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OPTICAL CONDUCTIVITY IN THE $t-J$ MODEL

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1 Introduction

The discovery of high-T_c superconductivity by Bednorz and Müller [1] in copper oxides has renewed interest to models with strong corelations, the Hubbard model [2] and its version in the limit of strong coupling, the $t-J$ model [3], since many unconventional properties of these materials are proved to be due to strong electron correlations. Among them linear temperature dependence of resistivity and anomalous frequency dependence of optical conductivity, the so-called midinfrared absorption, etc. (see, e.g.,[4, 5]).

Several rigorous results for the optical conductivity has been obtained for the one-dimensional Hubbard and $t-J$ models (see, e.g., [6] - [14]). By using the Bethe ansatz exact solution the charge stiffness, or the Drude weight D, was calculated near the Mott-Hubbard metal-insulator transition. However, to obtain the frequency dependence of the optical conductivity some approximations or numerical calculations for finite chains were used.

There have been so far only few analytical calculations of optical conductivity in two- and three-dimensional microscopical models with strong electron correlations. Among them are the early theoretical studies of the optical conductivity in the framework of Kubo linear response theory [15] done by the high-temperature expansion [16] and by the equation of motion method for the Green functions in the Hubbard I [17] and Hubbard III approximations [18, 19].

Later on the $t-J$ model was considered by Rice and Zhang [20] in the limit of small J when the hole motion has a diffusive character described by an incoherent spectrum for single particle excitations. They obtained a non-Drude conductivity, $\sigma(\omega) \propto 1/\omega$, in the high-frequency limit, $\omega \gg J$. A detailed discussion of optical and photoemission sum rules for one- and two-dimensional Hubbard models has been given by Eskes et al. [21] by comparing a strong coupling perturbation theory in powers of t/U with numerical calculations.

Recently the optical conductivity has been investigated within the dynamical mean-field approach which becomes exact in the limit of infinite dimensions [22, 23]. The symmetrical Hubbard model at half-filling has been considered near metal-insulator transition and a semiquantitative agreement with experiments has been observed. However, for the realistic two- or three-dimensional Hubbard model nonlocal (q - dependent) corrections to the transport vertices and the self-energy could be important.

Most extensively the optical conductivity for two-dimensional models of the CuO_2 plane has been studied by numerical methods based on exact diagonalization for small clusters within the framework of the the Hubbard or the $t-J$ models (see, e.g., [24] - [34] and the review paper [35]). For example, in recent papers [34] a universal behavior, $\sigma(\omega) \propto (1-\exp(-\omega/T)/\omega$, has been claimed for the $t-J$ model in the limit of high temperature, $T > 0.1t \simeq 500$ K.

However, numerical investigations for small clusters have a poor frequency resolution and pronounced finite-size effects in the region of low temperature and low frequency to be quantitatively compared with experimental results. Therefore, an analytical self-consistent investigations of the one-particle and optical spectral functions in the strong coupling limit for the realistic two-dimensional Hubbard and $t-J$ models are required.

In my previous paper [36] a frequency dependent conductivity $\sigma(\omega)$ has been calculated for the asymmetric Hubbard model [37] by applying the memory function technique [38] in terms of the Hubbard operators. The asymmetric Hubbard model is described by two nonequal hopping integrals $t_{\alpha\beta}$ for the lower Hubbard band (LHB: $\alpha = \beta = 1$) and the upper Hubbard band (UHB: $\alpha = \beta = 2$). The generalized Drude formula was obtained for the conductivity with frequency and temperature dependent relaxation rates due to electron scattering on charge and spin fluctuations. It has been shown that the interband transitions ($\propto t_{12}$) are essential for the Drude current relaxation. The latter is proportional to $[(t_{\alpha\alpha})^2-(t_{12})^2]^2$ and cancels out for the conventional Hubbard model, $(t_{\alpha\beta}=t)$, which results in the $\delta(\omega)$-type Drude term. However, for copper oxides which can be described by the effective asymmetric Hubbard model (see, e.g., [39]) a finite relaxation rate is observed. In this respect the results of [36] are a generalization of the optical conductivity calculations [40] for the $p-d$ model. Numerical estimations for the relaxation rate caused by spin-fluctuation scattering in [40] have shown that its temperature and frequency dependence agree quite well with experimental results in copper oxides.

In the present paper we consider the optical conductivity for the $t-J$ model by applying the memory function method in terms of the Hubbard operators. Contrary to the conventional Hubbard model we obtain

a finite relaxation rate in the $t-J$ model. It has two contributions: the first one due to the kinematical interaction in the LHB as in the Hubbard model considered in [36] and the second one due to the exchange interaction that, however, do not compensate each other. So we conclude that the Hubbard model in the limit of strong correlations is not equvalent to the $t-J$ model concerning their transport properties. It was also pointed out in several numerical studies for finite clusters (see, e.g. [28]).

The employment of the Hubbard operator technique has a twofold advantage. First of all by using equations of motion for the Hubbard operators we automatically take into account scattering of electrons on spin and charge fluctuations due to strong correlations as it has first been pointed out by Hubbard [41]. In the Fermi liquid models (see, e.g. [42] - [45]) one has to introduce a phenomenological spin fluctuation scattering mechanism to obtain nonzero relaxation. To study the transport properties in the auxiliary field representation (see, e.g., [46] and the references therein) or in the gauge field technique [47, 48] one has to adopt a spin-charge separation condition which has been rigorously proved only for the one-dimensional Hubbard model.

By employing the Hubbard operator representation we can also preserve rigorously restriction for no double occupancy for the LHB considered in the $t-J$ model. In the auxiliary field and the gauge field techniques this restriction has to be imposed by the local constraint for the total number of fermions and bosons. The latter can be allowed for only approximately as, e.g., in the $1/N$ expansion technique with N being the spin-orbital degeneracy (see, e.g., [46]). However, it is difficult to give an unambiguous physical interpretation of the obtained results for a realistic value of $N=2$.

The paper is organized as follows. In Section 2 we introduce the $t-J$ model by a unitary transformation of the asymmetric Hubbard model written in terms of the Hubbard operators. In Section 3 a general expression for the frequency dependent conductivity in terms of the memory function is obtained. The calculation of the relaxation rate for the optical conductivity is given in Section 4. In the last Section 5 model estimations for the relaxation rate and the conductivity are presented and the obtained results are summarized.

2 The $t-J$ model

We start our consideration from a two-band $p-d$ model reduced by a cell-perturbation method [39] to a singlet-hole asymmetric Hubbard model with the LHB occupied by one-hole Cu-d like states and the UHB occupied by two-hole $p-d$ singlet states [37]. In terms of the Hubbard operators the asymmetric Hubbard model reads:

$$H = H_0 + H_t = E_1 \sum_{i\sigma} X_i^{\sigma\sigma} + E_2 \sum_i X_i^{22}$$

$$- \sum_{i\neq j\sigma} \{ t_{ij}^{11} X_i^{\sigma 0} X_j^{0\sigma} + t_{ij}^{22} X_i^{2\sigma} X_j^{\sigma 2} + 2\sigma t_{ij}^{12} (X_i^{2\bar{\sigma}} X_j^{0\sigma} + X_i^{\sigma 0} X_j^{\bar{\sigma} 2}) \}. \quad (1)$$

Here $2\sigma = \pm 1$, $\bar{\sigma} = -\sigma$ and we introduce energy levels $E_1 = E_0 - \mu$ and $E_2 = 2E_0 - 2\mu + U$ for singly and doubly occupied sites, respectively, where E_0 is a reference energy and μ is the chemical potential. The Hubbard operators are defined by the equation:

$$X_i^{pq} = |i,p\rangle\langle i,q| \,, \quad X_i^{pq} X_i^{rs} = \delta_{qr}\, X_i^{ps} \quad (2)$$

for 4 possible states at a lattice site i:

$$|i,p\rangle = |i,0\rangle \,, \quad |i,\sigma\rangle \,, \quad |i,\uparrow\downarrow\rangle$$

for an empty site, a singly occupied site by electron with spin $\sigma = (\uparrow,\downarrow)$ and for a doubly-occupied site, respectively. For these states a completeness relation for the Hubbard operators (2) holds

$$X_i^{00} + \sum_\sigma X_i^{\sigma\sigma} + X_i^{22} = 1 \,. \quad (3)$$

The hopping integrals have different values for the LHB (t_{ij}^{11}), the UHB (t_{ij}^{22}) and the interband transitions (t_{ij}^{12}). In the singlet-hole model the Coulomb repulsion energy U in the standard Hubbard model is substituted by the charge transfer energy $\Delta = \epsilon_p - \epsilon_d$ between p- and d-levels in CuO_2 plane.

In the strong coupling limit, $\Delta \gg |t_{ij}^{\alpha\beta}|$, which holds for the model (1) (see [37]) we can apply perturbation theory and further reduce the Hubbard model (1) to the one-band $t-J$-like model for the LHB (see, e.g.,

[21]). By using the canonical transformation, $\tilde{H} = \exp(-S)H\exp(S)$, we obtain the following expression for the one-band $t-J$ model:

$$H = H_t + H_J = E_1 \sum_{i\sigma} X_i^{\sigma\sigma} - \sum_{i\neq j\sigma} t_{ij}^{11} X_i^{\sigma 0} X_j^{0\sigma} + (1/2) \sum_{i\neq j\sigma} J_{ij} \left(X_i^{\sigma\bar{\sigma}} X_j^{\bar{\sigma}\sigma} - X_i^{\sigma\sigma} X_j^{\bar{\sigma}\bar{\sigma}} \right) \quad (4)$$

where the exchange energy in the second order is equal to $J_{ij} = 2(t_{ij}^{12})^2/\Delta$ and we neglect contributions given by the operators X_i^{02} and three-site terms (see [21]). Usually in the $t-J$ model only the nearest neighbor hopping is considered, $t_{ij}^{11} = t$, that gives also exchange interaction for the nearest neighbor sites J. However, in the model (4) reduced from the effective two-band Hubbard model (1) these two parameters, t and J, are independent ones since $t^{11} \neq t^{12}$.

In the $t-J$ model (4) only the singly occupied band (LHB) is considered that gives $X_i^{22} = 0$ and the completeness relation for the Hubbard operators (3) reads:

$$X_i^{00} + \sum_{\sigma} X_i^{\sigma\sigma} = 1 \,. \quad (5)$$

It should be pointed out that a number of important properties of the Hubbard model (1) as, e.g., weight transfer from the UHB to the LHB and changes of the spectral functions with doping (see [37, 21]) are lost in the $t-J$ model. However, as claimed in many publications, the low energy physics should be the same in both models. To check this statement for the Drude relaxation rate in the optical conductivity which is essentially the low energy physics we consider in the present paper the $t-J$ model and compare the obtained results with those one for the two-band Hubbard model [36].

3 Memory function

In the linear response theory of Kubo [15] the frequency dependent conductivity is defined by the current-current correlation function

$$\sigma_{xx}(\omega) = \frac{1}{V} \int_0^{\infty} dt e^{\imath\omega t} (J_x(t), J_x) \quad (6)$$

where V is the volume of the system, $\Im\omega > 0$, and

$$(A(t), B) = \int_0^{\beta} d\lambda \langle A(t - \imath\lambda) B \rangle \quad (7)$$

is the Kubo – Mori scalar product for the operators in the Heisenberg representation, $A(t) = \exp(\imath Ht) A \exp(-\imath Ht)$, and $\langle AB \rangle$ denotes equilibrium statistical averaging for a system with the Hamiltonian H, $\beta = 1/T$ (here $\hbar = k_B = 1$).

The real, absorptive part of the conductivity (6) can be written in the form :

$$\Re\sigma_{xx}(\omega) = \frac{1 - \exp(-\beta\omega)}{V\omega} \Re \int_0^{\infty} dt e^{\imath\omega t} \langle J_x(t) J_x \rangle \,. \quad (8)$$

This fluctuation-dissipation equation is often used in numerical calculations (see, e.g., [28, 34]). It is also convenient for estimations of high-frequency conductivity [20]. However, to obtain an interpolation formula for the dynamical conductivity being valid from the hydrodynamical to the optical frequency region it is much more convenient to employ the memory function method as has been discussed by Götze et al. [38]. Later in a vast literature it has been proved that Götze et al. formulation is very efficient in calculations of dynamical conductivity. Below we shortly formulate the memory function approach in a slightly different form to overcome the problem of perturbation calculation of the memory function.

To calculate the conductivity (6) we will use the equation of motion method for the retarded two-time Green functions (GF) [49, 50] for the scalar product (7)

$$((A|B))_{\omega} = -\imath \int_0^{\infty} dt e^{\imath\omega t} (A(t)), B) \quad (9)$$

and for the commutator GF

$$\langle\langle A|B \rangle\rangle_{\omega} = -\imath \int_0^{\infty} dt e^{\imath\omega t} \langle [A(t), B] \rangle \quad (10)$$

where $\Im\omega > 0$ and the operators have zero average values: $\langle A \rangle = \langle B \rangle = 0$. The conventional dynamical susceptibility is given by

$$\chi_{AB}(\omega) = -\langle\langle A|B \rangle\rangle_{\omega}. \quad (11)$$

The GF (9), (10) are coupled by the equation

$$\omega((A|B))_\omega = \langle\langle A|B\rangle\rangle_\omega - \langle\langle A|B\rangle\rangle_{\omega=0}. \tag{12}$$

We have also the following useful relations:

$$((\imath\dot{A}|B))_\omega = ((A|-\imath\dot{B}))_\omega = \langle\langle A|B\rangle\rangle_\omega \tag{13}$$

$$(\imath\dot{A}, B) = (A, -\imath\dot{B}) = \langle[A, B]\rangle \tag{14}$$

where $\imath\dot{A} = \imath dA/dt = [A, H]$.

By using the above given definitions and writing the current operator as the time derivative of the polarization operator of the system, $J_x = \dot{P}_x$, we obtain the following equivalent representation for the optical conductivity (6)

$$\sigma(\omega) = \frac{\imath}{V}((J|J))_\omega = \frac{1}{V}\langle\langle P|J\rangle\rangle_\omega = \frac{\imath}{V\omega}[\chi_{JJ}(0) - \chi_{JJ}(\omega)] \tag{15}$$

where we have omitted the indexes for the operators J_x, P_x. By employing the standard dispersion relation [49] for the GF (10) or susceptibility (11) we readily get the sum rule for the real, or absorptive part of the conductivity (15):

$$\int_0^\infty d\omega \Re\sigma_{xx}(\omega) = \frac{1}{V}\int_0^\infty d\omega \frac{\Im\chi_{JJ}(\omega)}{\omega} = \frac{\pi}{2V}\Re\chi_{JJ}(0) = \frac{\imath\pi}{2V}\langle[J_x, P_x]\rangle. \tag{16}$$

The sum rules (16) has been extensively used by many authors to discuss the metal-insulator transitions in the Hubbard model (see, e.g., [6]-[13],[21, 26, 28, 30, 35]) since the **right-hand side of** (16) can be calculated from the static correlation functions.

To calculate the current-current correlation function for conductivity (15) it is convenient to employ the memory function approach of Mori [51] in the form slightly different from that one used by Götze et al. [38]. We define the memory function $M_{JJ}(\omega) \equiv M(\omega)$ by the equation

$$\Phi_{JJ}(\omega) = ((J|J))_\omega = \frac{\chi_0}{\omega + M(\omega)} \tag{17}$$

where $\chi_0 = \chi^0_{JJ}$ and

$$M(\omega \pm \imath\delta) = M'(\omega) \pm \imath M''(\omega).$$

Here $M'(\omega) = \Re M(\omega)$ and $M''(\omega) = \Im M(\omega)$ are real functions.

We calculate the memory function by using equation of motion for the GF

$$\Phi_{JJ}(t-t') = (\!(J(t); J(t'))\!).$$

By differentiating it in respect to time t and t' we readily get an equation for its Fourier transform (9):

$$\Phi(\omega) = \Phi_0(\omega) + \Phi_0(\omega)M_0(\omega)\Phi_0(\omega) \tag{18}$$

where

$$\Phi_0(\omega) = \frac{\chi_0}{\omega} \tag{19}$$

and the "scattering matrix"

$$M_0(\omega) = -(1/\chi_0)((F_x|F_x))_\omega(1/\chi_0) \tag{20}$$

is given by the correlation function for forces

$$F_x = \imath\dot{J}_x = [J_x, H]. \tag{21}$$

We have also used the relation of orthogonality for current and force:

$$(F_x, J_x) = (\imath\dot{J}_x, J_x) = \langle[J_x, J_x]\rangle = 0.$$

From eqs. (17), (18) we obtain the following relation for the memory function $M(\omega)$ and $M_0(\omega)$ (20):

$$M_0(\omega) = -[M(\omega)/\chi_0] - [M(\omega)/\chi_0]\Phi_0(\omega)M_0(\omega). \tag{22}$$

A formal solution of this equation by iteration shows that the memory function is just the irreducible part of the scattering matrix (20) which has no parts connected by single zero order GF $\Phi_0(\omega)$:

$$M(\omega) = ((F_x|F_x))^{(irred)}_\omega(1/\chi_0). \tag{23}$$

In the original formulation of the memory function approach in the calculation of the dynamical conductivity Götze et al. [38] have used a perturbation calculation for the memory function from eqs. (20), (22). However, in solving eq. (22) by perturbation expansion one should

be cautious since the solution is a non-analytical function in (ω, coupling constant) [52]. Evaluation of the memory function from the high-frequency expansion of the dynamical conductivity (15):

$$\sigma(\omega) \simeq (\imath\chi_0/V)(1/\omega - M(\omega)/\omega^2)$$

used in several papers (see, e.g. [6, 13]) can be also misleading since the frequency dependence of $M(\omega)$ may be nontrivial. Therefore our "exact" representation for the memory function in terms of the irreducible part of the force-force relaxation function, eq.(23), seems to be more convenient. Though, the exact meaning of the irreducibility is really given by Mori in his definition of the memory function in terms of the operators with the projected time evolution.

Now we can write the frequency dependent conductivity (15) by using the representation for the GF (17) in the form of the generalized Drude law:

$$\sigma(\omega) = \frac{\chi_0}{V} \quad \frac{m}{\tilde{m}(\omega)} \quad \frac{1}{\tilde{\Gamma}(\omega) - \imath\omega} \tag{24}$$

where the effective optical mass and the relaxation rate are given by

$$\frac{\tilde{m}(\omega)}{m} = 1 + \lambda(\omega), \quad \tilde{\Gamma}(\omega) = \frac{\Gamma(\omega)}{1 + \lambda(\omega)} \tag{25}$$

with

$$\lambda(\omega) = \frac{M'(\omega)}{\omega}, \quad \Gamma(\omega) = M''(\omega). \tag{26}$$

The real and imaginary parts of the memory function are coupled by the dispersion relation

$$M'(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} dz \quad \frac{M''(z)}{z - \omega}. \tag{27}$$

It is also convenient, by using the spectral representation for the GF, to write the relaxation rate given by eq.(26) in terms of the conventional time-dependent force-force correlation function:

$$\Gamma(\omega) = \frac{1 - \exp(\beta\omega)}{2\chi_0\omega} \int_{-\infty}^{\infty} dt e^{\imath\omega t} \langle F_x F_x(t) \rangle \tag{28}$$

where

$$\chi_0 = (J_x, J_x) = \imath\langle [J_x, P_x] \rangle \tag{29}$$

is the static susceptibility.

In the next Section we calculate the force-force correlation function and relaxation rate (28) for the $t - J$ model (4).

4 Relaxation rate

We start with a definition of the polarization operator for the $t-J$ model (4):

$$P_x = e \sum_i R_i^x N_i = e \sum_i R_i^x \sum_\sigma X_i^{\sigma\sigma} \tag{30}$$

where R_i^α are coordinates of electrons with charge e on a square lattice. From this definition the following expression for the current operator results

$$J_x = -\imath[P_x, H] = \imath e \sum_{i \neq j \sigma} (R_i^x - R_j^x)\, t_{ij}\, X_i^{\sigma 0} X_j^{0\sigma}. \tag{31}$$

By introducing the $\mathbf{q}$-representation for the Hubbard operators and the hopping integral

$$X_{\mathbf{q}}^{\alpha\beta} = \frac{1}{\sqrt{N}} \sum_i X_i^{\alpha\beta} e^{-\imath\mathbf{q}\mathbf{R}_i}, \quad t(\mathbf{q}) = \sum_{i \neq 0} t_{i0} e^{-\imath\mathbf{q}\mathbf{R}_i}, \tag{32}$$

the current operator (31) can be written as

$$J_x = e \sum_{q\sigma} v_x(q)\, X_q^{\sigma 0} X_q^{0\sigma} \tag{33}$$

where $v_x(q) = -\partial t(\mathbf{q})/\partial q_x$ is the electron velocity.

Now we calculate the force (21) for the current (33) which can be written as a sum of two terms:

$$F_x = F_x^t + F_x^J = [J_x, H_t] + [J_x, H_J]. \tag{34}$$

The first term comes from the kinematical interaction and has the following form:

$$F_x^t = -\imath e \sum_{i \neq j \neq l} \sum_{\sigma\sigma'} (R_i^x - R_j^x)(t_{ij}\, t_{jl}\, X_i^{\sigma 0} X_i^{0\sigma'} B_{j\sigma\sigma'}^l + \text{H. c.}). \tag{35}$$

The second term is proportional to the exchange interaction. It reads:

$$F_x^J = \imath e \sum_{i \neq j \neq l} \sum_{\sigma\sigma'} (R_i^x - R_j^x)(t_{ij}\, J_{jl}\, X_i^{\sigma 0} X_j^{0\sigma'} B_{l\sigma\sigma'}^J + \text{H. c.}). \tag{36}$$

Here the Bose-like operators have been introduced:

$$B_{j\sigma\sigma'}^t = (X_j^{00} + X_j^{\sigma\sigma})\delta_{\sigma'\sigma} + X_j^{\bar{\sigma}\sigma}\delta_{\sigma'\bar{\sigma}},$$

$$B^J_{l\sigma\sigma'} = -X^{\bar{\sigma}\bar{\sigma}}_l \delta_{\sigma'\sigma} + X^{\bar{\sigma}\sigma}_l \delta_{\sigma'\bar{\sigma}} = B^t_{l\sigma\sigma'} - \delta_{\sigma'\sigma} \, . \tag{37}$$

In the second line of this equation we have used the completeness relation, eq.(6), for the Hubbard operators. The Bose-like operators describe electron scattering on spin and charge fluctuations caused by the non-fermionic commutation relations (kinematical interaction) for the Hubbard operators and by the exchange spin-spin interaction. It can be demonstrated explicitly by using the following representation

$$-X^{\bar{\sigma}\bar{\sigma}}_j + X^{\bar{\sigma}\sigma}_j = -\frac{1}{2}\sum_\sigma X^{\sigma\sigma}_j + \frac{1}{2}(X^{\sigma\sigma}_j - X^{\bar{\sigma}\bar{\sigma}}_j) + X^{\bar{\sigma}\sigma}_j$$

$$= -\frac{1}{2}N_j + 2\sigma S^z_j + S^\sigma_j \tag{38}$$

where $S^z_j = \pm 1/2$ and $S^\sigma_j = S^\pm_j$ for $2\sigma = \pm 1$. From eq. (38) it follows that the operators (37) can be written in terms of the number N_j and spin S^α_j operators.

Now we can write the total force (34) in $\mathbf{q}$-representation as

$$F_x = -\frac{e}{\sqrt{N}}\sum_{k,q}\sum_{\sigma\sigma'} v_x(k)\,[t(k-q) - J(q)](X^{\sigma 0}_k X^{0\sigma'}_{k-q} B_{q\sigma\sigma'} + \text{H. c.}) \tag{39}$$

where we neglected the $\mathbf{q} = \mathbf{0}$ term in the functions (37) and introduce only one function $B_{q\sigma\sigma'} = B^t_{q\sigma\sigma'} = B^J_{q\sigma\sigma'}$ in the $\mathbf{q}$ representation

$$B_{q\sigma\sigma'} = \frac{1}{\sqrt{N}}\sum_i B_{i\sigma\sigma'} e^{-\imath \mathbf{q}\mathbf{R}_i} \, . \tag{40}$$

Now we can calculate the relaxation rate given by eq.(28):

$$\Gamma(\omega) = \frac{1 - \exp(\beta\omega)}{2\chi_0\omega}\int_{-\infty}^{\infty} dt e^{\imath\omega t}\langle F_x F_x(t)\rangle \, . \tag{41}$$

To calculate the many-particle time-dependent correlation functions in the right-hand side of eq. (41) we apply the mode-coupling approximation in terms of an independent propagation of electron-hole and charge-spin fluctuations. This approximation is essentially equivalent to the self-consistent Born approximation in which vertex corrections are neglected. The proposed approximation is defined by the following decoupling of the time-dependent correlation functions:

$$\langle X^{\sigma 0}_k X^{0\sigma'}_{k-q} B_{q\sigma\sigma'} | X^{s'0}_{k'-q'}(t) X^{0s}_{k'}(t)(B_{q'ss'}(t))^\dagger\rangle$$

$$\simeq \delta_{k,k'}\delta_{q,q'}\delta_{s,\sigma}\delta_{s',\sigma'}\langle X^{\sigma 0}_k X^{0\sigma}_k(t)\rangle\langle X^{0\sigma'}_{k-q} X^{\sigma' 0}_{k-q}(t)\rangle\langle B_{q\sigma\sigma'}(B_{q\sigma\sigma'}(t))^\dagger\rangle \, . \tag{42}$$

There are 4 correlation functions of the type given by eq.(42). However, by using the symmetry relations for the correlation functions in terms of the Bose-like operators (37) we can write the final result for the relaxation rate in a compact form:

$$\Gamma(\omega) = \frac{\exp(\beta\omega) - 1}{\chi_0\omega}\frac{2e^2}{N}\sum_{k,q} g^2_x(k, k-q)\int\int_{-\infty}^{\infty} d\omega_1 d\omega_2 n(\omega_1)[1 - n(\omega_2)]$$

$$N(\omega - \omega_1 + \omega_2)\chi''_{cs}(q, \omega - \omega_1 + \omega_2)A(k, \omega_1)A(k-q, \omega_2) \, , \tag{43}$$

where $n(\omega) = (\exp\beta\omega + 1)^{-1}$, $N(\omega) = (\exp\beta\omega - 1)^{-1}$ and the momentum dependent vertex is given by

$$g_x(k, k-q) = v_x(k)\,t(k-q) - v_x(k-q)\,t(k) + J(q)\,[v_x(k) - v_x(k-q)] \tag{44}$$

The charge-spin susceptibility $\chi''_{cs}(q,\omega) = \Im\chi_{cs}(q,\omega)$ is defined by the Fourier component of the retarded Green function (see [49]):

$$\chi_{cs}(q,\omega) = -\langle\langle\rho_{cs}(q)|\rho_{cs}(-q)\rangle\rangle_\omega = -\frac{1}{4}\langle\langle N_q|N_{-q}\rangle\rangle_\omega - \sum_\alpha\langle\langle S^\alpha_q|S^\alpha_{-q}\rangle\rangle_\omega \tag{45}$$

We introduced also the spectral function which does not depend on spin σ in the paramagnetic state:

$$A(q,\omega) = -\frac{1}{\pi}\Im\langle\langle X^{0\sigma}_q \mid X^{\sigma 0}_q\rangle\rangle_{\omega+\imath\delta} \, . \tag{46}$$

It defines the spectrum of electronic excitations by the one-electron (fully "dressed") Green function for the $t - J$ model.

To conclude this Section we calculate the static current-current susceptibility (29) in the denominator of eq.(43) that is also define the sum rule for the conductivity, eq.(16). By performing the commutation between the polarization operator (30) and the current (33) we readily get

$$\chi_0 = (J_x, J_x) = \imath\langle[J_x, P_x]\rangle = e^2\sum_{i\neq j,\sigma}(R^x_i - R^x_j)^2 t_{ij}\langle X^{\sigma 0}_i X^{0\sigma}_j\rangle \, . \tag{47}$$

For the model with only the nearest neighbors hopping, $(R^x_i - R^x_j)^2 = a^2_x$, the static susceptibility (47) is equal to the average kinetic energy, the hopping term H_t in eq.(4), multiplied by a constant:

$$\chi_0 = -e^2 a^2_x (1/2)\langle H_t\rangle \, . \tag{48}$$

The latter equation for the conventional Hubbard model (3) has been used by many authors to study the conductivity sum rule (16) (see, e.g., [6]- [14],[21, 26, 28, 30, 31]). In the **q**-representation the static susceptibility (47) reads:

$$\chi_0 = -e^2 \sum_{q,\sigma} \frac{\partial^2 t(q)}{\partial q_x^2} \langle X_q^{\sigma 0} X_q^{0\sigma} \rangle = \frac{Ne^2}{m_{eff}} \tag{49}$$

where the doping and temperature dependence of the effective mass m_{eff} is defined by the correlation function $\langle X_q^{\sigma 0} X_q^{0\sigma} \rangle$. The latter can be calculated by using the solution for the one-electron Green function for the $t-J$ model.

5 Results and Discussion

In the present paper we have derived a closed set of formulas for the frequency dependent conductivity for the $t-J$ model (4) in the form of the generalized Drude law (24) with the relaxation rate (43) depending on the kinematical and the exchange electron interactions (44) with spin and charge fluctuations. The static current-current susceptibility (49) defines the sum rule (16) and enter as a normalization factor in the definition of the relaxation rate (43).

In comparison with the relaxation rates for the two-band Hubbard model [36], where the kinematical interaction gives a contribution proportional to $[(t_{\alpha\alpha})^2 - (t_{12})^2]^2$ and cancels out for the conventional Hubbard model, $(t_{\alpha\beta} = t)$, in the present paper we observe a finite kinematical contribution in the $t-J$ model. It is defined by the first part of the vertex (44), $g_t(k, k-q) = v_x(k)\ t(k-q) - v_x(k-q)\ t(k) \propto t^4$. However, in the $t-J$ model we have an additional spin-exchange scattering, $g_J(k, k-q) = J(q)\ [v_x(k) - v_x(k-q)] \propto t^2 J^2 \propto t^6$ since $J \propto t_{12}^2/\Delta$ in (4). So we observe a nonequivalence of the two-band Hubbard model in the strong correlation limit to the $t-J$ model (even with allowing for the three-site terms omitted in (4) [21]). We can also suggest that higher order in t^2/Δ contributions in the conventional Hubbard model could give a final Drude relaxation rate depending on the spin-exchange scattering.

Now we estimate the relaxation rate (43) and the conductivity (24) by adopting some approximations for the one-electron spectral function (46)

and the charge spin susceptibility (45). As it has been proved in many calculations for the $t-J$ model (see, e.g. [35]), the spectral function for one-hole excitations can be written as a sum of a coherent contribution from the quasiparticle propagation in a narrow band of the order $2J$ and an incoherent part due to diffusive motion of holes in a broad band of the order $2W \simeq 8t$. So we can use the following approximation:

$$A(k,\omega) = Z_k \delta(\omega + \mu - \epsilon_k) + A_{inc}(k,\omega) \tag{50}$$

where Z_k is the quasiparticle weight for excitations with the dispersion ϵ_k. For the second, incoherent contribution we write:

$$A_{inc}(k,\omega) \simeq N_{inc}\ \theta(W - |\omega + \mu|)\ . \tag{51}$$

Here the incoherent density of states $N_{inc} \simeq (1 - Z_k)/2W$. The total density of states (50) obeys the sum rule

$$\frac{1}{N} \sum_k \int_{-\infty}^{\infty} d\omega A(k,\omega) = \langle X_i^{00} + X_i^{\sigma\sigma} \rangle = 1 - \frac{n}{2} \tag{52}$$

where $n = 2\langle X_i^{\sigma\sigma} \rangle$ is the occupation number. In this approximation we get estimations for the chemical potential $\mu/W \simeq (1 - 3\delta)/(1 + \delta)$ and for the static current-current susceptibility (49) $\chi_0/N \simeq e^2 a^2 W \delta$ with $\delta = 1 - n$.

By using eqs.(50), (51) we get the following expressions for the relaxation rate (43) depending on the coherent contribution

$$\Gamma_{coh}(\omega) = \frac{\exp(\beta\omega) - 1}{\chi_0 \omega} \frac{2e^2}{N} \sum_{k,q} g_x^2(k, k-q) Z_k\ Z_{k-q}$$

$$n(\epsilon_k)[1 - n(\epsilon_{k-q})] N(\omega - \epsilon_k + \epsilon_{k-q}) \chi''_{cs}(q, \omega - \epsilon k + \epsilon_{k-q}) \tag{53}$$

and the incoherent contribution

$$\Gamma_{inc}(\omega) = \frac{\exp(\beta\omega) - 1}{\chi_0 \omega} \frac{2e^2}{N} \sum_{k,q} g_x^2(k, k-q) \int_{-W-\mu}^{W-\mu} \int_{-W-\mu}^{W-\mu} d\omega_1 d\omega_2$$

$$N_{inc}^2 n(\omega_1)[1 - n(\omega_2)] N(\omega - \omega_1 + \omega_2) \chi''_{cs}(q, \omega - \omega_1 + \omega_2)\ . \tag{54}$$

The coherent part of the relaxation rate (53) has the conventional form for the Drude relaxation rate calculated in the Born approximation. However, the relaxation rate and the conductivity (24) have quite

a complicated temperature and doping dependence due to a very specific dependence of the one-electron spectra ϵ_k and spin susceptibility on that parameters. The formula (53) has been evaluated in [40] numerically for the $p-d$ model and for the Millis-Monien-Pines spin susceptibility model [53]. It has been shown that the relaxation rate (53) has a crossover from T^2 at $T \to 0$ to a linear temperature dependence in the static limit, $(\omega \to 0)$, and a crossover from ω^2 at $\omega \to 0$ to a linear frequency dependence for low enough temperatures in agreement with experiments for copper oxides.

Therefore, in the present paper we do not discuss the coherent part or the relaxation rate (53) and consider only the incoherent part (54). By using the model for the spin susceptibility suggested in the numerical calculations [34]

$$\chi_s''(q,\omega) \simeq \chi_s(q)\,\chi_s''(\omega) \simeq \chi_s(q)\,\tanh\frac{\omega}{2T}\,\frac{1}{1+(\omega/\omega_s)^2} \qquad (55)$$

we get after some algebra the following estimation for the incoherent relaxation rate:

$$\Gamma_{inc}(\omega) = \omega_s\,\Gamma(\nu,\tau)\,A = \omega_s\,\Gamma(\nu,\tau)\,\frac{2e^2}{\chi_0 N}\sum_{k,q} g_x^2(k,k-q)N_{inc}^2\chi_s(q) \quad (56)$$

where the dimensionless function for the frequency $\nu = \omega/2T$ and temperature $\tau = T/\omega_s$ is given by

$$\Gamma(\nu,\tau) = 2\tau\,\gamma(\nu,\tau)$$

$$= 2\tau\frac{\tanh\nu}{\nu}\int_0^{+\infty}\frac{dx}{(1+4\tau^2x^2)}\,\frac{1}{\cosh^2 x}\,\frac{\nu\tanh\nu - x\tanh x}{\tanh^2\nu - \tanh^2 x}. \qquad (57)$$

It is remarkable that for the incoherent spectrum (51) the $q-$ and frequency dependence is **factorized** as the product of the scaling function $\gamma(\nu,\tau)$ in (57) and the integral over scattering vectors, the constant A in (56). Therefore, the problem of hot spots on the Fermi surface discussed by Hlubina and Rice [54] is irrelevant for the incoherent scattering.

The frequency and temperature dependence of the relaxation rate $\Gamma(\nu,\tau)$ is shown **in Fig.1** for several temperatures $\tau = T/\omega_s$ where $\omega_s \simeq J \simeq 1500$ K.

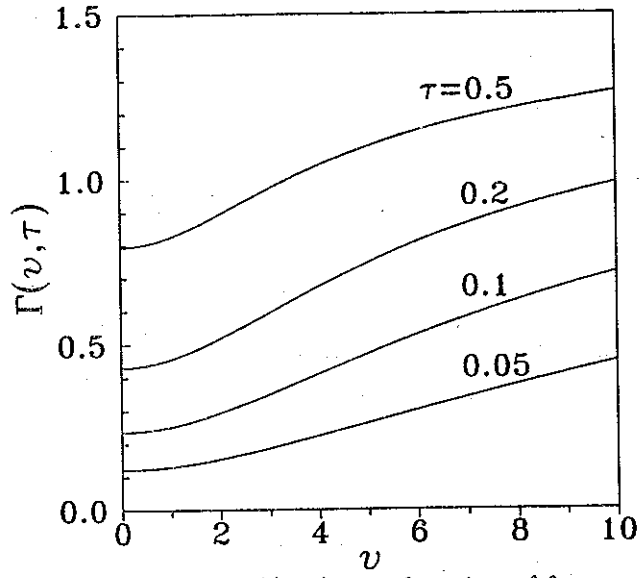

Fig.1. Relaxation rate $\Gamma(\nu,\tau)$ as a function of frequency $\nu = \omega/2T$ and temperature $\tau = T/\omega_s$

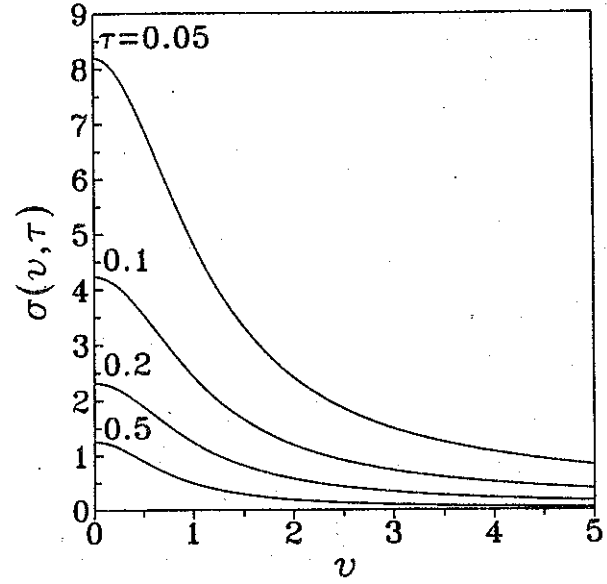

Fig.2. Dimensionless conductivity $\sigma(\nu,\tau)/(\chi_0/V\omega_s)$ as a function of frequency $\nu = \omega/2T$ and temperature $\tau = T/\omega_s$

A linear frequency dependence is observed for $\nu = \omega/2T \geq 1$ for low temperatures, $\tau \leq 0.2$ ($T \leq 300$ K). In the static limit, $\nu = 0$, we have a linear temperature dependence even in the low temperature limit, $T \to 0$, since

$$\Gamma(\nu = 0, \tau \to 0) \simeq \tau \int_0^{+\infty} \frac{x\, dx}{\sinh x} = \tau \frac{\pi^2}{2} . \qquad (58)$$

This low temperature linear behavior is due to the specific temperature dependence of the model spin susceptibility (55).

For the absorptive, real part of the conductivity, neglecting the optical mass renormalization, $\lambda(\omega) \simeq 1$, we obtain from (24), (56) the following representation for the incoherent scattering:

$$\sigma(\nu, \tau) = \frac{\chi_0}{V\omega_s} \frac{1}{2\tau} \frac{A\gamma(\nu, \tau)}{\nu^2 + (A\gamma(\nu, \tau))^2} . \qquad (59)$$

The frequency and temperature dependence of the dimensionless conductivity $\sigma(\nu, \tau)/(\chi_0/V\omega_s)$ for $A = 1$ in eq.(59) is shown in Fig.2 for several temperatures. Since $\Gamma(\nu, \tau) \propto \nu$ in the low temperature region (see Fig.1) the conductivity $\sigma(\nu, \tau) \propto 1/\nu$ in a wide frequency range including the midinfrared band. This universal $1/\omega$ behavior was observed by Rice et al. in the model with purely incoherent spectrum of holes [20] and also in numerical calculations for small clusters [34] and in the Hubbard model in the limit of infinite dimension [22, 23]. So we can confirm by our analytical calculations that midinfrared absorption or the universal $(1/\omega)$ dependence of the optical conductivity can be explained by the diffusive character of the hole motion in systems with strong correlations, as in the copper oxides. In the static limit, $(\nu \to 0)$, we obtain a linear temperature dependence for the resistivity, $\rho(\tau) = 1/\sigma(\tau)$, in the limit of low temperature which readily follows from (58).

The results presented in this section have been obtained for the model incoherent spectrum (51) and the model spin-fluctuation susceptibility (55). Both the coherent part in (50) in the form of the undamped quasiparticle spectrum and the incoherent part (51) in the form of the frequency independent contribution are really very crude approximations to one-hole ARPES spectra observed in copper oxides (see, e.g., [55]).

However, since relaxation rate (43) depends only on the averaged values of the spectral functions we can argue that a qualitative behaviour of the relaxation rate should not depend on the details of the former. Therefore, the main result of our analytical study – the non-Drude behaviour of the relaxation rate caused by the diffusive character of the hole motion in systems with strong correlations, seems to be justified. To improve our estimations that enables us to compare them with experiments one has to solve numerically the full self-consistent system of equations for the one-electron Green function for doped holes and for the spin susceptibility for the $t - J$ model. These very involved numerical calculations are planned to consider later.

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