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MAGNETOELASTIC INTERACTION IN THE MODEL OF HEISENBERG MAGNET

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INTRODUCTION

Investigation of nonlinear properties of magnet crystals attracts a great attention in the past decades [1 – 10]. Mainly, this interest has been initiated both by rapid development of the theory of nonlinear differential equations (such branches as inverse scattering method, algebraic and geometrical methods of integrating, numerical experiments and so on), new experimental data, and the possibility of their wide application in the different branches of applied science and technology.

It should be mentioned that the most popular model used in the investigations of nonlinear particle – like excitations in magnets is the model introduced by Landau and Lifshitz [9]. The Landau – Lifshitz equation (LLE) could be written, in particular isotropic case, in the following form

$$i\hbar S_t = \frac{1}{2}\left[S, S_{xx}\right], \tag{1}$$

where we introduce

$$S = \begin{pmatrix} S^z & S^- \\ S^+ & -S^z \end{pmatrix} = \overrightarrow{S} \cdot \overrightarrow{\sigma}, \tag{2}$$

here $\overrightarrow{S} = (S^x, S^y, S^z)$ is the vector of magnetization $S^\pm = S^x \pm iS^y$ are the corresponding value of the projection of the classical spin vector, or exactly, of the magnetization vector (i.e. we allow that $\overrightarrow{S} \equiv \overrightarrow{M}$),

$$\widehat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \widehat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \widehat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

are Pauli operators, and the brackets [...] ({...}) correspond to (anti)commutator.

This equation (1) give us the macroscopic description of magnetization dynamics in ferromagnets and represents the equation of motion of the magnetization vector in non — dissipative media.

On the other hand, it is well known, that in the microscopic level the most popular models for description of magnetic properties of a crystal are the spin models of Heisenberg — Frenkel magnet

$$\widehat{H} = -\sum_{i,k} J_{ik}\, \widehat{\overrightarrow{S}}_i \widehat{\overrightarrow{S}}_k, \tag{3}$$

where J_{ik} are exchange integrals, $\widehat{\overrightarrow{S}}_j$ are the spin operators of the atom in the j- th site.

The problem of the correspondence between classical and quantum concepts and the transition procedure from quantum — mechanical description of ferromagnet (3) to the description on the classical, macroscopic level (1) is not trivial. This problems has been discussed by many authors (see for example, [1] and the papers cited there). At the same time, the presence in the spin Hamiltonian (3) of such physical parameters as exchange integrals, constants of anisotropy, values of atom spin and so on, makes more favorable to investigate magnets starting from quantum — mechanical Hamiltonian (3). The present study is devoted to investigation of the particle — like (localized) excitation in magnets describing by the soliton solutions of nonlinear differential equations. Namely the presence of particle — like excitations in spin systems can explain a number of peculiarities of the slow neutron scattering on magnets, dynamic structure form factors and so on [10 — 12] in the presence of interaction of spin subsystem with phonon one.

The derivation of the equation of motion of the magnetization vector (or, "the vector of classical spin"), starting from microscopical description is not trivial. Concerning the formal procedure of transition from spin Hamiltonian (3) to the quasiclassical description it should be noted, that this procedure could be based, for example, on the bozonization of spin Hamiltonian (by use of Holstein — Primakoff transformation [13, 14] or other [13]) and after that the obtained bozonized Hamiltonian should be averaged using the Glauber coherent states [4, 6]. As the result of carrying out of the above mentioned procedure we obtain a classical Hamiltonian of the model. As it was shown in the paper [6] this approach is permissible (and correct) for the magnets with sufficiently large values of spins ($S \gg 1$, or for $\delta S \gg 1$ [14], δ is the constant of anisotropy). It should be noted that in this approach the appearance of nonphysical degrees of freedom due to the truncation of the infinite expanding series could lead to uncontrollable mistake [6].

In the case of magnets with the spin value $S = 1/2$, and also for magnets with the spin value $S > 1/2$ in the presence of exchange anisotropy only (i.e. when the multipole spin dynamics is frozen), direct use of the generalized coherent states which is constructed on the operators of $SU(2)$ group becomes possible. In this case it is not necessary to carry out the bozonization procedure of the spin Hamiltonian, because both the Hamiltonian and the coherent state are constructed on the operators of the same group. Also it should be mentioned, that in the case of magnets with the spin value $S = 1/2$, the correct use of exact Wigner — Seitz transformation allow us to write the Hamiltonian (3) in terms of Fermi operators and then using the transition procedure we obtain generalized nonlinear Schroedinger equation for the probability amplitudes of the spin excitations [11].

Investigation of the magnets with the spin values $S \geq 1$ taking into consideration the single—ion and some other types of anisotropy in the spin Hamiltonian (3) is more complicated due to the exciting of the multipole spin dynamics. In this case number of quasiclassical parameters required for the full macroscopic description of magnet grows up to $4S$, and the procedure of derivation of the equation of classical spin and multipole dynamics should be based on the generalized coherent states constructed on operators of $SU(2S+1)$ group (see details in papers [6, 7, 15]).

In the present paper the spin — phonon interaction in quasi — one — dimensional magnet crystals with the spin value $S = 1/2$ is investigated in the scope of the SU(2) generalized coherent states technique. As we have mentioned above this

investigation is correct not only for the magnets with the spin value $S = 1/2$, but also for the magnets with the spin value $S > 1/2$ if we take into consideration exchange anisotropy only, neglecting single — ion — anisotropy, and for the case of magnets with spin value $S \gg 1$ [6 - 8].

QUANTUM AND CLASSICAL MODELS OF MAGNETS WITH MAGNETOELASTIC INTERACTION

Let us consider the model of the Heisenberg ferromagnet with single — axis anisotropy in the presence of oscillation of sites of the crystal lattice

$$\widehat{H} = \widehat{H}_s + H_p, \tag{4}$$

where

$$\widehat{H}_s = -\sum_{j=1}^{N}\left\{\frac{J^0}{2}\left(\widehat{S}_j^+\widehat{S}_{j+1}^- + \widehat{S}_j^-\widehat{S}_{j+1}^+\right) + J^z\widehat{S}_j^z\widehat{S}_{j+1}^z\right\} \equiv$$
$$\equiv -J^0\sum_{j=1}^{N}\left\{\widehat{\vec{S}}_j\widehat{\vec{S}}_{j+1} + \Delta\widehat{S}_j^z\widehat{S}_{j+1}^z\right\}, \tag{5}$$

is the spin part of the Hamiltonian,

$$H_p = \sum_{j=1}^{N}\left\{\frac{p_j^2}{2m} + \frac{k}{2}\left(y_{j+1} - y_j\right)^2\right\} \tag{6}$$

is the phonon part of the Hamiltonian. Here $\Delta = \left(J^z - J^0\right)/J^0$ is the constant of exchange anisotropy, m and p are the momentum and mass of the atom, correspondingly, $|y_{j+1} - y_j|$ is the displacement of the j- th atom from the equilibrium position, k is the elastic constant, j is the summation index. In the expression (6) we take into consideration the harmonic oscillations of the crystal lattice only.

Let us pass over to the classical description. In order to make this we average $\widehat{H}_s$ by use of the $SU(2)$ generalized coherent states (GCS) [4, 5]. Let us remind that $SU(2)$ GCS in complex parameterization has the form

$$|\xi\rangle = \prod_j |\xi_j\rangle = \prod_j \left(1 + |\xi_j|^2\right)^{-k} \exp\left\{\xi_j\widehat{S}_j^+\right\}|k, -k\rangle, \tag{7}$$

here k is the number of representation, ξ_j is the parameter of quasiclassical description. Spin operators averaged by use of the $SU(2)$ GCS get the following form

$$S_j^+ = \overline{S_j^-} = \left\langle \widehat{S}_j^+ \right\rangle = \frac{2\xi_j}{1 + |\xi_j|^2}, S_j^z = \left\langle \widehat{S}_j^z \right\rangle = \frac{1 - |\xi_j|^2}{1 + |\xi_j|^2}. \tag{8}$$

Note, that the parameterization via more habitual angle variables is possible. In this case the values of the averaged spin operators have the form

$$\overrightarrow{S} = s(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta). \tag{9}$$

The relationship between the complex parameters ξ and the angle one θ, φ is given by stereographycal projection

$$\xi_j = \tan\left(\frac{\theta}{2}\right)\exp\{i\varphi_j\}, \tag{10}$$

where the values of angle parameters are restricted as $0 \le \theta \le \pi$ and $0 \le \varphi < 2\pi$. By use of the vector (7) we carry out the averaging procedure of the spin Hamiltonian $\widehat{H}_s$. We have .

$$H_s = \left\langle \xi \left| \widehat{H_s} \right| \xi \right\rangle =$$

$$= -s^2 \sum_{j=1}^{N} \left\{ \frac{2J^0\left(\xi_j\overline{\xi_{j+1}} + \overline{\xi_j}\xi_{j+1}\right) + J^z\left(1-|\xi_j|^2\right)\left(1-|\xi_{j+1}|^2\right)}{\left(1+|\xi_j|^2\right)\left(1+|\xi_{j+1}|^2\right)} \right\},$$

and the Hamiltonian of the system takes the form

$$H = \left\langle \xi \left| \widehat{H} \right| \xi \right\rangle = H_s + H_p. \tag{11}$$

In order to obtain continual limit of the Hamiltonian (11) we assume

a). In the expansion of exchange integral J^z we take into account the linear terms only (we assume, that isotropic exchange integral do not depend on the lattice deformation, i.e. $J^0 = const$)

$$J^z = J^3 + \overline{J^3}\,|y_{j+1} - y_j|, \tag{12}$$

b). In the expansion of ξ_{j+1} we take into account terms no more than quadratic in a_0

$$\xi_{j+1} = \xi_j + a_0\xi_{jx} + \frac{a_0^2}{2}\xi_{jxx} + \dots$$

$$y_{j+1} = y_j + a_0 y_{jx} + \dots$$

Then rewriting Hamiltonian (11) in spherical variables we obtain

$$H = -s^2 \sum_{j=1}^{N} \left\{ \frac{J^0}{2}\left(S_j^+S_{j+1}^- + S_j^-S_{j+1}^+\right) + J^z S_j^z S_{j+1}^z \right\} + H_p. \tag{13}$$

This Hamiltonian in the continual limit takes the form

$$H = H_s + H_p + H_{sp}, \tag{14}$$

where

$$H_s = s^2 J^0 \frac{a_0^2}{2} \int \frac{dx}{a_0}\left[\frac{1}{2}\left((S_x^x)^2 + (S_x^y)^2 + (S_x^z)^2\right) - \delta\,(S^z)^2\right], \tag{14.a}$$

$$H_{sp} = -s^2 J^0 \chi \int \frac{dx}{a_0} y_x\,(S^z)^2, \tag{14.b}$$

$$H_p = \int \frac{dx}{a_0}\left(\frac{p^2}{2m} + \frac{ka_0^2}{2}y_x^2\right), \tag{14.c}$$

here $\delta = 2\Delta/a_0^2$ is the constant of single-axis anisotropy,

$$J^z = J^0 + J^{z'} + \overline{J^3}\,|y_{j+1} - y_j| = J^3 + \overline{J^3}\,|y_{j+1} - y_j| = J^3 + \chi J^0 \frac{|y_{j+1} - y_j|}{a_0},$$

and $\chi = \overline{J^3}a_0/J_0\,(\gamma = 2\chi/a_0^2)$is the dimensionless constant of spin-phonon interaction. Note that in the expression (14) we neglect the constant terms

$$-J^0 s^2 N + \int \frac{dx}{a_0}\left[-ay_x\overline{J^3}\left(a_0 S^z S_x^z + \frac{a_0^2}{2}S^z S_{xx}^z\right)\right] = const.$$

Introducing Poison brackets in the following form (see [5])

$$\{A, B\} = \int \left\{ -\varepsilon_{ijk}\frac{\delta A}{\delta S^i}\frac{\delta B}{\delta S^j}S^k - \frac{\delta A}{\delta p}\frac{\delta B}{\delta y} + \frac{\delta A}{\delta y}\frac{\delta B}{\delta p} \right\} dx, \tag{15}$$

we derive the equation of dynamics of magnetization vector coupled with the lattice (chain) oscillations

$$\hbar S_t^l = \{H, S^l\} = -\varepsilon_{ilk}\frac{\delta H}{\delta S^i}S_k,$$

$$y_t = -\frac{\delta H}{\delta p} = -\frac{p}{m},$$

$$p_t = \frac{\delta H}{\delta y} = ka_0^2 y_{xx} - \chi s^2 J^0\left[(S^z)^2\right]_{xx}$$

or, rewriting this system of equations in the matrix form we obtain

$$i\hbar S_t + s^2 J^0 \frac{a_0^2}{2}[S, S_{xx}] + s^2 J^0 \frac{(\Delta + \chi u)}{2}[S, \widehat{\sigma}^z]\{S, \widehat{\sigma}_z\} = 0, \tag{16.a}$$

$$u_{tt} - \frac{ka_0^2}{m}u_{xx} + \chi\frac{s^2 J^0}{m}\left[(S^z)^2\right]_{xx} = 0, \tag{16.b}$$

here we put $u = y_x$,

$$S = \begin{pmatrix} S^z & S^- \\ S^+ & -S^z \end{pmatrix},$$

and brackets $[...,...]$ ($\{...,...\}$) correspond to (anti)commutator.

After simple scale transformation

$$x' = bx, t' = at$$

where

$$b^2 = \frac{k\hbar^2}{ma_0^2 (s^2 J^0)^2}, a = \frac{k\hbar}{ms^2 J^0},$$

we obtain the following system of equations

$$iS_t + \frac{1}{2}[S, S_{xx}] + G\frac{(\Delta + \chi u)}{2}[S, \hat{\sigma}_z]\{S, \hat{\sigma}_z\} = 0, \qquad (17.a)$$

$$u_{tt} - u_{xx} + \lambda \left[(S^z)^2\right]_{xx} = 0, \qquad (17.b)$$

where

$$G = \left(\frac{s^2 J^0}{\omega_0 \hbar}\right)^2,$$

$$\omega_0 = \sqrt{\frac{k}{m}},$$

$$\lambda = \frac{s^2 J^0}{k a_0^2}.$$

MODELS WITH PHONON UNHARMONICITY

Let us now take into consideration terms of higher order in the expansion of potential energy of interaction between the atoms of the crystal lattice (chain). The energy of lattice oscillations takes the form

$$H_p = \sum_{j=1}^{N} \frac{p_j^2}{2m} + U, \qquad (18.a)$$

where

$$U = \sum_{j=1}^{N} \varphi(y_{j+1} - y_j). \qquad (18.a)$$

After making the corresponding transition procedure as above, we obtain the system of coupled Landau — Lifshitz and Boussinesq equations describing nonlinear spin excitations accompanied with the nonlinear sound mode propagating in ferromagnet

$$i\hbar S_t + s^2 J^0 \frac{a_0^2}{2}[S, S_{xx}] + s^2 J^0 \frac{(\Delta + \chi u)}{2}[S, \hat{\sigma}^z]\{S, \hat{\sigma}_z\} = 0,$$

$$u_{tt} - v_s^2 u_{xx} - \Lambda \left(u^2\right)_{xx} - a_0^2 B u_{xxxx} + \chi \frac{s^2 J^0}{m}\left[(S^z)^2\right]_{xx} = 0,$$

where

$$\Lambda = \frac{1}{2}\varphi'''(0)\, a_0^3,$$

$$B = \frac{v_s^2}{2},$$

$$v_s^2 = \frac{\varphi''(0)\, a_0^2}{m} = \frac{k a_0^2}{m}$$

is the sound velocity, k is the constant of elasticity. This system of equations after scale transformation can be written as

$$iS_t + \frac{1}{2}[S, S_{xx}] + G\frac{(\Delta + \chi u)}{2}[S, \hat{\sigma}_z]\{S, \hat{\sigma}_z\} = 0, \qquad (19.a)$$

$$u_{tt} - u_{xx} - \alpha \left(u^2\right)_{xx} - \beta u_{xxxx} + \chi\lambda \left[(S^z)^2\right]_{xx} \qquad (19.b)$$

where

$$\alpha = \frac{m}{k a_0^2}\Lambda, \beta = \frac{\hbar^2 B}{a_0^3 (s^2 J^0)^2}.$$

Let us consider now multisublattice model consisting of p sublattices of ferromagnet type and q sublattices of antiferromagnet type ($M = p + q$). Interaction between the sublattices is defined by the phonon subsystem. This model can be written in the form

$$\hat{H} = \sum_{j=1}^{M} \hat{H}_{sj} + H_p, \qquad (20)$$

where

$$\hat{H}_{sj} = -\sum_{i=1}^{N}\left\{J_i^0\left(\hat{S}_{ji}^x \hat{S}_{ji+1}^x + \hat{S}_j^y \hat{S}_{ji+1}^y\right) + J_i^z \hat{S}_{ji}^z \hat{S}_{ji+1}^z\right\}$$

and H_p is defined by the formulae (18). Carrying out the procedure as above, we get in long wave limit approximation the following system of equations of spin — phonon interaction

$$iS_{jt} + \frac{1}{2}[S_j, S_{jxx}] + G_j \frac{(\Delta_j + \chi_j u)}{2}[S_j, \hat{\sigma}_z]\{S_j, \hat{\sigma}_z\} = 0, \qquad (21.a)$$

$$u_{tt} - u_{xx} - \alpha \left(u^2\right)_{xx} - \beta u_{xxxx} + \left(\sum_{k=1}^{p} \chi_k \lambda_k (S^z)^2 + \sum_{k=p+1}^{M} \chi'_k \lambda_k (S^z)^2\right)_{xx} = 0, \qquad (21.b)$$

here

$$J_k^z = J_k^3 + \chi_k' J^0 \frac{|y_{j+1} - y_j|}{a_0}$$

and χ_k' is the dimensionless constant of interaction of the antiferromagnet subsystem with the phonon one.

Now let us take into consideration exchange interaction between the ferromagnet and the antiferromagnet sublattices

$$\widehat{H}_{int} = -\sum_{k,j=1}^{M}\sum_{i=1}^{N} I_{kj}\widehat{S}_{ji}^z\widehat{S}_{ki}^z, \tag{22}$$

where I_{kj} is the integral of exchange between k- th and j- th sublattices and the Hamiltonian of the model is

$$\widehat{H} = \sum_{j=1}^{M} \widehat{H}_{sj} + \widehat{H}_{int} + H_p$$

where $\widehat{H}_{sj}$ is defined by eq.(20) and H_p is determined by eq.(18). In this case the system of coupled Landau — Lifshitz and Boussinesq equations takes the form

$$iS_{jt} + \frac{1}{2}[S_j, S_{jxx}] +$$

$$+G_j[S_j,\widehat{\sigma}_z]\left(\chi_j u\{S_j,\widehat{\sigma}_z\} + \sum_{k=1}^{p}\Delta_{kj}\{S_k,\widehat{\sigma}_z\} + \sum_{k=p+1}^{M}\Delta_{kj}'\{S_k,\widehat{\sigma}_z\}\right) = 0, \tag{23.a}$$

$$u_{tt} - u_{xx} - \alpha\left(u^2\right)_{xx} - \beta u_{xxxx} + \left(\sum_{k=1}^{p}\chi_k\lambda_k\left(S_k^z\right)^2 + \sum_{k=p+1}^{M}\chi_k'\lambda_k\left(S_k^z\right)^2\right)_{xx} = 0, \tag{23.b}$$

here

$$\Delta_{kj} = \frac{[I_{ik} + \delta_{ik}(I_{ii} + J^3 - J_i^0)]}{J^0},$$

and for the antiferromagnet sublattice the same expression for Δ_{kj}' has been introduced, and $S_j = \begin{pmatrix} S_j^z & S_j^- \\ S_j^+ & -S_j^z \end{pmatrix}$ is magnetization vector.

Below we discuss some limit cases of the system of equations (23).

1). Quasistationary limit $v \ll 1, u_{tt} \ll u_{xx}$.

In order to simplify our consideration we put $\alpha = \beta = 0$, then the eq.(23.b) give us

$$u(x,t) = \sum_{k=1}^{p}\lambda_k\left[(S_k^z)^2 - 1\right] + \sum_{k=p+1}^{M}\chi_k'\lambda_k\left[(S_k^z)^2 - 1\right]. \tag{24}$$

Here the constants of integration defined by the boundary conditions

$$\left(S_k^z(\pm\infty,t)\right)^2 = 1, u(\pm\infty,t) = 0.$$

Substituting eq.(24) to eq.(23.a) we get

$$S_{jt} + \frac{1}{2}[S_j, S_{jxx}] +$$

$$+\chi_j G_j[S_j,\widehat{\sigma}_z]\left(\left(\sum_{k=1}^{p}\chi_k\lambda_k\left[(S_k^z)^2 - 1\right] + \sum_{k=p+1}^{M}\chi_k'\lambda_k\left[(S_k^z)^2 - 1\right]\right)\{S_j,\widehat{\sigma}_z\} +\right.$$

$$+G_j[S_j,\widehat{\sigma}_z]\left(\sum_{k=1}^{p}\Delta_{kj}\{S_k,\widehat{\sigma}_z\} + \sum_{k=p+1}^{M}\Delta_{kj}'\{S_k,\widehat{\sigma}_z\}\right) = 0. \tag{25}$$

In the simplest case $M = p = 1$, $q = 0$ from eq.(25) we have

$$iS_t + \frac{1}{2}[S, S_{xx}] + \frac{G}{2}\left[\chi^2\lambda\left[(S_k^z)^2 - 1\right] + \Delta\right][S,\widehat{\sigma}_z]\{S,\widehat{\sigma}_z\} = 0, \tag{26}$$

here $S \equiv S_1$.

Note, that the Hamiltonian of this equation is defined by the expression

$$H = s^2J^0\int\frac{dx}{a_0}\left[\frac{a_0^2}{2}\left(\overrightarrow{S_x}\right)^2 + \left[\chi^2\frac{s^2J^0}{ka_0^2} - \Delta\right](S^z)^2 - \chi^2\frac{s^2J^0}{ka_0^2}(S^z)^4\right]. \tag{27}$$

2). Nearsonic limit $v \sim 1$.

In this case using the standard procedure

$$\partial_t^2 - \partial_x^2 \simeq -2(\partial_t + \partial_x)\partial_x$$

we can replace this operator in the equation (23.b). Integrating this equation with vanishing boundary condition we obtain

$$u_t + u_x + \frac{\alpha}{2}\left(u^2\right)_x + \frac{\beta}{2}u_{xxxx} - \frac{1}{2}\left(\sum_{k=1}^{p}\chi_k\lambda_k\left(S_k^z\right)^2 + \sum_{k=p+1}^{M}\chi_k'\lambda_k\left(S_k^z\right)^2\right)_x = 0. \tag{28}$$

In the harmonic phonon approximation the constants $\alpha = \beta = 0$ and this equation can be written in the form

$$u_t + u_x - \frac{1}{2}\left(\sum_{k=1}^{p}\chi_k\lambda_k\left(S_k^z\right)^2 + \sum_{k=p+1}^{M}\chi_k'\lambda_k\left(S_k^z\right)^2\right)_x = 0. \tag{29}$$

3). Small amplitude approximation.

In this case we assume, that deviations of classical spin vector from the equilibrium position (classical vacua) is sufficiently small $|S^+|^2 \ll 1$. We introduce the function $\varphi_j(x,t) = S_j^+(x,t)$, so $|\varphi_j|^2 \ll 1$, then $S_j^z = 1 - \frac{1}{2}|\varphi_j|^2$. Substituting this relation to eq.(23) we derive

$$i\varphi_t - \varphi_{xx} + 2\chi G u\varphi + 2\Delta G\varphi - \Delta G(\overline{\varphi}\varphi)\varphi = 0 \tag{30.a}$$

$$u_{tt} - u_{xx} - \alpha(u^2)_{xx} - \beta u_{xxxx} + 2\chi\lambda(\overline{\varphi}\varphi)_{xx} = 0, \tag{30.b}$$

where

$$\varphi = (\varphi_1, ..., \varphi_M)^t, \Delta = (\Delta_1, ..., \Delta_p, \Delta'_{p+1} ... \Delta'_M),$$

$$\chi = (\chi_1, ..., \chi_p, \chi'_{p+1} ... \chi'_M),$$

$$\overline{\varphi}\varphi = \sum_{k=1}^{p} |\varphi_k|^2 + \sum_{k=p+1}^{M} |\varphi_k|^2,$$

$$\Delta_j = \sum_{k=1}^{p} \Delta_{kj} + \sum_{k=p+1}^{M} \Delta'_{kj}.$$

We have obtained the system (30) neglecting the following terms

$$\varphi_j |\varphi_j|^2_{xx} + \frac{1}{2}\varphi_{jxx}|\varphi_j|^2 + \chi u |\varphi_j|^2 \varphi_j$$

as a terms of higher order of nonlinearity.

In harmonic approximation $\alpha = \beta = 0$, and from eq.(30) we derive

$$i\varphi_t - \varphi_{xx} + 2\chi G u\varphi + 2\Delta G\varphi - \Delta G(\overline{\varphi}\varphi)\varphi = 0 \tag{31.a}$$

$$u_{tt} - u_{xx} + 2\chi\lambda(\overline{\varphi}\varphi)_{xx} = 0. \tag{31.b}$$

Note the most remarkable case, which is the nearsonic limit $v \sim 1$ that reduces eq.(31) to nonlinear Schroedinger equation with Yajima — Oikawa potential (see [16]) ($M = 1$)

$$i\varphi_t - \varphi_{xx} + 2\chi G u\varphi + 2\Delta G\varphi - \Delta G|\varphi|^2\varphi = 0, \tag{32.a}$$

$$u_t + u_x + \chi\lambda(|\varphi|^2)_{xx} = 0. \tag{32.b}$$

Here we take into account relation

$$\partial_t^2 - \partial_x^2 \simeq -2(\partial_t + \partial_x)\partial_x.$$

It should be mentioned, that the regular method of constructing of multisoliton solutions of eq.(23) is proposed in ref. [17].

DOMAIN-WALL AND SOLITON-LIKE SOLUTIONS

In order to derive and investigate soliton solutions of the obtained nonlinear quasiclassical models it is convenient to pass over the angle variables for the classical spin (magnetization) vector. Then the Hamiltonian averaged by use of $SU(2)$ GCS in the continual limit takes the following form

$$H = H_s + H_{sp} + H_p, \tag{33}$$

where

$$H_s = s^2 J_0 \int \frac{dx}{a_0}\left[\frac{a_0^2}{2}\left(\overrightarrow{S_x}\right)^2 - \Delta(S^z)^2\right] \tag{33.a}$$

is the spin part of Hamiltonian, (δ is the constant of exchange anisotropy),

$$H_p = \int \frac{dx}{a_0}\left(\frac{p^2}{2m} + \frac{a_0^2 k}{2} y_x^2\right) \tag{33.b}$$

is the phonon part of Hamiltonian, and

$$H_{sp} = -s^2 J_0 \chi \int \frac{dx}{a_0} y_x (S^z)^2 \tag{33.c}$$

is the Hamiltonian of spin — phonon interaction.

In terms of angle variables the magnetization vector (classical spin) is

$$\overrightarrow{S} = s(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta). \tag{34}$$

Let us remind that the Hamiltonian equation of motion obtained for $SU(2)$ GCS in ref. [5] by path integral method, have the following form

$$\hbar\varphi_t = -\frac{1}{\sin\theta}\frac{\delta H}{\delta\theta}, \tag{35.a}$$

$$\hbar\theta_t = \frac{1}{\sin\theta}\frac{\delta H}{\delta\varphi}, \tag{35.b}$$

and supplementing this equations by obvious equations

$$y_t = -\frac{\delta H}{\delta p} = -\frac{p}{m}, \tag{35.c}$$

$$p_t = \frac{\delta H}{\delta y} \tag{35.d}$$

we obtain the full set of classical equations of motion. Using the equations (35) and the Hamiltonian density

$$h = s^2 J_0 \left\{\frac{a_0^2}{2}(\theta_x^2 + \sin\theta\varphi_x^2) - [\Delta + \chi y_x]\cos^2\theta\right\} + \frac{p^2}{2m} + \frac{a_0^2 k}{2} y_x^2, \tag{36}$$

derived from eq.(33) we obtain the following system of equations of spin — phonon dynamics

$$a_0^2\theta_{xx} - \left[a_0^2\varphi_x^2 + 2\left[\Delta + \chi u\right]\right]\sin\theta\cos\theta + \frac{\hbar}{J_0 s^2}\sin\theta\varphi_t = 0, \tag{37.a}$$

$$a_0^2\left(\sin^2\theta\varphi_x\right)_x - \frac{\hbar}{J_0 s^2}\sin\theta\theta_t = 0, \tag{37.b}$$

$$u_{tt} - \frac{ka_0^2}{m}u_{xx} - \chi\frac{s^2 J_0}{m}\left(\cos^2\theta\right)_{xx} = 0. \tag{37.c}$$

In (37.c) we introduce the designation $u = y_x$. It should be mentioned that in order to obtain solution with physical sense of the system (37) we assume that the boundary conditions for $\theta(x,t)$ and $\varphi(x,t)$ are defined by the minima of the classical Hamiltonian (33) of easy — axis magnet ($\delta > 0$) (i.e. by the classical vacua of the system)

$$\theta = 0, \pi, x \to \pm\infty. \tag{38}$$

In order to simplify the system (37) it is convenient to use dimensionless variables

$$z = bx, \tau = at, \tag{39}$$

where

$$b^2 = \frac{k\hbar^2}{ma_0^2\left(s^2 J_0\right)^2}, a = \frac{k\hbar}{ms^2 J_0}. \tag{40}$$

Then we can rewrite eq. (37) in the form

$$\theta_{zz} - \left[\varphi_z^2 + 2\left(\frac{s^2 J_0}{\omega_0\hbar}\right)^2\{\Delta + \chi u\}\right]\sin\theta\cos\theta + \sin\theta\varphi_\tau = 0, \tag{41.a}$$

$$\left(\sin^2\theta\varphi_z\right)_z - \sin\theta\theta_\tau = 0, \tag{41.b}$$

$$u_{\tau\tau} - u_{zz} - \chi\frac{s^2 J_0}{ka_0^2}\left(\cos^2\theta\right)_{zz} = 0, \tag{41.c}$$

here

$$\omega_0 = \sqrt{\frac{k}{m}}.$$

We shall obtain the soliton solution of system (41) in the following form

$$\theta\left(z - v\tau\right), u = u\left(z - v\tau\right), \varphi = \psi\left(z - v\tau\right) + \omega\tau, \tag{42}$$

and with the boundary condition (easy — axis magnet)

$$\theta = \theta\left(z,\tau\right)|_{z(x)\to\pm\infty} = 0, \pi, \tag{43}$$

we derive

$$\theta_{\xi\xi} - \left[\varphi_\xi^2 + 2\left(\frac{s^2 J_0}{\omega_0\hbar}\right)^2\{\Delta + \chi u\}\right]\sin\theta\cos\theta + \{-v\psi_\xi + \Omega\}\sin\theta = 0, \tag{44.a}$$

$$\left(\sin^2\theta\varphi_\xi\right)_\xi + v\sin\theta\theta_\xi = 0, \tag{44.b}$$

$$\left(v^2 - 1\right)u_{\xi\xi} + \chi\frac{s^2 J_0}{ka_0^2}\left(\cos^2\theta\right)_{\xi\xi} = 0. \tag{44.c}$$

Note that in the case $\chi = 0$ the system (44) reduces to the system of equation (8.10), (8.16) (see [1]). Integrating the equation (44.c) taking into account boundary condition (43) and

$$u = 0, \xi \to \pm\infty, \tag{45}$$

we obtain

$$u = \chi\frac{s^2 J_0}{ka_0^2}\frac{\sin^2\theta}{v^2 - 1}, \tag{46}$$

or

$$u = \chi\frac{s^2 J_0}{ka_0^2}\frac{\sin^2\theta}{v^2 - v_s^2}. \tag{47}$$

Here v is the velocity of the magnetic soliton in the system with spin — phonon interaction. Sound velocity in our designation (40) is

$$v_s = a_0\sqrt{\frac{k}{m}} \equiv 1 \tag{48}$$

and

$$\omega_0 = \sqrt{\frac{k}{m}}. \tag{49}$$

Thus in the case $v = v_s$ solution (46) becomes singular. This singularity we define as magnet- acoustic resonance, which means that in the case of motion of magnetic soliton with nearsonic velocities the pumping of energy of magnetic soliton to the phonon subsystem takes place in the system.

We can integrate the second equation of the system (44)

$$\left(\sin^2\theta\psi_\xi - v\cos\theta\right)_\xi = 0 \tag{50}$$

if we take into account boundary conditions

$$\xi \to \pm\infty, \frac{d\psi}{d\xi} < 0, \theta = 0. \tag{51}$$

Then we derive the following relation

$$\psi_\xi = -\frac{v}{2}\frac{1}{\cos^2\theta/2}. \tag{52}$$

The boundary conditions (51) correspond to nonlinear excitation of bell — soliton types.

Substituting relationships (52) and (46) into (44.a) we obtain

$$\left(\frac{\theta}{2}\right)_{\xi\xi}+\sin\frac{\theta}{2}\left\{\frac{v^2}{4}\cos^{-3}\left(\frac{\theta}{2}\right)+\omega\cos\left(\frac{\theta}{2}\right)\right\}-$$

$$-\left[a_1\Delta-4\chi^2\frac{a_1a_2}{1-v^2}\sin^2\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)\right]\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\theta=0, \qquad (53)$$

here

$$a_1=2\left(\frac{s^2J_0}{\omega_0\hbar}\right)^2=2G, \qquad (54)$$

$$a_2=\frac{s^2J_0}{ka_0^2}=\lambda. \qquad (55)$$

First integrating of eq.(53) with boundary conditions (51) gives

$$\left(\frac{\theta}{2}\right)_{\xi}^2=-\frac{v^2}{4}\tan^2\frac{\theta}{2}+a_1\Delta\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}-\omega\sin^2\frac{\theta}{2}+2\chi^2A\cos^4\frac{\theta}{2}\sin^4\frac{\theta}{2}, \qquad (56)$$

where

$$A=\frac{a_1a_2}{1-v^2}. \qquad (57)$$

Integrating equation (56) we obtain the elliptic integral of the following form

$$\xi-\xi_0=\frac{1}{2}\int\frac{ydy}{(y-1)\sqrt{-\frac{v^2}{4}y^4-\omega y^3+a_1\Delta y^2+2\chi^2A\,(y-1)}}, \qquad (58)$$

where

$$y=\tan^2\frac{\theta}{2}+1. \qquad (59)$$

Integral in the eq.(58) can be expressed through the elliptic integral of Weierstrasse of I and III types and then the numerical calculations must be done. However we can consider some limit cases of solution of the system (41), which can be expressed by the elementary function.

a). Let us consider magnetic solitons moving with velocity v, $v^2\ll v_s^2,(v_s=1)$. Then $\chi^2A\to 0$ and assuming $\chi^2A=0$ (i.e. magnetic solitons do not feel the deformation of lattice), we have the solution

$$\tan^2\frac{\theta}{2}=\frac{\mu^2}{\Omega\cosh^2\mu\xi-[\Omega-\Omega_1]/2}, \qquad (60.a)$$

where $\xi=z-v\tau,\mu=\sqrt{a_1\Delta-\omega-\left(\frac{v}{2}\right)^2}$, and $\Omega=\sqrt{\Omega_1^2+4\gamma^2\,(v/2)^2}$ is the parameter, that in laboratory frame of reference performs the dimensionless frequency of precession of magnetic moment in nonlinear spin wave with the parameters v and ω, and $\Omega_1=\omega+(v/2)^2$ defines the dimensionless frequency of the precession of magnetic moment in the soliton with the same parameters v and ω in the laboratory frame of reference. Integrating eq.(52) and taking into account (60.a) and (42) we obtain

$$\varphi=\omega\tau-\frac{v}{2}\xi+\arctan\left[\sqrt{\frac{\Omega-\Omega_1}{\Omega+\Omega_1}}\tanh\gamma\xi\right]. \qquad (60.b)$$

The solution (60.a,b) taking into account our designation (39) completely coincides with the solution obtained in ref.[1]. Solution for the deformation wave accompanying magnetic soliton we get from eq.(46) using the solution (60.a- b)

$$u=4\chi\frac{s^2J_0}{ka_0^2}\frac{4\mu^2}{1-v^2}\frac{\Omega\cosh^2\mu\xi-[\Omega-\Omega_1]/2}{\{\Omega\cosh^2\mu\xi-[\Omega-\Omega_1]/2+\mu\}}. \qquad (60.c)$$

b). In the case of the magnet solitons moving with the velocity $v^2\gg v_s^2$ as in (57) we have $A\to 0$ and assuming $A=0$ we have solution (60).

Thus both the supersonic magnetic solitons and the solitons moving with the velocities less than velocity of sound do not feel the lattice deformation

c). Let us consider solutions of the system of equation (44) in the small amplitude spin deviations approximation. We rewrite the eq.(58) as

$$\xi-\xi_0=\frac{1}{2}\int\frac{(x+1)\,dx}{x\sqrt{-\frac{v^2}{4}(x+1)^4-\omega\,(x+1)^3+a_1\Delta\,(x+1)^2+2\chi^2Ax}}, \qquad (61)$$

where

$$x=\tan^2\frac{\theta}{2}$$

and assuming that the deviation of the classical spin from the equilibrium position (i. e. from the ground states of the classical model) is sufficiently small $\theta\ll 1$, we take into account in eq.(61) terms $O(x^2)$ and neglect the terms of higher order. Then the eq.(61) can be reduced to the following sum of integrals, which can be easily integrated

$$\xi-\xi_0=\frac{1}{2}\left[\int\frac{dx}{\sqrt{R}}+\int\frac{dx}{x\sqrt{R}}\right], \qquad (62)$$

where

$$R=a+bx+cx^2,$$

$$a=\mu^2=a_1\Delta-\omega-\frac{v^2}{4},$$

$$b = 2\chi^2 A + 2a_1\Delta - 3\omega - v^2,$$
$$c = a_1\Delta - 3\omega - \frac{3v^2}{2}.$$

Integrating the eq. (62) we obtain the solution in the following form

$$\xi - \xi_0 = \frac{2}{\sqrt{c}} \operatorname{arcsinh} \frac{2cx + b}{\sqrt{D}} - \frac{2}{\mu} \operatorname{arcsinh} \frac{2\mu^2 + bx}{x\sqrt{D}}, \tag{63}$$

where $D = \sqrt{4\mu^2 c - b^2}$.

Finally let us find the domain — wall type solutions of the system (44). We put

$$\varphi = \varphi_0 = const,$$

and find the solution of eq.(44.a) in the following form

$$\theta = \theta(\xi), \xi = z - v\tau \tag{64}$$

Substituting eq.(64) to the equation (44.a) we derive

$$\theta_{\xi\xi} - a_1 [\Delta + \chi u] \sin\theta \cos\theta = 0. \tag{65}$$

Integrating eq.(65) with the vanishing boundary conditions (we consider the easy — axis model) leads us to the following expression

$$\theta_\xi^2 - a_1\Delta \sin^2\theta - \chi^2 \frac{\Gamma}{2} \sin^4\theta = 0. \tag{66}$$

Solution of this equation can be expressed trough the integrals

$$2(\xi - \xi_0) = \int \frac{dx}{x\sqrt{R}} + \int \frac{dx}{\sqrt{R}}, \tag{67}$$

where

$$\Gamma = \frac{s^2 J_0}{k a_0^2} \frac{1}{v^2 - 1},$$
$$R = a + bx + cx^2,$$

$$a = c = a_1\Delta,$$
$$b = 2(a_1\Delta + \chi^2\Gamma),$$
$$x = \tan^2\frac{\theta}{2}.$$

One can easily integrate eq. (67) and get the solution in the following form

$$2\sqrt{a}(\xi - \xi_0) = \ln\left| x \frac{2\sqrt{aR} + 2a(x+1) + 2\chi^2\Gamma}{2\sqrt{aR} + 2a(x+1) + 2\chi^2\Gamma x} \right|. \tag{68}$$

It is obvious, that in the case $\chi^2 = 0$ the solution (68) takes the form of well — known domain wall

$$\theta = 2\arctan\exp\left\{\sqrt{a}(\xi - \xi_0)\right\}. \tag{69}$$

If we put $\chi^2 \ll 1$ expanding the solution (68) we obtain the domain wall type solution in the following form

$$\tan^2\frac{\theta}{2} \simeq \frac{1}{1 - \chi^2\Gamma} \exp\left\{2\sqrt{a}(\xi - \xi_0)\right\}. \tag{70}$$

CONCLUSION

Thus, in this paper we show, that magnetic solitons in deformable crystal lattice is accompanied by the deformation wave. In the case of motion of the magnetic soliton with the near — sonic velocities the effect of resonance due to the interaction of spin subsystem with the phonon one takes place in the system. We define this phenomenon as magnet — acoustic resonance, and here the energy pumping from the magnetic soliton to the deformation wave takes place. At the same time the linear approximation for the sound equation used to derive solutions (60) and (61) is doubtfully based. Neglected nonlinear terms could play a leading role in the case of near — sonic velocities. This fact can be amplified also by the requirement to take into consideration effects of dissipation of the energy of magnetic soliton. More realistic model should take into account also the presence of two additional transversal sound modes, and, consequently, the possibility of resonance by interaction of magnetic soliton with transversal waves.

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