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NORMAL FERMILY LIQUID BEHAVIOUR  
OF QUASIHOLE IN THE SPIN-POLARON MODEL  
FOR COPPER OXIDES

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Understanding of the quasiparticle (QP) characteristics of charge carriers forming the normal-state electronic properties of high- $T_c$  superconductors (HTSC's) is an issue of current interest. [1] In particular, it is under debate now whether these compounds can be described within the normal Fermi liquid (FL) approach, or a more exotic scenario, for instance, the marginal FL (MFL) concept, [2] should be involved.

In an attempt to understand the QP properties of HTSC's one has to take into account a strong difference between an intermediate and a low doping regime as it follows from the angle resolved photoemission (ARPES) experiments. [3] Actually, at the intermediate level of doping ARPES indicates a large Fermi surface (FS), while the reference insulator compounds [4] show a hole dispersion that is compatible with a small four-pocket shape of FS at low doping. In the latter regime, a hole propagation is strongly affected by the presence of antiferromagnetic (AFM) correlations in the spin subsystem. The essential features of this problem are described by the  $t - J$  model.

In the present paper, we investigate the regime of low doping based on the  $t - J$  model in the slave-fermion Schwinger boson representation. We are mainly interested in a QP hole behavior near FS. We show that at zero temperature the imaginary part of the hole self-energy  $\text{Im}\Sigma(\mathbf{k}, \omega) \propto \omega^2 \ln \omega$ , which indicates a conventional FL behavior of quasiholes. Our result is at variance with one reported in [5], where the MFL-behavior of quasiholes is obtained even at  $T = 0$ . The reason of this contradiction is discussed below.

By using the slave-fermion Schwinger boson factorization for electron operators, the  $t - J$  model with the Neel ground state can be mapped onto the so-called spin-polaron [6, 7, 8, 9] model with the Hamiltonian given by

$$H = \sum_{\mathbf{q}} \omega_{\mathbf{q}} \alpha_{\mathbf{q}}^{\dagger} \alpha_{\mathbf{q}} - \mu \sum_{\mathbf{k}} h_{\mathbf{k}}^{\dagger} h_{\mathbf{k}} + \frac{zt}{\sqrt{N}} \sum_{\mathbf{k}, \mathbf{q}} h_{\mathbf{k}}^{\dagger} h_{\mathbf{k}-\mathbf{q}} [M_1(\mathbf{k}, \mathbf{q}) \alpha_{\mathbf{q}} + M_2(\mathbf{k}, \mathbf{q}) \alpha_{-\mathbf{q}}^{\dagger}], \quad (1)$$

where  $\omega_{\mathbf{q}} = zJ/2\sqrt{1 - \gamma_{\mathbf{q}}^2}$  with  $\gamma_{\mathbf{k}} = \frac{1}{2}(\cos k_x + \cos k_y)$ ,  $\mu$  is the chemical potential of holes,  $M_1(\mathbf{k}, \mathbf{q}) = M_2(\mathbf{k} - \mathbf{q}, -\mathbf{q}) = (u_{\mathbf{q}}\gamma_{\mathbf{k}-\mathbf{q}} + v_{\mathbf{q}}\gamma_{\mathbf{k}})$ , and  $z = 4$  for the square lattice;  $N$  is the total number of sites, the lattice spacing is taken to be unity, and  $u_{\mathbf{q}}, v_{\mathbf{q}}$  are the usual parameters of the Bogoliubov  $u - v$  transformation. In the Hamiltonian (1)  $h_{\mathbf{k}}$  ( $h_{\mathbf{k}}^\dagger$ ) are canonical spinless fermion operators and  $\alpha_{\mathbf{q}}$  ( $\alpha_{\mathbf{q}}^\dagger$ ) are canonical boson operators.

We introduce the Fourier transformed two-time retarded Green function (GF)  $G(\mathbf{k}, \omega) = \ll h_{\mathbf{k}} | h_{\mathbf{k}}^\dagger \gg_\omega$  for holes and the matrix GF  $D(\mathbf{q}, \omega) = \ll A_{\mathbf{q}} | A_{\mathbf{q}}^\dagger \gg_\omega$  for magnons,  $A_{\mathbf{q}}$  is the two-component operator and  $A_{\mathbf{q}}^\dagger = (\alpha_{\mathbf{q}}^\dagger, \alpha_{-\mathbf{q}})$ . By applying the irreducible GF method [10] and using a decoupling procedure, which is equivalent to the self-consistent Born approximation (SCBA), both for  $G(\mathbf{k}, \omega)$  and  $D(\mathbf{q}, \omega)$ , we obtain

$$G^{-1}(\mathbf{k}, \omega) = \omega + \mu - \Sigma(\mathbf{k}, \omega), \quad (2)$$

$$D(\mathbf{q}, \omega) = \frac{1}{\mathcal{D}_{\mathbf{q}}(\omega)} \begin{pmatrix} \omega_{\mathbf{q}} + \omega + \Pi_{22}(\mathbf{q}, \omega) & -\Pi_{12}(\mathbf{q}, \omega) \\ -\Pi_{21}(\mathbf{q}, \omega) & \omega_{\mathbf{q}} - \omega + \Pi_{11}(\mathbf{q}, \omega) \end{pmatrix}, \quad (3)$$

where  $\mathcal{D}_{\mathbf{q}}(\omega) = [\omega - \Pi^-(\mathbf{q}, \omega)]^2 - [\omega_{\mathbf{q}} + \Pi^+(\mathbf{q}, \omega)]^2 + \Pi_{12}(\mathbf{q}, \omega)\Pi_{21}(\mathbf{q}, \omega)$  with  $\Pi^\pm(\mathbf{q}, \omega) = 1/2[\Pi_{11}(\mathbf{q}, \omega) \pm \Pi_{22}(\mathbf{q}, \omega)]$ . The elements of the polarization operator  $\Pi(\mathbf{q}, \omega)$  for the magnon GF has the form

$$\Pi_{\alpha\beta}(\mathbf{q}, \omega) = \frac{(zt)^2}{N} \sum_{\mathbf{k}} g_{\alpha\beta}(\mathbf{k}, \mathbf{q}) \iint_{-\infty}^{\infty} d\omega_1 d\omega_2 [n(\omega_2) - n(\omega_1)] \frac{\rho_{\mathbf{k}}(\omega_1)\rho_{\mathbf{k}-\mathbf{q}}(\omega_2)}{\omega - \omega_1 + \omega_2 + i\eta}, \quad (4)$$

where  $g_{\alpha\beta}(\mathbf{k}, \mathbf{q}) = M_\alpha(\mathbf{k}, \mathbf{q})M_\beta(\mathbf{k}, \mathbf{q})$  and  $\rho_{\mathbf{k}}(\omega) = -1/\pi \text{Im}G(\mathbf{k}, \omega)$  is the hole spectral function. The hole self-energy  $\Sigma(\mathbf{k}, \omega)$  is given by

$$\Sigma(\mathbf{k}, \omega) = \frac{(zt)^2}{N} \sum_{\mathbf{q}} \iint_{-\infty}^{+\infty} d\omega_1 d\omega_2 [N(\omega_2) + 1 - n(\omega_1)] \frac{\rho_{\mathbf{k}-\mathbf{q}}(\omega_1)\chi_{\mathbf{k}, \mathbf{q}}(\omega_2)}{\omega - \omega_1 - \omega_2 + i\eta}, \quad (5)$$

where  $n(\omega) = (e^{\beta\omega} + 1)^{-1}$  and  $N(\omega) = (e^{\beta\omega} - 1)^{-1}$ . In (5) we have introduced an effective spectral function  $\chi_{\mathbf{k}, \mathbf{q}}(\omega)$  as

$$\chi_{\mathbf{k}, \mathbf{q}}(\omega) = -\frac{1}{\pi} \sum_{\alpha\beta} g_{\alpha\beta}(\mathbf{k}, \mathbf{q}) \text{Im}D_{\alpha\beta}(\mathbf{q}, \omega), \quad (6)$$

for spin fluctuations coupled to a particular hole  $\mathbf{k}$ -state.

For the single-hole problem at zero temperature [7, 8] the self-energy (5) is reduced and only the contribution due to  $D_{11}(\mathbf{q}, \omega)$  remains. By considering the case of finite hole concentration  $\delta$ , the full form (5) is adopted here, in contrast to [5]. In a proper analysis, Eqs. (2)–(5) should be treated self-consistently, the problem which, to our knowledge, can be solved only numerically. Here, we are interested in a particular question of calculating the damping of QP hole states in an analytical way based on the well-established numerical results of SCBA.

Actually, spectral characteristics of a hole propagating in an AFM background at low level of doping have been investigated in many works [11]. Those results led to the consensus that the hole spectrum involves a narrow QP band of coherent states at low energies and a broad continuum of incoherent states. The corresponding spectral function is then represented as  $\rho_{\mathbf{k}}(\omega) = \rho_{\mathbf{k}}^{\text{coh}}(\omega) + \rho_{\mathbf{k}}^{\text{inc}}(\omega)$  with

$$\rho_{\mathbf{k}}^{\text{coh}}(\omega) = Z_{\mathbf{k}}\delta(\omega + \mu - E_{\mathbf{k}}). \quad (7)$$

The QP weight  $Z_{\mathbf{k}}$  and the bandwidth  $W$  are estimated to be  $Z_{\mathbf{k}} \simeq J/t \equiv Z$  and  $W \simeq 2J$ . The QP dispersion  $E_{\mathbf{k}}$  in the vicinity of its minima  $\mathbf{k}_i = (\pm\pi/2, \pm\pi/2)$  can be expanded as  $E_{\mathbf{k}_i + \mathbf{k}'} \simeq E_{\mathbf{k}_i} + k_{\parallel}^2/2m_{\parallel} + k_{\perp}^2/2m_{\perp}$ . [8, 9] Here  $k_{\parallel}$  and  $k_{\perp}$  are the component of  $\mathbf{k} - \mathbf{k}_i$  in the (1, -1) and (1, 1) directions in the Brillouin zone (BZ) for  $\mathbf{k}_i = (\pi/2, \pi/2)$ . For instance, the anisotropy factor  $a = m_{\parallel}/m_{\perp}$  is calculated to be  $a = 6$  for  $J = 0.3t$ . [8] This anisotropy can be absorbed by the following transformation  $\mathbf{k}' \rightarrow (ak'_{\parallel}, k'_{\perp})$ , which does not change our final results. Hence, we further consider the case  $m_{\parallel} = m_{\perp} = m(\sim J^{-1})$  [7, 8, 9]

From the above results one may expect that the filling of QP states leads to a four-pocket FS. Some arguments have also been given in [9, 12], that the fraction of BZ covered by these pockets at  $T = 0$ , is equal to the hole concentration  $\delta$ . This leads to the following estimations for the Fermi momentum  $k_F = \sqrt{\pi\delta}$  and the chemical potential  $\mu = E_{k_i} + k_F^2/2m$ .

The nearly structureless incoherent part  $\rho_{\mathbf{k}}^{inc}(\omega)$  is distributed predominantly above the QP band and can be approximated as [13]

$$\rho_{\mathbf{k}}^{inc}(\omega') = (1/2\Gamma)\theta(|\omega'| - J)\theta(2\Gamma - \omega'), \quad (8)$$

where  $2\Gamma \simeq 2zt$  and  $\omega'$  is measured from the middle of the QP band. In Eq.(8) the negative energy cutoff  $\omega_c \simeq -J - 2\Gamma(1 - Z_k)\delta$  is implied. It is provided by the sum rule  $N^{-1} \sum_{\mathbf{k}} \int d\omega n(\omega) \rho_{\mathbf{k}}(\omega) = \delta$  taken at  $T = 0$  with the value of the chemical potential  $\mu$  defined above. Numerical analysis [9, 12] of the model allows us to conclude that  $\omega_c$  does not depend on  $T$  (for  $T \ll J$ ).

In the above formulated scheme (2)–(5) the QP damping is due to scattering by spin-waves. For QP states near FS renormalization of low-lying long-wavelength spin excitations ( $\omega \ll \varepsilon_F = k_F^2/2m$ ,  $q \leq 2k_F \ll 1$ ) is of crucial importance. This renormalization is due to the coupling of spin-waves to "particle-hole" pair excitations and is described by the polarization operator (4) which contains three contributions. The first part  $\Pi^{c-c}(\mathbf{q}, \omega)$  is due to the transitions within the narrow QP band, when both  $\rho_{\mathbf{k}}(\omega_1)$  and  $\rho_{\mathbf{k}-\mathbf{q}}(\omega_2)$  in Eq.(4) are replaced by  $\rho^{coh}$ . The remaining two terms  $\Pi^{c-i}(\mathbf{q}, \omega)$  and  $\Pi^{i-i}(\mathbf{q}, \omega)$  are provided by the coherent-incoherent and incoherent-incoherent transitions.

First, considering  $\Pi^{c-c}(\mathbf{q}, \omega)$  we come to the following expression for small  $|\mathbf{q}| \ll$

$$\Pi_{\alpha\beta}^{c-c}(\mathbf{q}, \omega) = \frac{(zt)^2}{N} \sum_{i, \mathbf{k}'} Z_{\mathbf{k}_i}^2 M_{\alpha}(\mathbf{k}_i + \mathbf{k}', \mathbf{q}) M_{\beta}(\mathbf{k}_i + \mathbf{k}', \mathbf{q}) \frac{[n(\varepsilon_{\mathbf{k}'-\mathbf{q}}) - n(\varepsilon_{\mathbf{k}'})]}{\omega - \varepsilon_{\mathbf{k}'} + \varepsilon_{\mathbf{k}'-\mathbf{q}} + i\eta}, \quad (9)$$

where  $\varepsilon_{\mathbf{k}'} = (k'^2/2m - \varepsilon_F)$  is a hole energy referred to the Fermi level  $\varepsilon_F$  and the summation over  $i$  is due to the presence of four equivalent minima. Since for small momentum  $|\mathbf{q}| \ll 1$  the vertex function  $M_{1,2}(\mathbf{k}, \mathbf{q})$  is proportional to  $\sqrt{q}$ , we make an approximation  $M_{1,2}(\mathbf{k}_i + \mathbf{k}', \mathbf{q}) = \pm \tilde{M}(\mathbf{k}_i, \mathbf{q})$ , where

$$\tilde{M}(\mathbf{k}, \mathbf{q}) = 2^{-5/4} q^{-1/2} (q_x \sin k_x + q_y \sin k_y), \quad (10)$$

to keep the leading contributions in Eq.(9). This leads to the following relations between the elements of the polarization operator:  $\Pi_{11}^{c-c}(\mathbf{q}, \omega) = \Pi_{22}^{c-c}(\mathbf{q}, \omega) = -\Pi_{12}^{c-c}(\mathbf{q}, \omega) = -\Pi_{21}^{c-c}(\mathbf{q}, \omega) \equiv \Pi_{\mathbf{q}}(\omega)$ . We also note that the summation over  $i$  in Eq.(9) introduces an effective interaction  $\sum_i |\tilde{M}(\mathbf{k}_i, \mathbf{q})|^2 = q/\sqrt{2}$ . Then, for  $T = 0$  we obtain the following expressions for the real and imaginary parts of  $\Pi_{\mathbf{q}}(\omega)$ ,

$$\text{Re}\Pi_{\mathbf{q}}(\omega) = C \left\{ -q + \text{sgn}[\eta(q, \omega)] [\nu(q, \omega)]^{1/2} + \text{sgn}[\eta(q, -\omega)] [\nu(q, -\omega)]^{1/2} \right\}, \quad (11)$$

$$\text{Im}\Pi_{\mathbf{q}}(\omega) = C \left\{ [-\nu(q, \omega)]^{1/2} - [-\nu(q, -\omega)]^{1/2} \right\}, \quad (12)$$

where  $\eta(q, \omega) = m\omega/q + q/2$ ,  $\nu(q, \omega) = \eta^2(q, \omega) - (k_F)^2$  and  $C = 4\sqrt{2}mt^2 Z^2/\pi \sim 4\sqrt{2}J/\pi$ . The step- $\Theta$ -functions insuring the positivity of the arguments of the square roots are implied in Eqs.(11) and (12).

For further purposes we fix also the asymptotic,  $\omega \rightarrow 0$ , behavior of  $\Pi_{\mathbf{q}}(\omega)$  for  $q < 2k_F$ :

$$\text{Im}\Pi_{\mathbf{q}}(\omega) = 0, \quad \text{Re}\Pi_{\mathbf{q}}(\omega) = \frac{C\pi\delta q^3}{2m^2\omega^2}, \quad (13)$$

for  $\omega/q > k_F/m$ , while for  $\omega/q < k_F/m$  one has

$$\text{Im}\Pi_{\mathbf{q}}(\omega) = \frac{-2Cm\omega}{\sqrt{(2k_F)^2 - q^2}}, \quad \text{Re}\Pi_{\mathbf{q}}(\omega) = -Cq. \quad (14)$$

The limiting case (13) is important in calculating of the renormalized spin-wave velocity, which will shortly be discussed below. The limit (14) corresponds to that

region of the spin fluctuation spectrum, generated by "particle-hole" excitations, which produces finite damping of quasiholes near FS. In this respect, we note the conventional linear  $\omega$ -dependence of  $\text{Im}\Pi_q(\omega)$  in Eq. (14) in contrast to a marginal ( $q$ - and  $\omega$ -independent, at  $T = 0$ ) form of  $\text{Im}\Pi_q(\omega)$  in [5] (see Eq.(19) there). The reason of this difference can be explained as follows. Due to the incorrectly defined limits of integration in Eq.(14) of [5], the authors lost part of the polarization operator (which is presented by the second term in the curly brackets in Eq. (12) in this paper), which led to wrong subsequent approximations.

Considering the remaining contributions  $\Pi_{\alpha\beta}^{c-i}(\mathbf{q}, \omega)$  and  $\Pi_{\alpha\beta}^{i-i}(\mathbf{q}, \omega)$  we note that each of them is characterized by a threshold energy  $\Delta$  for creating a "particle-hole" pair excitation. Namely,  $\Delta = \varepsilon_F$  for processes involved in  $\Pi_{\alpha\beta}^{c-i}(\mathbf{q}, \omega)$  and  $\Delta = 2J$  for  $\Pi_{\alpha\beta}^{i-i}(\mathbf{q}, \omega)$ . Therefore,  $\text{Im}\Pi_{\alpha\beta}^{inc} = 0$ , where  $\Pi_{\alpha\beta}^{inc} = \Pi_{\alpha\beta}^{c-i} + \Pi_{\alpha\beta}^{i-i}$ , for frequencies  $\omega < \varepsilon_F$  we are interested in. Evaluation of  $\text{Re}\Pi_{\alpha\beta}^{inc}$  requires the summation over all virtual processes, which gives a finite estimate for these quantities even as  $\omega \rightarrow 0$ . To the lowest order in  $q$  and  $\delta$  for  $\text{Re}\Pi_{\alpha\beta}^{inc}$  we obtain  $\text{Re}\Pi_{11,22}^{inc}(q, \omega) \simeq -\text{Re}\Pi_{12,21}^{inc}(q, \omega) \simeq -Aq\delta$  where  $A$  is positive and can be estimated as  $A \simeq t/\sqrt{2}\{\ln(zt/J) + z^2(1 - Z_k)[\ln(2J/zt\delta) + 1]\}$ .

A position of the pole in the spin-wave GF in the long-wavelength limit is now determined as

$$\tilde{\omega}_{\mathbf{q}} = \omega_{\mathbf{q}} \sqrt{1 - 2[Aq\delta - \text{Re}\Pi_q(\omega)]/\omega_{\mathbf{q}}} + O(\delta^2). \quad (15)$$

Since the unrenormalized spin-wave velocity  $u = \sqrt{2}J$ , is much larger than the Fermi velocity  $v_F = \sqrt{\pi}\delta J$  one has to take in Eq. (15) the limit (13) which gives  $\text{Re}\Pi_q(\omega_q) \simeq \omega_q\delta$ . Thus the renormalized spin-wave velocity now reads  $\tilde{u}(\delta) = u\sqrt{1 - 2(A/u - 1)\delta}$ . For the actual values of  $\delta$  it holds  $A/u \gg 1$  and hence, one obtains, in accordance with [9, 13], a spin-wave softening due to the presence of the incoherent part in the hole spectrum.

The above estimation for  $\tilde{u}$  is valid up to the critical hole concentration  $\delta_c$  which

is defined as  $\tilde{u}(\delta_c) = v_F (= \sqrt{\pi}\delta_c J)$ . In particular, for  $J/t = 0.3$  we estimated  $\delta_c \simeq 0.04$ . For higher concentrations  $\delta > \delta_c$ , by taking the corresponding limit (14) one can see from Eq.(15) that the pole  $\tilde{\omega}_{\mathbf{q}}$  becomes purely imaginary. So; the long-wavelength magnons, with  $q \leq 2k_F$  lose their identity and can not be now detached from the incoherent part of the spectrum produced by pair excitations. Disappearance of the long-wavelength magnons due to dilution of the AFM state with holes was connected by several authors [14] with the occurrence of a phase transition into a disordered magnetic phase. The applicability of the spin-polaron model in the disordered phase will be discussed below.

Let us consider the effective spectral function (6). By using the approximated vertex function (10) for the momentum near FS,  $k' \sim k_F$  ( $k' = k - k_i$ ), we obtain

$$\chi_{k_i+k', \mathbf{q}}(\omega) \simeq -1/\pi |\tilde{M}(\mathbf{k}_i, \mathbf{q})|^2 \text{Im}\tilde{D}(\mathbf{q}, \omega), \quad (16)$$

where  $\tilde{M}(\mathbf{k}_i, \mathbf{q}) = 2^{-3/4}q^{1/2}\hat{\mathbf{q}}\hat{\mathbf{k}}_i$  and  $\tilde{D}(\mathbf{q}, \omega) = D_{11} - 2D_{12} - D_{22}$ . Taking into account the relation between the elements of the polarization operator  $\Pi(\mathbf{q}, \omega)$ , that has been obtained above, we write

$$\text{Im}\tilde{D}(\mathbf{q}, \omega) = 4\omega_{\mathbf{q}}^2 \text{Im}\Pi_q(\omega)/|\mathcal{D}_{\mathbf{q}}(\omega)|^2. \quad (17)$$

For the actual region of the  $\omega$ - and  $q$ -variables, defined as  $\omega/q < v_F$ , where the part of spin fluctuation spectrum responsible for the quasihole damping is located, one has  $|\mathcal{D}_{\mathbf{q}}(\omega)|^2 = [\omega^2 + c^2q^2]^2 + [2\omega_{\mathbf{q}}\text{Im}\Pi_q(\omega)]^2$  with  $c = u\sqrt{2(A+C)/u-1} \gg v_F$ . This strong inequality allows us to take the static limit,  $\omega \rightarrow 0$ , for  $\mathcal{D}_{\mathbf{q}}(\omega)$  in Eq. (17) that results in

$$\chi_{k_i+k', \mathbf{q}}(\omega) \simeq -1/\pi\sqrt{2}(u/c^2)^2 \text{Im}\Pi_q(\omega)(\hat{\mathbf{q}}\hat{\mathbf{k}}_i)^2 q^{-1}. \quad (18)$$

Inserting (18) into (5) one obtains for the imaginary part of the hole self-energy

$$\text{Im}\Sigma(\mathbf{k}', \varepsilon) \propto \iint dq \cos^2 \theta d\theta \int_0^{\varepsilon} d\omega \text{Im}\Pi_q(\omega) \delta(\varepsilon - \omega - \varepsilon_{\mathbf{k}'-\mathbf{q}}), \quad (19)$$

where  $\cos \theta = \hat{\mathbf{q}} \cdot \hat{\mathbf{k}}_i$ ,  $\mathbf{k}' = \mathbf{k} - \mathbf{k}_i$ ,  $\varepsilon$  is the hole energy referred to the Fermi level and  $\text{Im}\Pi_q(\omega)$  is defined in Eq. (14).

Like in the conventional considerations of the 2-dimensional (2D) FL, [15] the major contribution to  $\text{Im}\Sigma$  is given by scattering processes with the momentum transfer  $\mathbf{q}$  almost parallel to  $\mathbf{k}'$ . These processes result in the following dependence for  $\text{Im}\Sigma$  familiar for the 2D FL: [15]

$$\text{Im}\Sigma(\mathbf{k}', \varepsilon) \propto f(|\hat{\mathbf{k}}' \hat{\mathbf{k}}_i|)(\varepsilon^2/\varepsilon_F) \ln(\varepsilon/\varepsilon_F). \quad (20)$$

Here, a  $\mathbf{k}'$ -dependence of  $\text{Im}\Sigma$  is due to the anisotropy of the vertex (10) and is given by  $f(|\hat{\mathbf{k}}' \hat{\mathbf{k}}_i|)$  which is a positively defined smooth function of its variable.

Up to now we have considered the scattering processes retaining a quasihole in the vicinity of the same hole-pocket. There exist, however, processes in which the hole scatters from a given hole-pocket to the opposite or neighboring one, with momentum transfer  $\mathbf{q} \sim \mathbf{Q}$  ( $\mathbf{Q}$  is the AFM wave vector) and  $\mathbf{q} \sim \mathbf{Q}' = (\pi, 0)$ , respectively. Since the symmetry of the problem provides the equivalence of  $q \ll 1$  and  $q' = |\mathbf{Q} - \mathbf{q}| \ll 1$ , the first process gives just an extra factor 2 in  $\text{Im}\Sigma$ . Further, the vertex function  $\tilde{M}(\mathbf{k}_i, \mathbf{q})$  falls much faster at  $\mathbf{q} \simeq \mathbf{Q}'$  than at  $\mathbf{q} \simeq 0$  (or  $\mathbf{Q}$ ) and the second kind of processes gives higher order corrections in  $\omega$ . So, the conventional 2D FL behavior is expected for quasiholes at low doping.

Being originally formulated for a state with the AFM ordered spin subsystem, the spin-polaron model requires some justification if one tries to extend it to a disordered phase, *i.e.* either to  $\delta > \delta_c$  at  $T = 0$  or  $T > 0$ . Actually, [16] the hole spectrum is weakly affected by the absence of the long-range order, provided that the AFM correlations with the radius  $\xi \gg R_p$  survive ( $R_p$  is the size of the spin-polaron associated with a hole). It means that hole propagation over the same sublattice dominates and the four-pocket FS survives as well.

Connecting the magnetic phase transition at  $\delta = \delta_c$  with the disappearance of long-wavelength magnons with  $q \leq 2k_F$ , we did not find, however, any abrupt

change in that low-lying part of spin fluctuation spectrum which is responsible for the quasihole damping. Therefore, one may expect that not only the QP dispersion relation but also the character of the quasihole damping (20) do not change for  $\delta$  slightly above  $\delta_c$ .

This picture breaks down with further dilution of the magnetic subsystem, when the magnetic correlation length becomes comparable with the size of the spin-polaron. In this case, the nearest-neighbor hole hopping becomes dominant and a transition to a large FS takes place. However, this regime is beyond the scope of the present consideration.

Considering a possible effect of finite  $T$  (low enough to provide  $\xi \gg R_p$ ) we point out the existence of a characteristic temperature  $T_d(\delta)$  above which one may expect different behavior of a quasihole subsystem as compared to the low temperature case,  $T \ll T_d(\delta)$ . Actually, the Fermi-ensemble of quasiholes goes over into the strongly nondegenerate regime when the temperature  $T_d(\delta) \approx \varepsilon_F \approx 1.5J\delta$  is reached (for instance,  $T_d \approx 100\text{K}$  at  $\delta = 0.05$  for  $J = 1500\text{K}$ ). That is a result of the strong renormalization of the chemical potential  $\mu$  with  $T$ , which is naturally inherent in a fermion system of low density. Really, our analytical estimations, as well as numerical calculations, [12] show that  $\mu$  crosses the bottom of the QP band at  $T \simeq T_d$  and for  $T > T_d$  lies in the low energy incoherent part of the hole spectrum. This results in a dramatic change in the momentum distribution function  $N(k)$ : the four-pocket structure existing at  $T < T_d$  is almost washed out at  $T > T_d$ . [12] The onset of this strongly nondegenerate regime for hole carriers should manifest itself in a strong change of the thermodynamic and transport properties of the system, the problem which requires further theoretical and experimental studies.

In summary, we have investigated the quasihole damping in the low doping regime,  $\delta \ll 1$ , of the 2D  $t - J$  model. The self-energy parts for the hole and magnon GF are derived within the self-consistent Born approximation. Based on

the well established results [11] for the spectral density function of a hole moving in the AFM background, we first have calculated renormalization of spin-wave excitations due to the presence of holes. With increasing hole concentration  $\delta$ , softening of the long-wavelength spin-waves followed by their overdamping at  $\delta > \delta_c (\approx 0.04)$  has been obtained. The renormalized spectrum of spin excitations was incorporated to calculate the imaginary part  $\text{Im}\Sigma(k, \epsilon)$  of the hole self-energy. It has been shown that  $\text{Im}\Sigma$ , as  $\epsilon \rightarrow 0$ , possesses the form (20) characteristic of the conventional 2D FL.

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## References

- [1] W. Brening, Phys. Rep. **251**,153 (1995).
- [2] C. M. Varma, Int. J. Mod. Phys. B **3**, 2083 (1990).
- [3] Z.-X. Shen and D. S. Dessau, Phys. Rep. **253**, 1 (1995)
- [4] B. O. Wells *et al.*, Phys. Rev. Lett. **74**, 964, (1995).
- [5] Jian-Xin LI and Chang-De Gong, Phys. Rev. B **51**, 6343 (1995).
- [6] S. Schmitt-Rink, C. M. Varma and A. E. Ruckenstein, Phys. Rev. Lett. **60**, 2793 (1988).
- [7] C.L. Kane, P.A. Lee and N. Read, Phys. Rev. B **39**, 6880 (1989).
- [8] G. Martinez and P. Horsch, Phys. Rev. B **44**, 317, (1991).
- [9] J. Igarashi and P. Fulde, Phys. Rev. B **45**, 12357 (1992).
- [10] Yu. A. Tserkovnikov, Theor. Mat. Fiz. **7**, 250 (1971); **49**, 219 (1981).
- [11] See, for instance, the review by E. Dagotto, in Rev. Mod. Phys. **66**, 763 (1994).
- [12] N. M. Plakida, V. S. Oudovenko, and V. Yu. Yushankhai, Phys. Rev. B **50**, 6431 (1994).
- [13] G. Khaliulin and P. Horsch, Phys. Rev. B **47**, 463 (1993).
- [14] A.Sherman, M.Schreiber, Phys. Rev. B **48**, 7492 (1993); Phys. Rev. B **50**, 12887 (1994), F. P. Onufrieva, V. P. Kushnir and B. P. Toperverg, *ibid.*, **50**, 12935 (1994);
- [15] C. Hodges, H. Smith and J. Wilkins, Phys. Rev. B **4**, 302 (1971);

- [16] E. Dagotto *et al.*, Phys. Rev. B **41**, 2585 (1990). J. Igarashi and P. Fulde, *ibid.*, **48**, 998 (1993). R. Hayn, J.-L. Richard and V.Yu. Yushankhai, Solid State Commun., **93**, 127, (1995).

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