{-1}$$

consist in the fact that the sum of diagonal (positive) matrix elements γ_{nn} increases as increasing of sum limit, while the sum of the off-diagonal elements $\gamma_{nn'}$ with $\pm$ signs changes slowly. Therefore, the densities $\rho^<$ $\rho^>$ increase when the level number diminishes, so that $\rho^< \gg \rho^>$. We can estimate the ratio Y of Bose-condensate densities in a trap with $\rho|_g$ and without gravity $\rho|_0$ for the given temperature interval .

$$Y = \frac{\rho|_g}{\rho|_0} = \frac{(\rho^< + \rho^>)|_g}{(\rho^< + \rho^>)|_0} \simeq \frac{\rho^<|_g}{\rho^<|_0}\left(1 - \frac{\rho^>|_0}{\rho^<|_0} + \frac{\rho^>|_g}{\rho^<|_g}\right), \quad \rho^<|_{0,g} \gg \rho^>|_{0,g}.$$

In the case of a "strong" gravity field $(U_- - U_g) \ll U_0$ the inequality $\rho^<|_g > \rho^<|_0$ follows after the evaluation of ratios $S^<|_g(S^<|_0)^{-1}$ for the three numbers of the upper limit of a sums 75, 50, 25 ($g \neq 0$) and for a limit $n_0 = 100$ ($g = 0$)

$$\frac{S^<_{75}}{S^<_{100}} \simeq 1,03, \quad \frac{S^<_{50}}{S^<_{100}} \simeq 1,08, \quad \frac{S^<_{25}}{S^<_{100}} \simeq 1,11.$$

Then, as $\rho^>|_g = 0$ and $\rho^>|_0(\rho^<|_0)^{-1} \sim 10^{-3}$, sufficiently small ω and h leads to

$$Y = \frac{\rho^<|g}{\rho^<|_0}\left(1 - \frac{\rho^>|_0}{\rho^<|_0}\right) > 1.$$

In the case of a "weak" gravity field $(U_- - U_g) \sim U_0$ the equations $\rho^<|_g = \rho^<|_0$ and $\rho^>|_g > \rho^>|_0$ are valid, so that

$$Y = \left(1 - \frac{\rho^>|_0 - \rho^>|_g}{\rho^<|_0}\right) > 1.$$

The three-dimensional structure of a trap - the contribution of the overlap integrals over the x and y axes into $\gamma_{nn'}$ - changes the above estimates on $2\% - 3\%$. One can predict the most accuracy of the given estimates for the "weak" gravity (compared with "strong" one), as the defect via the deviation of atoms from Bose-statistics is inversely proportional to a cut-off number of levels.

Taking into account, that the large Bose-condensate density corresponds the large critical temperature, we note, that the gravity field increases the critical temperature T_c of Bose-condensation in a trap. It means, that if the experiments [3,4] were done in Space and on the Earth, the smaller temperature T_c would be found in the first case. The obvious interpretation of this effect is as follows: for the fixed atom and condensate densities the decreasing of an atom level number increases their number at each of the rest levels and therefore the temperature of the system. The value of potential barrier turns to zero at the point $Z = -h/2$ under condition $g = \omega^2 h/4$ (destruction of the trap). The described phenomena is

a) mesoscopic phenomena because of its dependence of the geometry of the system

b) nonperturbative phenomena, as the parameters of perturbation theory by gravity field

$$\frac{mg}{\hbar\omega}\left(\frac{\hbar}{m\omega}\right)^{1/2} \sim 100, \quad \frac{mg}{GR}\left(\frac{\hbar}{m\omega}\right)^{1/2} \sim 1.$$

are so big that its presence leads to the radical reconstruction of the system (3,4).

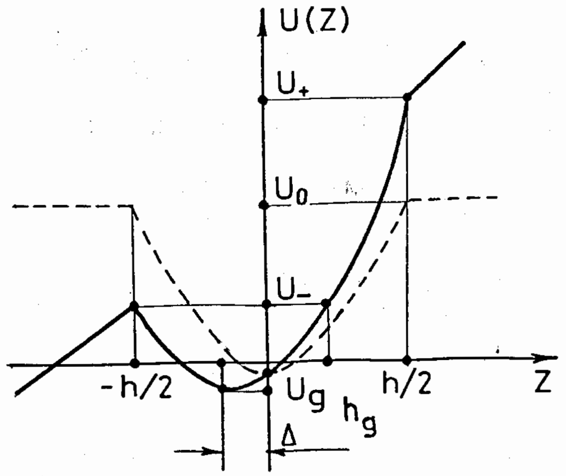

Fig.1 The dashed and solid lines for a trap potential in a free Space (U_0) and in a gravity field (U_-, U_+)

Note, that the shift of the critical temperature T_c caused by the gravity field is opposite to one has been found experimentally [11] (and considered theoretically [12]) in the form of ratio

$$\frac{T_c(^4He\, in\, porous\, glasses)}{T_c|_0(^4He\, in\, volume)} \simeq 0,998.$$

As the influence of the gravity field on the critical temperature of superfluid Helium is concerned, we should say that the essential effect may be expected for Helium in porous media, where diameter of pores should be chosen in the scale of gravitational perturbation.

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