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DYNAMICAL SPIN SUSCEPTIBILITY IN THE $t-J$ MODEL

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1 Introduction

While the reference **antiferromagnetic (AFM)** insulator compounds of high-T_c superconductors (HTSC's) are well understood in terms of 2-dimensional (2D) isotropic Heisenberg model the nature of anomalous spin-dynamics in the doped samples still requires proper understanding [1].

One of the simplest models invoked to describe the HTSC's is the $t-J$ model, which contains the essential physics of CuO_2 planes in the superconducting cuprates. The $t-J$ (or its extension $t-t'-J$) model is the low energy effective model obtained from the Hubbard model by projecting out doubly occupied sites. As a result the $t-J$ model is formulated in terms of the so-called Hubbard operators (HO's), which are neither Fermi nor Bose operators. This particularity makes it difficult to treat the $t-J$ model within the conventional field-theory methods.

The various approaches, e.g., slave-boson [2, 3] or slave-fermion [4, 5, 6] methods and diagrammatic technique for HO's [7], have been used to study the spin dynamics within the $t-J$ model. In the slave-field approaches the local constraint is usually replaced by the global one restricting the validity of the approach. Whereas in any approximation formulated in terms of HO's the constraint of no doubly occupancy can be rigorously preserved.

Recently in Ref. [8] the diagrammatic technique for HO's has been used to calculate the spin susceptibility within the Larkin equation [9] for the $t-J$ model. However, in the particular case $J=0$ (corresponding to $U\to\infty$ limit of the Hubbard model) the contribution from the irreducible part in the denominator of Larkin equation (see Eq. (9) of Ref. [8]) vanishes, which indicates that the so-called kinematical interaction is not properly taken into account.

In the present paper we study the dynamical spin susceptibility for the $t-J$ model by applying memory function technique [16] in term of HO's which has been applied recently for calculation of the optical conductivity for this model [10]. We show that there exist two different in nature contributions to the memory function. The first one is due to the kinematical interaction and comes from the particle-hole excitations in the itinerant hole subsytem. While the second one comes from the localized spin fluctuations due to the Heisenberg interaction. The existence of these two contributions explicitly shows that there is a competition between itinerant and localized magnetism as it has been pointed out in Refs. [7, 8] and observed experimentally (see, e. g. Ref. [11]).

It is found that the low energy ($\omega\to 0$) spin dynamics has a diffusive character. While in the high energy limit ($\omega\to\infty$) the spin-wave-like excitations are regained. The mean-filed-like (MFL) expression obtained earlier [12] by Kondo and Yamaji's (KY's) theory [13] is recovered in this limit.

The paper is organized as follows. In the next section we formulate the $t-J$ model in terms of HO's. In Sec. 3 . the general formalism of the memory function approach is presented and within the mode coupling approximation the memory function is calculated. The self-consistency of the presented approach is discussed in Sec. 5. Sec. 6 summarizes our main results. In the Appendix the expression for static spin susceptibility is derived.

2 The model

The Hamiltonian of the $t-J$ model reads

$$H = H_t + H_J = \sum_{i,j,\sigma} t_{ij} X_i^{\sigma 0} X_j^{0\sigma} + \sum_{i,j} J_{ij} \{ \mathbf{S}_i \mathbf{S}_j - \frac{1}{4} n_i n_j \}. \tag{1}$$

The first term in the right-hand side of Eq. (1) describes an electron hopping between the nearest ($t_{ij} = t$) and next-nearest ($t_{ij} = t'$) neighbor lattice sites. The second term describes the exchange interaction of localized spins $\mathbf{S}_i$ between the nearest neighbor sites ($J_{ij} = J$). The HO's in Eq. (1) are defined as

$$X_i^{\alpha\beta} = |i,\alpha\rangle\langle i,\beta| \tag{2}$$

for three possible states at the lattice site i

$$|i,\alpha\rangle = |i,0\rangle, \quad |i,\sigma\rangle, \tag{3}$$

for an empty site and for a singly occupied site by electron with spin σ. In the $t-J$ model only singly occupied sites are retained and the completeness relation for the HO's reads as

$$X_i^{00} + \sum_{\sigma} X_i^{\sigma\sigma} = 1. \tag{4}$$

The spin and density operators in Eq. (1) are expressed by HO's as

$$S_i^{\sigma} = X_i^{\sigma\bar{\sigma}}, \quad S_i^z = \frac{1}{2}\sum_{\sigma} \sigma X_i^{\sigma\sigma}, \quad n_i = \sum_{\sigma} X_i^{\sigma\sigma}, \tag{5}$$

where $\bar{\sigma} = -\sigma$. The HO's obey the following multiplication rules

$$X_i^{\alpha\beta} X_j^{\gamma\delta} = \delta_{ij}\delta_{\beta\gamma} X_i^{\alpha\delta} \tag{6}$$

and commutation relations

$$\left[X_i^{\alpha\beta} X_j^{\gamma\delta}\right]_{\pm} = \delta_{ij}\left(\delta_{\beta\gamma} X_i^{\alpha\delta} \pm \delta_{\delta\alpha} X_i^{\gamma\beta}\right). \tag{7}$$

In Eq.(7) the upper sign stands for the case when both HO's are Fermi-like ones (as, e. g., $X_i^{0\sigma}$). The spin and density operators (5) are Bose-like and for them the lower sign in Eq.(7) should be taken. The HO's are neither Fermi nor Bose operators, the are projected operators. These unconventional commutation relations (7) makes impossible to treat the model within the conventional diagrammatic technique. To use the latter one needs to introduce slave particles with the constraint of no doubly occupancy. While in the treatment within the HO's the constraint is automatically fulfilled. Therefore we treat the problem in terms of HO's and use the memory function formalism to determine the dynamical spin susceptibility.

3 Memory Function

3.1 General Formalism

The dynamical spin susceptibility is defined as

$$\chi_{\mathbf{q}}(\omega) = -\langle\langle S_{\mathbf{q}}^+ | S_{\mathbf{q}}^- \rangle\rangle_\omega = -\sum_{\mathbf{R}_{ij}} e^{-i\mathbf{q}\mathbf{R}_{ij}} \langle\langle S_i^+ | S_j^- \rangle\rangle_\omega, \tag{8}$$

where $\mathbf{R}_{ij} = \mathbf{R}_i - \mathbf{R}_j$ and $\langle\langle A|B\rangle\rangle_\omega$ denotes the Fourier transformed two-time retarded commutator Green function (GF) [14, 15]

$$\langle\langle A|B\rangle\rangle_\omega = -i\int_0^\infty dt e^{i\omega t}\langle[A(t), B]\rangle, \tag{9}$$

where $\mathrm{Im}\omega > 0$, $A(t) = \exp(iHt)A\exp(-iHt)$, and $\langle AB\rangle$ denotes the equilibrium statistical average.

In the paramagnetic state with zero sublattice magnetization an average of the commutator $\langle[S_i^+, S_j^-]\rangle = 2\delta_{ij}\langle S_i^z\rangle$ equals zero. Since just this quantity enters as an initial condition ($t = 0$) in the equation of motion for the GF (8), it is more convenient to construct the self-consistent equation for the Kubo-Mori relaxation function (see, e.g., Ref. [16])

$$((A|B))_\omega = -i\int_0^\infty dt e^{i\omega t}(A(t), B), \tag{10}$$

where $(A(t), B)$ is the Kubo-Mori scalar product defined as

$$(A(t), B) = \int_0^\beta d\lambda \langle A(t - i\lambda)B\rangle, \tag{11}$$

where $\beta = 1/T$ ($h = k_B = 1$).

The GFs (9) and (10) are coupled by the equation

$$\omega((A|B))_\omega = \langle\langle A|B\rangle\rangle_\omega - \langle\langle A|B\rangle\rangle_{\omega=0}. \tag{12}$$

There are also following useful relations which can be obtained from the definitions (9)-(11)

$$((i\dot{A}|B))_\omega = ((A| - i\dot{B}))_\omega = \langle\langle A|B\rangle\rangle_\omega, \tag{13}$$

$$(i\dot{A}, B) = (A, -i\dot{B}) = \langle[A, B]\rangle, \tag{14}$$

$$(A, B) = -\langle\langle A|B\rangle\rangle_{\omega=0}, \tag{15}$$

where $i\dot{A} = idA/dt = [A, H]$.

By using the above formulas for the dynamical spin susceptibility we obtain

$$\chi_{\mathbf{q}}(\omega) = \chi_{\mathbf{q}} - \omega\Phi_{\mathbf{q}}(\omega), \tag{16}$$

where $\chi_{\mathbf{q}} \equiv \chi_{\mathbf{q}}(0)$ is the static spin susceptibility and $\Phi_{\mathbf{q}}(\omega) \equiv ((S_{\mathbf{q}}^+|S_{\mathbf{q}}^-))_\omega$. To calculate the spin-spin relaxation function $\Phi_{\mathbf{q}}(\omega)$ it is convenient to employ the memory function approach of Mori (see, e.g. Ref. [16]). We define the memory function $M(\mathbf{q},\omega)$ by the equation

$$\Phi_{\mathbf{q}}(\omega) = \frac{\chi_{\mathbf{q}}}{\omega - M(\mathbf{q},\omega)/\chi_{\mathbf{q}}}. \tag{17}$$

To calculate the memory function we use the equation of motion for the relaxation function (10)

$$\omega((S_{\mathbf{q}}^+|S_{\mathbf{q}}^-))_\omega = \chi_{\mathbf{q}} + ((i\dot{S}_{\mathbf{q}}^+|S_{\mathbf{q}}^-))_\omega, \tag{18}$$

and similarly $((i\dot{S}_{\mathbf{q}}^+|S_{\mathbf{q}}^-))_\omega$ obey the following equation of motion

$$\omega((i\dot{S}_{\mathbf{q}}^+|S_{\mathbf{q}}^-))_\omega = (i\dot{S}_{\mathbf{q}}^+, S_{\mathbf{q}}^-) + ((i\dot{S}_{\mathbf{q}}^+| - i\dot{S}_{\mathbf{q}}^-))_\omega. \tag{19}$$

From Eq.(14) we have $(i\dot{S}_{\mathbf{q}}^{+}, S_{\mathbf{q}}^{-}) = \langle[S_{\mathbf{q}}^{+}, S_{\mathbf{q}}^{-}]\rangle = 2/\sqrt{N}\langle S_{\mathbf{q}}^{z}\rangle\delta_{\mathbf{q}0}$ which is zero in the paramagnetic phase, that results

$$\omega\Phi_{\mathbf{q}}(\omega) = \chi_{\mathbf{q}} + \frac{1}{\omega}((i\dot{S}_{\mathbf{q}}^{+}| - i\dot{S}_{\mathbf{q}}^{-}))_{\omega}. \tag{20}$$

By introducing the zero order GF $\Phi_{\mathbf{q}}^{0}(\omega) = \chi_{\mathbf{q}}/\omega$ we rewrite Eq. (20) as follows

$$\Phi_{\mathbf{q}}(\omega) = \Phi_{\mathbf{q}}^{0}(\omega) + \Phi_{\mathbf{q}}^{0}(\omega)T_{\mathbf{q}}(\omega)\Phi_{\mathbf{q}}^{0}(\omega) \tag{21}$$

where we have introduced the scattering matrix

$$T_{\mathbf{q}}(\omega) = \frac{1}{\chi_{\mathbf{q}}}((i\dot{S}_{\mathbf{q}}^{+}| - i\dot{S}_{\mathbf{q}}^{-}))_{\omega}\frac{1}{\chi_{\mathbf{q}}}. \tag{22}$$

By comparing (22) to the definition of the memory function (17) we get the following relation between the scattering matrix and the memory function

$$T_{\mathbf{q}}(\omega) = \frac{M(\mathbf{q},\omega)}{\chi_{\mathbf{q}}^{2}} + \frac{M(\mathbf{q},\omega)}{\chi_{\mathbf{q}}^{2}}\Phi_{\mathbf{q}}^{0}(\omega)T_{\mathbf{q}}(\omega). \tag{23}$$

A formal solution of the Eq. (23) by iteration shows that the quantity $M(\mathbf{q},\omega)/\chi_{\mathbf{q}}(\omega)$ is just the irreducible part of the scattering matrix which has no parts connected by the single zero order GF $\Phi_{\mathbf{q}}^{0}(\omega)$:

$$M(\mathbf{q},\omega) = \chi_{\mathbf{q}}^{2}T_{\mathbf{q}}^{irr}(\omega) = ((i\dot{S}_{\mathbf{q}}^{+}| - i\dot{S}_{\mathbf{q}}^{-}))_{\omega}^{irr}. \tag{24}$$

Finally, the dynamical spin susceptibility in terms of the memory function can be written as

$$\chi_{\mathbf{q}}(\omega) = -\chi_{\mathbf{q}}\frac{M(\mathbf{q},\omega)/\chi_{\mathbf{q}}}{\omega - M(\mathbf{q},\omega)/\chi_{\mathbf{q}}}. \tag{25}$$

3.2 Mode-coupling approximation

First we express the memory function in terms of the irreducible current-current time-dependent correlation function by using the spectral representation for the GF

$$M(\mathbf{q},\omega) = \sum_{\mathbf{R}_{ij}} e^{-i\mathbf{q}\mathbf{R}_{ij}}\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega'\frac{e^{\beta\omega'} - 1}{\omega'(\omega - \omega' + i\eta)}\int_{-\infty}^{\infty} dt e^{-i\omega' t}\langle J_i(t)|J_j^{+}\rangle^{irr}, \tag{26}$$

where the current operator in the site representation is defined as $J_i = -i\dot{S}_i^{-}$, $J_i^{+} = i\dot{S}_i^{+}$. Current operator can be written as a sum of two terms

$$J_i = J_i^{t} + J_i^{J} = [S_i^{-}, H_t] + [S_i^{-}, H_J]. \tag{27}$$

In Eq.(27) the first term comes from the so-called kinematical interaction which is due to the unconventional commutation relation for the HO's operators (7). This term is proportional to the hopping integral and reads as

$$J_i^{t} = -\sum_{m} t_{im}(X_m^{-0}X_i^{0+} - X_i^{-0}X_m^{0+}). \tag{28}$$

The second term in Eq.(27) comes from the exchange interaction between localized spins and has the form

$$J_i^{J} = 2\sum_{m} J_{im}(S_i^{z}S_m^{-} - S_m^{z}S_i^{-}). \tag{29}$$

To calculate the irreducible time-dependent correlation function in the right-hand side of Eq. (26) we employ the mode-coupling approximation in terms of an independent propagation of the dressed particle-hole and spin fluctuations (see, e. g., Götze et al., [17]). This scheme is essentially equivalent to the self-consistent Born approximation in which the vertex corrections are neglected. The proposed scheme is defined by the following decoupling of the time-dependent correlation functions:

$$\langle X_m^{-0}(t)X_i^{0+}(t)X_j^{+0}X_l^{0-}\rangle \simeq \langle X_m^{-0}(t)X_l^{0-}\rangle\langle X_i^{0+}(t)X_j^{+0}\rangle, \tag{30}$$

$$\langle S_i^{z}(t)S_m^{-}(t)S_j^{z}S_l^{+}\rangle \simeq \langle S_i^{z}(t)S_j^{z}\rangle\langle S_m^{-}(t)S_l^{+}\rangle. \tag{31}$$

The cross-correlations like $\langle J_i^{t}|(J^{J})_j^{+}\rangle$ are ignored within the proposed approximation and they will be omitted.

By using the above defined decoupling scheme (30) and (31) and the spectral representation for the GF, the memory function (26) can be written as

$$M(\mathbf{q},\omega) = M_t(\mathbf{q},\omega) + M_J(\mathbf{q},\omega), \tag{32}$$

where $M_t(\mathbf{q},\omega)$ is the contribution from the itinerant hole subsystem and reads as

$$M_t(\mathbf{q},\omega) = \frac{1}{N}\sum_{\mathbf{k}} t_{\mathbf{kq}}^{2}\iint_{-\infty}^{\infty} d\omega_1 d\omega'[n(\omega_1 - \omega') - n(\omega_1)]\frac{A_{\mathbf{k}}(\omega_1)A_{\mathbf{k}-\mathbf{q}}(\omega_1 - \omega')}{\omega'(\omega - \omega' + i\eta)}, \tag{33}$$

where $n(\omega) = (e^{\beta\omega} + 1)^{-1}$, $t_{\mathbf{kq}} \doteq t_{\mathbf{k}} - t_{\mathbf{k}-\mathbf{q}}$ with $t_{\mathbf{k}} = z(t\gamma_{\mathbf{q}} + t'\gamma_{\mathbf{q}}')$, $z = 4$, $\gamma_{\mathbf{q}} = 1/2[\cos q_x + \cos q_y]$ and $\gamma_{\mathbf{q}}' = \cos q_x \cos q_y$ for 2D square lattice (the lattice constant is taken to be unity) and $A_{\mathbf{k}}(\omega) = -1/\pi \mathrm{Im}\langle\langle X_{\mathbf{q}}^{0\sigma}X_{\mathbf{q}}^{\sigma 0}\rangle\rangle_{\omega}$ is the one particle spectral function which is spin independent in the paramagnetic phase.

The second contribution $M_J(\mathbf{q},\omega)$ in Eq.(32) comes from the localized spin subsystem and is given by

$$M_J(\mathbf{q},\omega) = \frac{2}{\pi^2 N}\sum_{\mathbf{k}} J_{\mathbf{kq}}^{2}\iint_{-\infty}^{\infty} d\omega_1 d\omega'[N(\omega_1 - \omega') - N(\omega_1)]\frac{\mathrm{Im}\chi_{\mathbf{k}}(\omega_1)\mathrm{Im}\chi_{\mathbf{k}-\mathbf{q}}(\omega_1 - \omega')}{\omega'(\omega - \omega' + i\eta)}, \tag{34}$$

where $N(\omega) = (e^{\beta\omega} - 1)^{-1}$ and $J_{\mathbf{kq}} = J_{\mathbf{k}} - J_{\mathbf{k}-\mathbf{q}}$, and $J_{\mathbf{q}} = zJ\gamma_{\mathbf{q}}$. In obtaining (34) relation $\langle\langle S_{\mathbf{q}}^{z}|S_{\mathbf{q}}^{z}\rangle\rangle_{\omega} = 1/2\langle\langle S_{\mathbf{q}}^{+}|S_{\mathbf{q}}^{-}\rangle\rangle_{\omega}$ which is valid in the rotationally invariant system has been used.

The real, $\mathrm{Re}M(\mathbf{q},\omega)$, and imaginary, $\mathrm{Im}M(\mathbf{q},\omega)$, parts of the memory function are odd and even functions of ω, respectively, and they are coupled by the dispersion relation

$$\mathrm{ReM}(\mathbf{q},\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega'\frac{\mathrm{Im}M(\mathbf{q},\omega')}{\omega' - \omega}. \tag{35}$$

Therefore, only imaginary part of the memory function should be evaluated.

3.3 Asymptotic behavior of $\chi_{\mathbf{q}}(\omega)$

Now we examine asymptotic behavior of the dynamical spin susceptibility. First we consider the hydrodynamic limit $\mathbf{q}\to 0$ and $\omega\to 0$. In this limit, $\mathrm{Re}M(\mathbf{q},\omega)$ being an odd function of ω vanishes while $\mathrm{Im}M(\mathbf{q},\omega)$ remains finite. By using Eqs.(33) and (34) we can express it as $\mathrm{Im}M(\mathbf{q},\omega)\simeq -Dq^2$ with $D=D_t+D_J$ where

$$D_t=\frac{\pi}{N}\sum_{\mathbf{k}}(\hat{\mathbf{q}}\nabla_{\mathbf{k}}t_{\mathbf{k}})^2\lim_{\omega\to 0}\lim_{\mathbf{q}\to 0}\int_{-\infty}^{\infty}d\omega_1 n'(\omega_1)A_{\mathbf{k}}(\omega_1)A_{\mathbf{k}-\mathbf{q}}(\omega_1-\omega),$$

$$D_J=\frac{2}{\pi N}\sum_{\mathbf{k}}(\hat{\mathbf{q}}\nabla_{\mathbf{k}}J_{\mathbf{k}})^2\lim_{\omega\to 0}\lim_{\mathbf{q}\to 0}\int_{-\infty}^{\infty}d\omega_1 N'(\omega_1)\mathrm{Im}\chi_{\mathbf{k}}(\omega_1)\mathrm{Im}\chi_{\mathbf{k}-\mathbf{q}}(\omega_1-\omega), \qquad (36)$$

where $\hat{\mathbf{q}}=\mathbf{q}/q$, $\nabla_{\mathbf{k}}=dt_{\mathbf{k}}/d\mathbf{k}$, $n'(\omega)=dn(\omega)/d\omega$, and $N'(\omega)=dN(\omega)/d\omega$. Finally in the hydrodynamic limit the dynamical spin-susceptibility can be expressed in the usual form (see Ref. [16]) as

$$\chi_{\mathbf{q}}(\omega)=\chi_0\frac{i\tilde{D}q^2}{\omega+i\tilde{D}q^2} \qquad (37)$$

where $\tilde{D}=D/\chi_0$ is the spin diffusion coefficient and χ_0 is the static uniform susceptibility.

Unlike to the hydrodynamic limit, in the high energy limit, $\omega\to\infty$ the dominant contribution to the memory functions comes from the real part: $M(\mathbf{q},\omega)\simeq m_{\mathbf{q}}/\omega$ where $m_{\mathbf{q}}$ is the first nonvanishing moment in $1/\omega$ expansion of the memory function defined as

$$m_{\mathbf{q}}=\frac{1}{\pi}\int_{-\infty}^{\infty}\mathrm{Im}M(\mathbf{q},\omega)=\langle[i\dot{S}^{+}_{\mathbf{q}},S^{+}_{\mathbf{q}}]\rangle\,. \qquad (38)$$

Thus in the high energy limit dynamical susceptibility takes the form

$$\chi_{\mathbf{q}}(\omega)=-\chi_{\mathbf{q}}\frac{\omega_{\mathbf{q}}^2}{\omega^2-\omega_{\mathbf{q}}^2} \qquad (39)$$

where $\omega_{\mathbf{q}}^2=m_{\mathbf{q}}/\chi_{\mathbf{q}}$. Let us note, that Eq. (39) with expressions of $\chi_{\mathbf{q}}$ and $m_{\mathbf{q}}$ derived in the Appendix [see Eqs. (62) and (63)] reproduces the result for the spin susceptibility obtained for the $t-J$ model in Ref. [12] by KY's theory [13] which is essentially self-consistent MFL approximation.

4 Self-consistency of the problem

The equations (25),(33), and (34) are the self-consistent integral equations for dynamical spin susceptibility which is obtained by using only mode coupling approximation. These equations should be solved numerically by iteration procedure. The static spin susceptibility at each iteration step should be calculated by Eq. (50) with $M(\mathbf{q},\omega)$ and $\chi_{\mathbf{q}}(\omega)$ from the preceding iteration. However some ansatz for $\chi_{\mathbf{q}}(\omega)$ as the starting point of iteration procedure should be defined. Moreover we need to know the hole spectral function entering into Eq. (33) for the memory function.

According to well known results for hole spectral function obtained within $t-J$ model [18] $A_{\mathbf{k}}(\omega)$ can be modeled as

$$A_{\mathbf{k}}(\omega)=Z_{\mathbf{k}}\delta(\omega+\mu-\varepsilon_{\mathbf{k}})+A^{inc}_{\mathbf{k}}(\omega), \qquad (40)$$

where $Z_{\mathbf{k}}$ is the quasiparticle weight for the excitations with the dispersion $\varepsilon_{\mathbf{k}}$ in a narrow band of the order J. The second part $A^{inc}_{\mathbf{k}}(\omega)$ is due to the diffusive motion of holes in a broad band with bandwidth $2W$ (of order $8t$ for 2D square lattice). We model it as

$$A^{inc}_{\mathbf{k}}=N_{inc}\theta(W-|\omega+\mu|), \qquad (41)$$

where N_{inc} is the density of state for the incoherent continuum and it is coupled to $Z_{\mathbf{k}}$ by the sum rule

$$\frac{1}{N}\sum_{\mathbf{k}}\int_{-\infty}^{\infty}A_{\mathbf{k}}(\omega)=\langle X_i^{00}+X_i^{\sigma\sigma}\rangle=1-\frac{n}{2}. \qquad (42)$$

By using Eq.(40) the contribution from the hole coherent motion to the imaginary part of the memory function can be expressed as

$$\mathrm{Im}M_t^{c-c}(\mathbf{q},\omega)=\frac{\pi}{N}\sum_{\mathbf{k}}t^2_{\mathbf{kq}}Z_{\mathbf{k}}Z_{\mathbf{k}-\mathbf{q}}\frac{n(\varepsilon_{\mathbf{k}-\mathbf{q}})-n(\varepsilon_{\mathbf{k}})}{\omega}\delta(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\mathbf{q}}-\omega)\,. \qquad (43)$$

To evaluate the second term in the memory function (34) as the starting point for $\chi_{\mathbf{q}}(\omega)$ one can use the MFL expression (39), with $m_{\mathbf{q}}$ and $\omega_{\mathbf{q}}$ defined by (54) and (62), respectively. As a result we obtain

$$\mathrm{Im}M_J(\mathbf{q},\omega)=\frac{\pi}{2N}\sum_{\mathbf{k}}B_{\mathbf{kq}}\{P_{\mathbf{kq}}(\omega)+P_{\mathbf{kq}}(-\omega)\} \qquad (44)$$

where

$$B_{\mathbf{kq}}=J^2_{\mathbf{kq}}\frac{m_{\mathbf{k}}m_{\mathbf{k}-\mathbf{q}}}{\omega_{\mathbf{k}}\omega_{\mathbf{k}-\mathbf{q}}} \qquad (45)$$

is an effective vertex function and

$$P_{\mathbf{kq}}(\omega)=\left\{\frac{N(\omega_{\mathbf{k}})-N(\omega_{\mathbf{k}-\mathbf{q}})}{\omega}\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}-\mathbf{q}}-\omega)-\frac{1+N(\omega_{\mathbf{k}})+N(\omega_{\mathbf{k}-\mathbf{q}})}{\omega}\delta(\omega_{\mathbf{k}}+\omega_{\mathbf{k}-\mathbf{q}}-\omega)\right\}. \qquad (46)$$

Eqs.(43) and Eq.(44) can be considered as the first iteration for the memory function. The hole parameters entering into expressions (54) and (62) for $m_{\mathbf{q}}$ and $\omega_{\mathbf{q}}$ can be calculated from the hole spectral function (40). Whereas χ_1 and χ_2 defined by Eqs. (56) and (64) should be evaluated self-consistently from the dynamical spin susceptibility (39). By using the above expressions (43) and (44) for $\mathrm{Im}M(\mathbf{q},\omega)$ and the dispersion relation (35) the dynamical spin susceptibility within the first iteration can be calculated from Eq.(25).

Using the obtained results one can evaluate the spin fluctuation part of the memory (34) and the static spin susceptibility (50)-(52) for the next iteration procedure. The iteration procedure should be continued until the convergency will be reached.

5 Summary

To summarize, based on the $t-J$ model and memory function approach we have derived a general representation for dynamical spin-susceptibility (25) in terms of the memory function (26). Our approach is formulated in terms of HO's and therefore the constraint of no doubly occupancy is rigorously preserved. The memory function is calculated by using the equation of motion method for two-time retarded GF's [14] within the mode coupling approximation (32-34). The two contributions to the memory function is obtained. The first one (33) comes from the itinerant hole subsystem and is due to the kinematical interaction. The second one (34) comes from the localized interacting spin subsytem. In the limit of small concentration of doped holes the latter one gives the main contribution which describes spin dynamics characteristic for the Heisenberg model. Whereas in the opposite limit of large hole concentration particle-hole excitations characteristic to the itinerant magnetism give the main contribution to spin dynamics. We have shown that in the paramagnetic phase there are two regimes in the spin dynamics. In the hydrodynamic limit ($q \to 0$, $\omega \to 0$) the spin susceptibility (37) describes diffusion spin dynamics with the diffusion coefficient (36), which has essentially two contributions. While in the high-frequency limit ($\omega \to \infty$) spin-wave-like excitations described by Eq. (39) are observed. Their dispersion, Eq. (62), obtained in the mode coupling like approximation for the equal time correlation function (60) recovers the earlier MFL result [12] obtained within the KY's [13]theory.

To compare our results with that obtained by diagrammatic methods we would like to point out that our approach, based on the general representation for the spin susceptibility (25), is equivalent to summation of infinite series of diagrams generated by the memory function (26). The latter one, being calculated in the mode coupling approximation ,Eqs. (30) and (31), can be schematically represented by two loop-diagrams: the first one of order t^2 due to the particle-hole loop and the second one of order J^2 due to the spin fluctuation loop. In Ref. [8] all the contributions in the denominator of the Larkin equation are proportional to J and therefore disappears in the limit $J=0$ ($U \to \infty$). While in our approach contribution due to the first loop (for which the kinematical interaction is responsible) remains. Whereas, in Ref. [7] the spin fluctuation contribution given by our second loop is neglected while several other diagrams beyond our simple one-loop diagram due to the kinematical interaction are taken into account.

At present time it is difficult to justify any of discussed scheme, including our mode coupling approximation. To check the validity of our approximation one has to solve numerically self-consistent equations and compare the obtained results with the experimental data. This will be done in a forthcoming publication.

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Appendix

In this Appendix we evaluate the static spin susceptibility $\chi_{\mathbf{q}}$ following Tserkovnikov [19]. For this purpose, by using Eqs.(17), (19), and (20) we can rewrite the memory function or the irreducible part of the current-current correlation function in the following way proposed by Tserkovnikov [15]:

$$((i\dot{S}^+_{\mathbf{q}}|-i\dot{S}^-_{\mathbf{q}}))^{irr}_\omega = ((i\dot{S}^+_{\mathbf{q}}|-i\dot{S}^-_{\mathbf{q}}))_\omega - ((i\dot{S}^+_{\mathbf{q}}|S^-_{\mathbf{q}}))_\omega((S^+_{\mathbf{q}}|S^-_{\mathbf{q}}))^{-1}_\omega((S^+_{\mathbf{q}}|-i\dot{S}^-_{\mathbf{q}}))_\omega. \qquad (47)$$

Likewise we define the irreducible part of the force-force correlation function as

$$((i^2\ddot{S}^+_{\mathbf{q}}|i^2\ddot{S}^-_{\mathbf{q}}))^{irr}_\omega = ((i^2\ddot{S}^+_{\mathbf{q}}|i^2\ddot{S}^-_{\mathbf{q}}))_\omega - ((i^2\ddot{S}^+_{\mathbf{q}}|S^-_{\mathbf{q}}))_\omega((S^+_{\mathbf{q}}|S^-_{\mathbf{q}}))^{-1}_\omega((S^+_{\mathbf{q}}|i^2\ddot{S}^-_{\mathbf{q}}))_\omega, \qquad (48)$$

and consequently the irreducible part of equal time force-force correlation function can be written as

$$(i^2\ddot{S}^+_{\mathbf{q}}, i^2\ddot{S}^-_{\mathbf{q}})^{irr} = (i^2\ddot{S}^+_{\mathbf{q}}, i^2\ddot{S}^-_{\mathbf{q}}) - (i^2\ddot{S}^+_{\mathbf{q}}, S^-_{\mathbf{q}})(S^+_{\mathbf{q}}, S^-_{\mathbf{q}})^{-1}(S^+_{\mathbf{q}}, i^2\ddot{S}^-_{\mathbf{q}}). \qquad (49)$$

By using Eq. (49) and identities (13)-(15) the static spin susceptibility can be expressed as

$$\chi_{\mathbf{q}} = \frac{\langle[i\dot{S}^+_{\mathbf{q}}, S^-_{\mathbf{q}}]\rangle^2}{(i^2\ddot{S}^+_{\mathbf{q}}, i^2\ddot{S}^-_{\mathbf{q}}) - (i^2\ddot{S}^+_{\mathbf{q}}, i^2\ddot{S}^-_{\mathbf{q}})^{irr}}. \qquad (50)$$

The first term in the denominator can be written as the second moment of the dynamical spin susceptibility

$$(i^2\ddot{S}^+_{\mathbf{q}}, i^2\ddot{S}^-_{\mathbf{q}}) = \langle[i^2\ddot{S}^+_{\mathbf{q}}, -i\dot{S}^-_{\mathbf{q}}]\rangle = \frac{1}{\pi}\int_{-\infty}^{\infty} \omega^2 \mathrm{Im}\chi_{\mathbf{q}}(\omega) \qquad (51)$$

and the latter one is equal to the second nonvanishing moment of the memory function

$$(i^2\ddot{S}^+_{\mathbf{q}}, i^2\ddot{S}^-_{\mathbf{q}})^{irr} = \frac{1}{\pi}\int_{-\infty}^{\infty} \omega^2 \mathrm{Im}M(\mathbf{q},\omega). \qquad (52)$$

The expression (50) is an exact representation for the static spin susceptibility.

However, to derive an approximate expression for $\chi_{\mathbf{q}}$ we start from the following identity

$$\langle[i\dot{S}^+_{\mathbf{q}}, S_{\mathbf{q}}]\rangle = (i^2\ddot{S}^+_{\mathbf{q}}, S^-_{\mathbf{q}}). \qquad (53)$$

We evaluate the left hand-side of Eq. (53) by using the commutation relation for HO's, that results

$$m_{\mathbf{q}} = \langle[i\dot{S}^+_{\mathbf{q}}|S^-_{\mathbf{q}}]\rangle = 4zJ(1-\gamma_{\mathbf{q}})\Big\{\frac{t}{2J}n_1 + \frac{t'}{2J}n'_1\lambda_{\mathbf{q}} - \chi_1\Big\}, \qquad (54)$$

with the following notations

$$n_1 = \frac{1}{N}\sum_{\mathbf{q}}\gamma_{\mathbf{q}}n_{\mathbf{q}}, \quad n' = \frac{1}{N}\sum_{\mathbf{q}}\gamma'_{\mathbf{q}}n_{\mathbf{q}}, \quad n_{\mathbf{q}} = \langle X^{\sigma 0}_{\mathbf{q}} X^{0\sigma}_{\mathbf{q}}\rangle, \qquad (55)$$

and

$$\chi_1 = \frac{1}{N}\sum_{\mathbf{q}}\gamma_{\mathbf{q}}\langle S^+_{\mathbf{q}} S^-_{\mathbf{q}}\rangle, \quad \lambda_{\mathbf{q}} = \frac{1-\gamma'_{\mathbf{q}}}{1-\gamma_{\mathbf{q}}}. \qquad (56)$$

To calculate the correlation function in the right-hand side of Eq. (53) we employ the decoupling scheme which is essentially equivalent to the mode coupling approximation but for the equal time correlation function. Due to the unconventional commutation relations for HO's it is more convenient to use the site representation

$$(i^2\ddot{S}^+_{\mathbf{q}}, S^-_{\mathbf{q}}) = \sum_{\mathbf{R}_{il}} e^{-i\mathbf{q}\mathbf{R}_{il}} (i^2\ddot{S}^+_i, S^-_l), \tag{57}$$

where second derivative of S^+_i reads

$$\begin{aligned} i^2\ddot{S}^+_i &= \sum_{j,n} t_{ij}\big\{t_{jn}\,[H_{ijn}+H_{nji}] - t_{in}\,[H_{jin}+H_{nij}]\big\} \\ &+ \sum_{j,n} J_{ij}\big\{J_{jn}\,[2P_{ijn}+\Pi_{nji}] - J_{in}\,[2P_{jin}+\Pi_{nij}]\big\}, \end{aligned} \tag{58}$$

with

$$\begin{aligned} H_{ijn} &= X^{+0}_i X^{+-}_j X^{0+}_n + X^{+0}_i (X^{00}_j + X^{--}_j) X^{0-}_n, \\ P_{ijn} &= S^z_i S^z_j S^+_n - S^z_i S^z_n S^+_j, \\ \Pi_{ijn} &= S^+_i S^-_j S^+_n - S^+_j S^-_i S^+_n. \end{aligned} \tag{59}$$

In obtaining (58) we have neglected terms proportional to tJ since they give no contribution within the adopted approximation.

In the sum (58) only two site indices can be equal. We extract those terms and by using the multiplication rules (6) replace the product of two HO's with the same site indices by one operator. On rearranging, in the sum there are no products of operators having the same site indices. Therefore in all products operators can be interchanged. (Of course in the case of two Fermi operators one has to change the sign of the product). Further, we substitute the properly rearranged right-hand side of eq. (58) into (57) and make the following decouplings

$$\begin{aligned} (X^{\sigma\sigma}_i S^+_j, S^-_l) &\simeq \langle X^{\sigma\sigma}_i\rangle (S^+_j, S^-_l) \qquad (i \neq j) \\ (S^+_i S^-_j S^+_n, S^-_l) &\simeq \langle S^+_i S^-_j\rangle (S^+_n, S^-_l) + \langle S^+_n S^-_j\rangle (S^+_i, S^-_l) \quad (i \neq j \neq n). \end{aligned} \tag{60}$$

In the above defined decoupling scheme the operators on the same lattice site is never decoupled. Therefore within the adopted approximation the local correlations are retained.

In the momentum space the above defined decoupling scheme results in the following expression:

$$(i^2\ddot{S}^+_{\mathbf{q}}, S^-_{\mathbf{q}}) \simeq \omega^2_{\mathbf{q}} (S^+_{\mathbf{q}}, S^-_{\mathbf{q}}) = \omega^2_{\mathbf{q}} \chi_{\mathbf{q}}, \tag{61}$$

where

$$\omega^2_{\mathbf{q}} = 4J^2 z^2 |\chi_1| (1-\gamma_{\mathbf{q}})[1 + \Delta + C\lambda_{\mathbf{q}} + \gamma_{\mathbf{q}}], \tag{62}$$

with the following notations:

$$\Delta = \frac{1}{|\chi_1|}\Big\{\chi_2 + \frac{1-z}{z}|\chi_1| + \alpha - \eta\Big\}, \quad C = \frac{\alpha' - \eta}{|\chi_1|}, \tag{63}$$

$$\chi_2 = \frac{1}{N}\sum_{\mathbf{q}} \gamma^2_{\mathbf{q}} \langle S^+_{\mathbf{q}} S^-_{\mathbf{q}}\rangle, \tag{64}$$

$$\alpha = \frac{t^2}{2NJ^2}\sum_{\mathbf{q}} \gamma^2_{\mathbf{q}} \tilde{n}_{\mathbf{q}}, \quad \alpha' = \frac{(t')^2}{2NJ^2}\sum_{\mathbf{q}} (\gamma'_{\mathbf{q}})^2 \tilde{n}_{\mathbf{q}}, \quad \eta = \frac{tt'}{2NJ^2}\sum_{\mathbf{q}} \gamma_{\mathbf{q}}\gamma'_{\mathbf{q}} n_{\mathbf{q}}, \tag{65}$$

and

$$\tilde{n}_{\mathbf{q}} = (1 - \frac{n}{2} - n_{\mathbf{q}}), \quad n = \langle X^{\sigma\sigma}_i \rangle. \tag{66}$$

Therefore, from Eq. (53) by using (54) and (61) for the static spin susceptibility we obtain the following representation

$$\chi_{\mathbf{q}} = \frac{\langle [i\dot{S}^+_{\mathbf{q}}, S^-_{\mathbf{q}}]\rangle}{\omega^2_{\mathbf{q}}} = \frac{m_{\mathbf{q}}}{\omega^2_{\mathbf{q}}}. \tag{67}$$

Essentially, the equation for static susceptibility (67) with expressions for $m_{\mathbf{q}}$ (54) and $\omega_{\mathbf{q}}$ (62) coincides with that one obtained in Ref.[12] and can be evaluated self-consistently from the one-particle GF.

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