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THE DEBYE — ONSAGER RELAXATION EFFECT IN FULLY IONIZED PLASMAS

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I. INTRODUCTION

One of the most interesting and enduring problem of plasma physics is the electrical conductivity. To calculate transport coefficients in nonideal, strongly coupled systems various methods have been elaborated [1]. In dense nonideal systems many-particle effects, like the Debye-Onsager relaxation effect, have to be taken into account and modify the known results of ideal systems. To receive a microscopic expressions for the electrical conductivity we use the generalized Zubarev method of linear response theory, which can be found in detail, e.g., in [2]. Using this approach transport coefficients are expressed through equilibrium correlation functions and a systematical treatment of many-particle effects is possible. The advantage of this method is, that known results of kinetic theory are reproduced without partial summations, as it is necessary in the Kubo approach.

We consider a fully ionized Hydrogen plasma consisting of a equal number of protons (mass M, charge $+e$) and electrons (mass m, charge $-e$). The Hamiltonian of the system

$$H_S = \sum_{c,k} E_c(k) a_c^+(k) a_c(k) + \frac{1}{2} \sum_{c,d,k,p,q} V_{cd}(q) a_c^+(k-q) a_d^+(p+q) a_d(p) a_c(k) \tag{1}$$

contains the kinetic energy $E_c(k) = \hbar^2 k^2/2m_c$ and the Coulomb interaction $V_{cd}(q) = e_c e_d/(\epsilon_0 \Omega q^2)$, where k denotes the single-particle variables momenta and spin, respectively. The indices c and d are used to identify the species (electron, proton), and Ω is the system volume under consideration. In the adiabatic case ($m \ll M$) the electrical conductivity σ is given by the linear relation between the mean electrical current $<\mathbf{j}>$ and the electrical field $\mathbf{E}$

$$<\mathbf{j}> = \left\langle \frac{1}{\Omega} \sum_k \frac{e}{m} \hbar \mathbf{k} \right\rangle = \sigma \mathbf{E}, \tag{2}$$

and is both a function of temperature and density of the system, $\sigma = \sigma(n, T)$. In plasma physics it is of use to introduce the dimensionless parameters

$$\Gamma = \frac{e^2}{4\pi\epsilon_0 k_B T} (4\pi n/3)^{1/3},$$
$$\Theta = \frac{2 m k_B T}{\hbar^2} (3\pi^2 n)^{-2/3}. \tag{3}$$

$\Gamma(n,T)$ describes the ratio between the mean potential energy and the kinetic energy and $\Theta(n,T)$ denotes the degree of degeneration of an ideal electron gas. Using these quantities the electrical conductivity can be read as

$$\sigma(n,T) = \frac{(k_B T)^{3/2}(4\pi\epsilon_0)^2}{m^{1/2}e^2}\sigma^*, \tag{4}$$

with a universal function $\sigma^*(\Gamma,\Theta)$ depending on the characteristic plasma parameters. The electrical conductivity of a fully ionized plasma with statically screened interaction is given by the Spitzer formula [3]

$$\sigma^*_{Sp} = 0.591\left[\frac{1}{2}\ln\left(\frac{3}{2}\Gamma^{-3}\right)\right]^{-1}, \tag{5}$$

which is valid in the low-density limit ($\Gamma \ll 1$). To improve this equation we have to include higher orders terms of the density expansion. A virial expansion for the electrical conductivity was given in [4]

$$\sigma^{-1}(n,T) = A(T)\ln n + B(T) + C(T)n^{1/2}\ln n + ..., \tag{6}$$

where $C(T)$ is related to the Debye-Onsager relaxation effect, which is known from the theory of electrolytes [5], see [6]. It describes the compensation of the electrical field, acting on a charged particle, caused by the formation of an asymmetric screening cloud. Klimontovich and Ebeling [7] investigated this effect in weakly ionized plasmas, where the assumption of a local Maxwellian distribution is well founded. It was shown, that the relaxation effect can only be described in a correct way, if the description contains two-particle correlations. In contrast to the Onsager result, contributions to the order $n^{1/2}\ln n$ are obtained in the kinetic theory [8,9], where only a pure one-particle picture is used. Hence, in the set of relevant observables the pair-distribution function has to be included. Within a quantum statistical approach, the relaxation effect was also investigated for fully ionized plasmas [4]. To arrive at the Onsager result only the lowest moments of relevant observables (one-particle occupation number, pair-distribution function) were used. This corresponds to a local Maxwellian distribution for the momenta. Whereas this assumption is well founded for a weakly ionized

plasma because of the high collision frequency between the charged and neutral particles, the distribution for the momenta in a fully ionized plasma can be different. As known from both the kinetic theory and from linear response theory, the inclusion of higher moments can modify results for the transport coefficients. To obtain a rigourous result for the virial coefficient $C(T)$ we want to include higher moments of the distribution functions, what is correspondent to distribution functions, which are not restricted to have a Maxwellian form.

II. LINEAR RESPONSE THEORY

We consider an open system, described by the system hamilton operator H_S (1) under the influence of a statical electrical field $\mathbf{E} = E\hat{\mathbf{e}}_z$. For the total Hamiltonian we write

$$H_{tot} = H_S - ER = H_S - E\sum_{c,i} e_c r^c_{i,z}. \tag{7}$$

Linear response theory starts with a modified equation of motion for the statistical operator of nonequilibrium ρ,

$$\frac{\partial\rho}{\partial t} - \frac{1}{i\hbar}[H_{tot},\rho] = -\epsilon(\rho - \rho_{rel}) \tag{8}$$

where the correct boundary condition, the weakening of initial correlations, is included in form of a source term. If we identify the relevant operator with the statistical operator of equilibrium we get the Kubo approach of linear response theory. The state of the system in the presence of the applied field is characterized through mean values of relevant observables $< B_\nu >$. These mean values are used to derive a generalized Gibbs state. In this form of linear response theory [10] we use a relevant operator of the quasi-equilibrium state

$$\rho_{rel} = \exp\left[-\sum_\nu \Phi_\nu B_\nu\right] / \mathrm{Tr}\exp\left[-\sum_\nu \Phi_\nu B_\nu\right], \tag{9}$$

following from the principle of maximizing the information entropy subject to constraints of given mean values $\mathrm{Tr}(\rho_{rel}B_\nu) = < B_\nu >$. These mean values fix the response parameters Φ_ν. Explicit expressions for the response parameters can be derived from the response equations (generalized Boltzmann equation)

$$EN[B_\nu] = \sum_{\nu'} \Phi_{\nu'} D[B_{\nu'}, B_\nu] \tag{10}$$

with the drift term

$$N[B_\nu] = (\dot{R}; B_\nu) + < \dot{R}(\epsilon); \dot{B}_\nu > \tag{11}$$

and the collision term

$$D[B_{\nu'}, B_\nu] = (\dot{B}_{\nu'}; B_\nu) + < \dot{B}_{\nu'}(\epsilon); \dot{B}_\nu > . \tag{12}$$

In the response equation (10) we have defined the correlation functions

$$< A(\epsilon); B > = \int_{-\infty}^{0} dt e^{\epsilon t} (A(t); B),$$
$$(A; B) = \frac{1}{\beta} \int_0^\beta d\tau \mathrm{Tr}[\rho_0 A(-i\hbar\tau) B], \tag{13}$$

where we take the observables in the Heisenberg picture, $\rho_0 = Z_0^{-1} \exp[-\beta(H_S - \sum_c \mu_c N_c)]$ denotes the statistical operator of equilibrium, and the limit $\epsilon \to +0$ has to be taken after the thermodynamic limit.

In order to determine transport coefficients we have to solve the two problems:

i) Using the methods of quantum statistics (thermodynamic Green functions) we have to calculate the correlation functions. By this way it is possible to include many-particle effects in a consequent way.

ii) Solving the generalized Boltzmann equation (10) we can determine the response parameter Φ_ν as a function of the electrical field.

If we have the response parameter Φ_ν at our disposal we can calculate the mean value of the current operator (2) with

$$< \mathbf{j} > = \beta \sum_\nu \Phi_\nu (B_\nu; \mathbf{j}). \tag{14}$$

After the evaluation of the response parameters in solving the response equations (10), the conductivity σ is expressed in terms of determinants using Cramer's rule.

III. CHOICE OF THE SET OF RELEVANT OBSERVABLES

For small densities it is sufficient to take as a relevant observable the single-particle occupation number $B_\nu = a_c^+(k)a_c(k) - < a_c^+(k)a_c(k) >_0$, which characterize the nonequilibrium single particle distribution function f_k in the homogeneous case. In this case, the response equations (10) are equivalent to the Boltzmann equation for the single particle distribution function, where the collision integral is expressed in terms of correlation functions. The ordinary Boltzmann collision integral is obtained by evaluating the correlation function in the ladder approximation (binary collisions) what is correct in the lowest order of density. Many particle effects beyond the T-matrix approximation contribute to higher orders in the density expansion, see [4]. In particular it can be shown that no contribution to the conductivity in the order $n^{1/2} \ln n$ arises.

In order to obtain the correct expression for the conductivity in the order $n^{1/2} \ln n$ with respect to the density we have to include the two-particle distribution function. This means in the homogeneous case that we have to enlarge the set of relevant observables B_ν by inclusion of the two-particle observables

$$\delta n_{cd}(pkq) = a_c^\dagger(k - q/2) a_d^\dagger(p + q/2) a_d(p - q/2) a_c(k + q/2). \tag{15}$$

Hence, the response equations (10) have the form of a coupled system of equations for the single and two-particle distribution function.

As already discussed for the single-particle distribution function, for the leading order in the density it is sufficient to evaluate the correlation functions arising in connection with the two-particle distribution function in Born approximation, where the Coulomb interaction is replaced by the statically screened Debye potential. A more detailed treatment of the collision term (dynamical screening, ladder summations) would result in corrections which are of higher order in the density. Because we are interested only in the coefficient $C(T)$ of the virial expansion of the conductivity (6) it is sufficient to consider all correlation functions in Born approximation, since the single particle distribution does not contribute in this order.

The full solution of the response equations (10) for the single- and two-particle distribution function is equivalent to the solution of a coupled system of integral equations if we consider the 'indices' k and kpq, respectively, as continuous variables. We will find an approximative solution of the response equations by considering only a finite number of moments of the distribution functions. In particular, instead of solving the response equations for the single-particle distribution we consider a finite number of moments

$$P_n = \sum_k \hbar k_z \left[\beta \frac{\hbar^2 k^2}{2m_c}\right]^{(n/2)} a_c^+(k) a_c(k). \tag{16}$$

For a sufficient high number of moments, the single-particle distribution function is approximated well, and the conductivity is obtained by the algebraic solution of the system of response equations. This procedure is identical with the Grad or the Chapman-Enskog approch for solving the linearized Boltzmann equation. As well-known from the Kohler variational principle, the inclusion of higher moments will improve the result for the conductivity (increase). It has been shown [4] that the inclusion of a finite number of low order moments gives a converging expression for the conductivity in the lowest order of density (term A in the virial expansion, eq.(6)).

The same argument can be applied for the two-particle distribution function. Instead of treating integral equations we introduce moments of the two-particle distribution according to

$$\delta n_{c,d}^{m,m'}(q) = \sum_{k,p} \mathbf{k}^m \mathbf{p}^{m'} a_c^\dagger(k - q/2) a_d^\dagger(p + q/2) a_d(p - q/2) a_c(k + q/2). \tag{17}$$

These moments refer to the momentum distributions of both particles. Since the dependence of q is taken in form of a variable, we are able to introduce the pair distribution function. In particular, the pair correlation function is given by

$$n_{c,d}(r, r') - n_c^0 n_d^0 = \int dq e^{iq(r-r')} \delta n_{c,d}^{0,0}(q). \tag{18}$$

Higher moments are related to correlations of currents at different positions etc.

In the following, we will proceed as follows: In a first step, we only consider the moment $\delta n_{c,d}^{0,0}(q)$ in combination with arbitrary moments P_n of the single-particle distribution. This will improve the coefficient A as well as C, where the most important contribution comes from P_2. In a second step we will confine us to P_0, P_2 and will improve the two-particle distribution by including the second moments.

IV. VARIATION OF THE SINGLE-PARTICLE DISTRIBUTION FUNCTION

Following the discussion in the previous section, the set of relevant observables has been enlarged by the moments of the single-particle distribution function P_n for the electrons and the pair-distribution function

$$\delta n_{cd}^{0,0}(q) = \sum_{p,k} a_c^\dagger(k - q/2) a_d^\dagger(p + q/2) a_d(p - q/2) a_c(k + q/2), \tag{19}$$

for the electron-proton and the electron-electron density. According to eq.(10), the system of coupled response equations is given by

$$EN[P_n] = \sum_{n'} \gamma_{n'} D[P_{n'}, P_n] + \sum_{c,d} \sum_{q \neq 0} \Phi_{cd}^{0,0}(-q) D[\delta n_{cd}^{0,0}(q), P_n] \tag{20}$$

$$EN[\delta n_{cd}^{0,0}(q)] = \sum_n \gamma_n D[P_n, \delta n_{cd}^{0,0}(q)] + \sum_{c',d'} \sum_{q' \neq 0} \Phi_{c'd'}^{0,0}(-q') D[\delta n_{c'd'}^{0,0}(q'), \delta n_{cd}^{0,0}(q)]. \tag{21}$$

This system of response equations serves for the determination of the response parameters $\{\gamma_n, \Phi_{cd}^{0,0}(-q)\}$. The relaxation effect is described using a pair-distribution function, which, as a consequence of the applied electrical field, has an asymmetric form. The deviation of the two-particle density from the equilibrium state is given from the response parameter $\Phi_{cd}^{0,0}(q)$. The most general form of this function can be read as

$$\Phi_{cd}^{0,0}(q) = \mathbf{E}\mathbf{q} f(|\mathbf{q}|), \tag{22}$$

where we have introduced a function $f(|\mathbf{q}|)$, which only depends on the distance between the particles of species c and d. This leads us to the symmetry relation

$$\Phi_{cd}^{0,0}(q) = -\Phi_{cd}^{0,0}(-q) = -\Phi_{dc}^{0,0}(q). \tag{23}$$

which should be reobtained from the calculations. We conlude, therefore, that the response parameter for particles of the same species $\Phi_{cc}^{0,0}(q)$ is zero, and the corresponding electron-electron pair distribution function gives no contribution to the relevant statistical operator.

Since we are only interested in the first correction term for the the virial expansion of the electrical conductivity, the correlation functions are evaluated in the Born approximation, and by using a statically screended Debye potential $V_S(q) = e^2/(\epsilon_0\Omega(q^2+\kappa^2))$. In order to solve the first response equation we us the results from the literature [4], which are given in Born approximation by

$$N[P_n] = (\dot{R}; P_n) = -\frac{eN}{\beta}\frac{\Gamma(\frac{5+n}{2})}{\Gamma(\frac{5}{2})} \tag{24}$$

and $D[P_{n'}, P_n] =< \dot{P}_{n'}(\epsilon); \dot{P}_n >$. The contribution of the lowest moment yields

$$< \dot{P}_0(\epsilon); \dot{P}_0 >= \frac{4}{3}(2\pi)^{-1/2}N^2\frac{1}{\Omega}m^{1/2}\beta^{3/2}\frac{e^4}{(4\pi\epsilon_0)^2}L_{BH} \tag{25}$$

where $L_{BH} = \frac{1}{2}\int_0^\infty x(x+\kappa^2)^{-2}\exp(-\beta\hbar^2 x/8m)dx$ denotes the Brooks-Herring type Coulomb logarithm. For higher order moments, also the electron-electron interaction contributes to $< \dot{P}_{n'}(\epsilon); \dot{P}_n >$, and we obtain

$$\begin{aligned} a_{n'n} &= < \dot{P}_{2n'}(\epsilon); \dot{P}_{2n} > / < \dot{P}_0(\epsilon); \dot{P}_0 >, \\ a_{10} &= a_{01} = 1, \quad a_{20} = a_{02} = 2, \quad a_{11} = 2+\sqrt{2}, \\ a_{21} &= a_{12} = 6 + 11/\sqrt{2}, \quad a_{22} = 24 + 157/\sqrt{8}. \end{aligned} \tag{26}$$

As a new term, a correlation function appears between the moments of the single-particle distribution and the pair-distribution function, $D[\delta n^{0,0}_{ep}(q), P_n] = \left(\delta\dot{n}^{0,0}_{ep}(q); P_n\right)$. This correlation function can be evaluated in the non-degenarate case and we get the result (see the Appendix)

$$\left(\delta\dot{n}^{0,0}_{ep}(q), P_m\right) = -\frac{\Gamma(\frac{5+m}{2})}{\Gamma(\frac{5}{2})}iq_z N^2\frac{e^2}{\epsilon_0\Omega(q^2+\kappa^2)} = -\frac{\Gamma(\frac{5+m}{2})}{\beta\Gamma(\frac{5}{2})}iq_z n_2(q). \tag{27}$$

The second response equation (21) is transformed via a partial integration in $N[\delta n^{0,0}_{ep}(q)]$ and $D[\delta n^{0,0}_{ep}(q'), \delta n^{0,0}_{ep}(q)]$. By this way, the dependence on the the single-particle moments is eliminated, and we have to solve

$$E\left(\dot{R}; \delta\dot{n}^{0,0}_{ep}(q)\right) = \sum_{q'\neq 0}\Phi^{0,0}_{ep}(-q')\left(\delta\dot{n}^{0,0}_{ep}(q'); \delta\dot{n}^{0,0}_{ep}(q)\right). \tag{28}$$

This serves for the determination of the response parameter $\Phi^{0,0}_{ep}(-q)$. The l.h.s. is calculated using the result (27) and the correlation function $\left(\delta\dot{n}^{0,0}_{ep}(q'); \delta\dot{n}^{0,0}_{ep}(q)\right)$ is transformed using the idendity $(\dot{A}; B) = \frac{i}{\hbar\beta} < [B, A] >_0$. The result is

$$\left(\delta\dot{n}^{0,0}_{ep}(q'); \delta\dot{n}^{0,0}_{ep}(q)\right) = -\frac{1}{\beta}\mathbf{q}\mathbf{q}'\delta_{q,-q'}\frac{1}{m}[N^2 - Nn_2(q)], \tag{29}$$

giving us the response parameter

$$\Phi^{0,0}_{ep}(q) = ieEq_z q^{-2}\frac{\beta}{\Omega}\frac{e^2}{\epsilon_0}\frac{1}{(q^2+\kappa^2/2)} \tag{30}$$

in explicit form. If we exchange the indices for the species in equation (28) we will find, that the response parameter $\Phi^{0,0}_{ep}(q)$ is indeed subjected to the supposed symmetry relation of eq.(23). Having the response parameter at our disposal, the set of coupled response equations is reduced to an algebraic system of equations for the determination of the reponse parameters γ_n. Using Cramer's rule, the electrical conductivity is related to the ratio of two determinants

$$\sigma = \frac{\beta}{\Omega}\frac{\begin{vmatrix} 0 & (\dot{R}; P_n) - \sum_{q\neq 0}\Phi_{ep}(-q)\left(\delta\dot{n}^{0,0}_{ep}(q); P_n\right) \\ (\dot{R}; P_{n'}) - \sum_{q\neq 0}\Phi_{ep}(-q)\left(\delta\dot{n}^{0,0}_{ep}(q); P_{n'}\right) & < \dot{P}_{n'}(\epsilon); \dot{P}_n > \end{vmatrix}}{|< \dot{P}_{n'}(\epsilon); \dot{P}_n >|}. \tag{31}$$

Taking the first three moments of the single-particle distribution function we obtain for the conductivity

$$\sigma^* = 0.583L^{-1}_{BH}\left[1 - \frac{1}{3(2+\sqrt{2})}\frac{\beta e^2\kappa}{4\pi\epsilon_0}\right]. \tag{32}$$

As discussed in [4], the first term is related to the Spitzer formula (5), and the correction factor coincides with the Onsager result. Obviously, it is sufficient to assume a local Maxwellian distribution (hydrodynamic approximation) for the single-particle distribution function in order to describe the relaxation effect. The used single-particle moments improve the kinetic part of the transport coefficient, but does not change the hydrodynamic contribution to the electrical conductivity.

V. VARIATION OF THE PAIR-DISTRIBUTION FUNCTION

To continue our discussion, we want to investigate, to what extent higher moments of the pair-distribution function influence the Debye-Onsager correction factor. The inclusion of the second moment of the single-particle distribution (the energy current) yields the main contribution to the improvement of the coefficients A and C in the virial expansion of the electrical conductivity. Therefore we will focus on the second moments of the pair distribution function. The set of relevant observables now consists of the variables $P_0, P_2, \delta n_{ep}^{0,0}(q)$ and $\delta n_{ep}^{m,n}(q)$. There exist several ways to construct these higher moments $\delta n_{ep}^{m,n}(q)$, in particular they can depend on 6 variables (momenta, scalar products). Since we were able to reproduce the Onsager result in the adiabatic case ($m \ll M$), we will neglect the momenta of the protons, which reduces the dependence on three variables. Using these we can then define the moments of the pair distribution function up to the second order, writing

$$\delta n_{ep}^{m,n}(q) = \sum_{l,h} B \; a_{l-q/2}^{+} a_{h+q/2}^{+} a_{h-q/2} a_{l+q/2} \tag{33}$$

with $B = \{1, \mathbf{lq}, l^2, 1, 1(\mathbf{lq})\}$. It has been shown by Kalashnikov [13], that if we include in the set of relevant observables the moment $\delta n_{ep}^{0,0}(q)$, it is not necessary to consider $\delta \dot{n}_{ep}^{0,0}(q)$, as it would not change the results for the transport coefficients. By this way we find, that we can restrict us to $\delta n_{ep}^{0,0}(q)$ and

$$\begin{aligned} \delta n_{ep}^{1,0}(q) &= \sum_{l,h} \mathbf{l} \; a_{l-q/2}^{+} a_{h+q/2}^{+} a_{h-q/2} a_{l+q/2} \\ \delta n_{ep}^{2,0}(q) &= \sum_{l,h} l^2 a_{l-q/2}^{+} a_{h+q/2}^{+} a_{h-q/2} a_{l+q/2} \end{aligned} \tag{34}$$

for the moments of the pair distribution function. Using this set of observables, the relevant statistical operator reads

$$\rho_{rel} = Z_{rel}^{-1} \exp\left[-\beta\left(H_S - \sum_c \mu_c N_c + \sum_n \gamma_n P_n + \sum_{m=0}^{2} \sum_{q\neq 0} \Phi_{ep}^{m,0}(-q)\, \delta n_{ep}^{m,0}(q)\right)\right]. \tag{35}$$

The related response equations (10), i.e., the coupled equations for the moments of the single-particle and the two-particle distribution function are given by

$$EN[P_n] = \sum_{n'} \gamma_{n'} D[P_{n'}, P_n] + \sum_{m=0}^{2} \sum_{q\neq 0} \Phi_{ep}^{m,0}(-q) D[\delta n_{ep}^{m,0}(q), P_n] \tag{36}$$

$$EN[\delta n_{ep}^{m,0}(q)] = \sum_{n} \gamma_n D[P_n, \delta n_{ep}^{m,0}(q)] + \sum_{m'=0}^{2} \sum_{q'\neq 0} \Phi_{ep}^{m',0}(-q') D[\delta n_{ep}^{m',0}(q'); \delta n_{ep}^{m,0}(q)] \tag{37}$$

and serve for the determination of the response parameter $\{\gamma_n, \Phi_{ep}^{m,0}(q)\}$. We start by investigating eq.(37) for $\delta n_{ep}^{1,0}(q)$. In particular, we have to calculate both the collision terms $< \delta \dot{n}_{ep}^{1,0}(q', \epsilon), \delta \dot{n}_{ep}^{1,0}(q) >$ and $< \delta \dot{n}_{ep}^{1,0}(q, \epsilon), \dot{P}_n >$. It can be shown that these terms are of higher order in the density. As a consequence of different tensor character the moment $\delta n_{ep}^{1,0}(q)$ does not couple with $\delta n_{ep}^{0,0}(q)$ and $\delta n_{ep}^{2,0}(q)$. Partial integrations in (37), as already discussed for the derivation of equation (28), leads to

$$EN[\delta \dot{n}_{ep}^{m,0}(q)] = \sum_{m'=0,2} \sum_{q'\neq 0} \Phi_{ep}^{m',0}(-q') D[\delta \dot{n}_{ep}^{m',0}(q'), \delta \dot{n}_{ep}^{m,0}(q)]. \tag{38}$$

Considering the drift term $N[\delta \dot{n}_{ep}^{m,0}(q)]$, we find that $< \dot{R}(\epsilon); \delta \ddot{n}_{ep}^{m,0}(q) >$ vanishes in Born approximation and we get the result

$$N[\delta \dot{n}_{ep}^{m,0}(q)] = -\frac{e}{m}\left(P; \delta \dot{n}_{ep}^{m,0}(q)\right) = i q_z \frac{e}{m} \frac{2\Gamma(\frac{3+m}{2})}{\beta\sqrt{\pi}} n_2(q), \tag{39}$$

which can be found using (27). Furthermore we have to evaluate the collision term $D[\delta \dot{n}_{ep}^{m',0}(q'); \delta \dot{n}_{ep}^{m,0}(q)]$. The correlation functions $< \delta \dot{n}_{ep}^{m',0}(q'); \delta \ddot{n}_{ep}^{m,0}(q) >$ are of order $O(q^3/\epsilon)$ and can therefore be omitted in the $q \to 0$ limit. Consequently, the contributions to the collision terms are given by

$$\begin{aligned} D[\delta \dot{n}_{ep}^{m',0}(q'); \delta \dot{n}_{ep}^{m,0}(q)] &= D_{m',m} = \left(\delta \dot{n}_{ep}^{m',0}(q'); \delta \dot{n}_{ep}^{m,0}(q)\right) \\ D_{2,0} = D_{0,2} = \frac{5}{2} D_{0,0} &\quad D_{2,2} = \frac{35}{4} D_{0,0}, \end{aligned} \tag{40}$$

see the equation (29). Again, the system of equations for the determination of the functions $\Phi_{ep}^{0,0}(q)$ und $\Phi_{ep}^{2,0}(q)$ is reduced to an algebraic system of equations and is given by

$$\begin{aligned} ieEq_z n_2(q) &= q^2\left[N^2 - N n_2(q)\right]\left(\Phi_1(q) + \frac{5}{2}\Phi_2(q)\right) \\ \frac{3}{2} ieEq_z n_2(q) &= q^2[N^2 - N n_2(q)]\left(\frac{5}{2}\Phi_1(q) + \frac{35}{4}\Phi_2(q)\right), \end{aligned} \tag{41}$$

which gives us the response parameters in explicit form. From eq.(41) we find

$$\Phi_{ep}^{0,0}(q) = 2\Phi_0(q) \quad \Phi_{ep}^{2,0}(q) = -\frac{2}{5}\Phi_0(q)$$
$$\Phi_0(q) = ieEq_z q^{-2}\frac{\beta}{\Omega}\frac{e^2}{\epsilon_0(q^2+\kappa^2/2)}. \tag{42}$$

The correlation functions in the response equation (36) are known from the discussion in Section IV. The new term is $D[\delta n_{ep}^{m,0}(q), P_n]$. Similar to the calculations in the Appendix we find

$$D[\delta n_{ep}^{m,0}(q), P_n] = \left(\delta \dot{n}_{ep}^{m,0}(q), P_n\right) = 2\left(\frac{3+n}{3}\right)\frac{\Gamma(\frac{3+m+n}{2})}{\sqrt{\pi}} i q_z \frac{e^2}{\epsilon_0\Omega(q^2+\kappa^2)}. \tag{43}$$

By using the results for the response parameters $\Phi_{ep}^{m,0}(q)$, we can solve the equations for the determination of γ_n. Having all the response parameters at our disposal we calculate the conductivity applying the scheme outlined in the previous section. The expression for the conductivity

$$\sigma^* = 0.578 L_{BH}^{-1}\left[1 - \frac{1}{3(2+\sqrt{2})}(1+0.078)\frac{\beta e^2\kappa}{4\pi\epsilon_0}\right] \tag{44}$$

contains a corrected Onsager coefficient. Hence, by inclusion of higher moments of the pair distribution function, we improve the Onsager correction factor. In difference to the result of eq.(32) we obtain another prefactor caused by the inclusion of only the second moment of the single-particle distribution function. We will reobtain the prefactor of eq.(32) by considering also the third moment.

VI. DISCUSSION

The work we have described allows a systematic treatment of the Debye-Onsager relaxation effect in fully ionized plasmas. Now we review on the principle ideas we have used and discuss them.

Our approach has been to consider higher moments of the relevant observables which were used to derive the Debye-Onsager correction factor in the virial expansion for the electrical conductivity. In order to derive this term it is sufficient to make a hydrodynamic

approximation for both the single-particle and the two-particle distribution function [7]. The inclusion of higher moments of the single-particle occupation number is related to distributions which are not restricted to have a local Maxwellian form. These moments yield a contribution to the coefficients $A(T)$ and $C(T)$. By this way we derived an improved expression in the kinetic dependence for the conductivity (prefactor in (32)). The hydrodynamic part is unchanged, and the calculations leads to the result, that the hydrodynamic approach for the single-particle distribution function in fully ionized plasmas is justified in order to describe the relaxation effect properly.

The inclusion of higher moments of the pair-distribution function is related to fluctuations of the density at different positions and improves the hydrodynamic approximation for the two-particle density. Considering the first two moments of both the single-particle distribution function and the pair-distribution function we improve the correction factor $C(T)$. Hence, the Onsager result has to be corrected in the case of a fully ionized plasma. By using the approach of linear response we were able to find corrections beyond the hydrodynamic approximation. Applying the Kohler variational principle we improve the contribution of the two-particle distribution function by the inclusion of higher moments.

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VII. APPENDIX

By using the idendity $(\dot{A}; B) = \frac{i}{\hbar\beta} < [B, A] >_0$ we find for the coorelation function (27)

$$\begin{aligned}\left(\delta \dot{n}_{ep}^{0,0}(q); P_n\right) &= -\frac{i}{\hbar\beta} < [\delta n_{ep}^{0,0}(q), P_n] > \\ &= -\frac{i}{\beta}\left[\frac{\beta\hbar^2}{2m}\right]^{n/2}\left(\frac{3+n}{3}\right)\sum_{l,h} q_z l^n < a^+_{l-q/2} a^+_{h+q/2} a_{h-q/2} a_{l+q/2} > .\end{aligned} \tag{45}$$

The correlation function $< a^+_{l-q/2} a^+_{h+q/2} a_{h-q/2} a_{l+q/2} >$ is associated with a Green function, which can be evaluated in ladder approximation including only one interaction line (Born approximation). Thus, we have to calculate

$$G(12,1'2') = G_0(11')G_0(22') + G_0(13)G_0(24)V(34,3'4')G_0(3'1')G_0(4'2'). \quad (46)$$

Applying the Matsubara technique of standard quantum statistics [12], the Green function $G(q,\omega_\lambda)$ is given in the limit of small densities ($f_k \ll 1$) by

$$G(q,\omega_\lambda) = \frac{V(q)}{\Delta E}\left[\frac{1}{\omega_\lambda - E_{h-q/2} - E_{l+q/2}} - \frac{1}{\omega_\lambda - E_{h+q/2} - E_{l-q/2}}\right] \quad (47)$$

with $\Delta E = E_{h+q/2} + E_{l-q/2} - E_{h-q/2} - E_{l+q/2}$. The connection between the spectral density $A(q,\omega)$ and the Green function is given via

$$A(q,\omega) = i[G(q,\omega+i\epsilon) - G(q,\omega-i\epsilon)]. \quad (48)$$

Exploiting the Dirac idendity, we get

$$A(q,\omega) = \frac{V(q)}{\Delta E}2\pi\left[\delta(\omega - E_{h-q/2} - E_{l+q/2}) - \delta(\omega - E_{h+q/2} - E_{l-q/2})\right]. \quad (49)$$

Composing all expressions we can calculate the correlation function

$$\begin{aligned} < a^+_{l-q/2}a^+_{h+q/2}a_{h-q/2}a_{l+q/2} > &= \int\frac{d\omega}{2\pi}\frac{1}{e^{\beta\omega}-1}A(q,\omega) \\ &= \frac{V(q)}{\Delta E}\left[n_B(E_{h-q/2}+E_{l+q/2}) - n_B(E_{h+q/2}+E_{l-q/2})\right] \\ &= \beta V_S(q)n^2\frac{(2\pi\beta\hbar^2)^3}{m^{3/2}M^{3/2}}\exp[-\beta(E_{h-q/2}+E_{l+q/2})], \end{aligned} \quad (50)$$

with the Bose distribution function $n_B(\omega) = [exp(\beta\omega) - 1]^{-1}$. For the calculation of (50) we took into account, that the Bose distribution function can be replaced by a Maxwell distribution at sufficient high temperatures. We end up with the determination of the correlation function

$$\left(\delta\dot{n}^{0,0}_{ep}(q); P_n\right) = -\frac{\Gamma(\frac{5+n}{2})}{\Gamma(\frac{5}{2})}iq_z N^2\frac{e^2}{\epsilon_0\Omega(q^2+\kappa^2)}. \quad (51)$$

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