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THE ULTRAVIOLET AND INFRARED POWER CORRECTIONS TO THE KOLMOGOROV SPECTRUM IN DEVELOPED TURBULENCE WITH SPONTANEOUS PARITY VIOLATION

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Гнатич М., Стеглик М. E17-96-340
УФ и ИК поправки к колмогоровскому спектру
развитой турбулентности со спонтанным нарушением четности

Рассчитаны ультрафиолетовая и инфракрасная степенные поправки к колмогоровскому спектру в статистической модели развитой турбулентности, основанной на принципе максимальной хаотичности поля скоростей. Степенные индексы $2\xi^+$, $2\xi^-$ аналогичны известным в теории критических явлений ультрафиолетовому поправочному индексу ω и инфракрасному индексу $(1-\alpha)$, соответственно. Получены значения $\xi^+ = 0{,}15$ и $\xi^- = -0{,}30$. Относительно небольшое значение индекса ξ^+ означает, что соответствующий поправочный член может при реальных значениях числа Рейнольдса повлиять на форму спектра кинетической энергии в инерционном интервале. Это приводит к отклонению получаемого в эксперименте фитированного значения от колмогоровского индекса.

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The Ultraviolet and Infrared Power Corrections
to the Kolmogorov Spectrum in Developed Turbulence
with Spontaneous Parity Violation

Ultraviolet and infrared power corrections to the Kolmogorov spectrum are calculated in statistical model of developed turbulence based on principle of maximal randomness of velocity field. The corresponding exponents $2\xi^+$, $2\xi^-$ are analogical to ultraviolet correction index ω and to infrared index $(1-\alpha)$ in theory of critical phenomena. The values $\xi^+ = 0.15$ and $\xi^- = -0.30$ were obtained. Rather small value of ξ^+ may cause that corresponding correction term bacomes to be relevant in the inertial interval for realistic Reynolds number. This can lead to deviation from the Kolmogorov exponent on fitting of experimental data by a single exponent.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1 Introduction

A significant progress on theory of the full developed turbulence has been achieved in the last years. In spite of seeming problems on application of the RNG method in the theory many valuable results has been reached (see, for example [1, 2, 3, 4]). The RNG method clearly demonstrates existence of infrared-stable fixed point, in which the Kolmogorov scaling is realized. An alternate way to statistical model construction of the developed turbulence that using a maximal randomness principle of a velocity field has been proposed in the papers [5, 6] on the base of self-consistent equations. But in those papers the investigation becomes substantially complicated by the fact that in contrast to the allied tasks of critical dynamics, the stationary equation of Fokker-Planck in the Wyld model has no closed solution. It does not allow, in particular, to get the system of equal-time self-consistent equations by means of standard methods. The attempts of getting such a system by means of subsidiary hypotheses conflict usually with the divergences of diagrams already in one-loop approximation. Independent statistical suppositions has been issued in the papers [7, 8].

Then the model has been investigated by authors [9] in one-loop approximation. They rigirously demonstrated that the equations are free of both the ultraviolet and infrared divergences and they possess solution with the Kolmogorov spectrum. However, the solution for a reflective symmetric medium is non-physical, because it leads to a negative value of the Kolmogorov constant. Their analysis shows that the turbulent system enforces spontaneously symmetry (parity) violation. Only that solution in frame of the model becomes physical (i.e. the Kolmogorov constant is positive) [9] and the energy spectrum then remains in the Kolmogorov form $E(k) \sim k^{-5/3}$ ($k \equiv | \mathbf{k} |$, $\mathbf{k}$ is wave vector). In present paper the model is extended and calculation of additive corrections to the Kolmogorov spectrum is performed.

Namely, as it is well known, experiments predict a possibility of a small deviation from the Kolmogorov exponent [10]. Here metod used on critical phenomena theory is taken over. In papers [11, 12] a pair correlation function $\chi(k) = \langle | \varphi(k) |^2 \rangle$ (φ is the order parameter) has been searched in the form

$$\chi(k) = Ak^{-2+\eta}[1 + a^+(kr_0)^{\alpha^+} + a^-(kr_c)^{-\alpha^-}]. \tag{1}$$

Here η is the Fisher index and r_0, r_c are molecular and correlation lengths, respectively. It has been shown in papers [11, 12] that ultraviolet correction index α^+ coincides with the well-known index ω, and infrared index α^- is connected with capacity index α by equality $\alpha^- = 1 - \alpha$. Thus the last term in expression (1) coincides with non-analytical temperature correction [13]. In papers [11, 12] the calculation is carried out for zero reduced temperature that corresponds to the inertial interval in our case. The corrections which has been obtained by this way are determined by structure of model in this range.

Analogously in our model of developed turbulence with spontaneous parity violation the additive corrective terms with exponents close to the Kolmogorov index

will be searched in the form

$$E(k) = \mathrm{C}_K W^{2/3} k^{-5/3}[1 + a^+(kr_d)^{2\xi^+} + a^-(kr_c)^{2\xi^-}]. \tag{2}$$

Here C_K is the Kolmogorov constant, W is the energy injection (pumping up energy power per mass unit), a^+ and a^- are some constants, r_d and r_c are micro- and macroscales of turbulence. The value of the correction index ξ^+ (or ξ^-) is proposed to be positive (negative); the multiplicator 2 is introduced for simplifying of record in calculation. The inequalities $kr_d \ll 1$ and $kr_c \gg 1$ hold in the inertial interval for developed turbulence, and this fact causes smallness of the corrective terms. The first of them is essential in ultraviolet region, the other in the infrared region. If at least one of the indices fulfils the inequality $\xi \ll 1$, the corresponding corrective term plays appreciable role in the inertial interval. Since it is not taken into account in data processing and the spectrum is fitted by a single effective exponent, it may differ from the Kolmogorov value. Here the values of the exponents ξ^+, ξ^- are calculated.

2 Basic assumptions

In previous paper [9] the equation of energy spectral balance was an starting-point that in the case of homogeneous stationary turbulence has the form

$$\frac{1}{2}\partial_t < E_{ij}(\mathbf{k}) > = 0 = < \dot{E}_{ij}(\mathbf{k}) > + d_{ij}(\mathbf{k}), \tag{3}$$

where

$$\begin{aligned} E_{ij}(\mathbf{k}) &= v_i(\mathbf{k}) v_j(-\mathbf{k}), \\ \dot{E}_{ij}(\mathbf{k}) &= -\nu k^2 E_{ij}(\mathbf{k}) - T_{ij}(\mathbf{k}), \\ T_{ij}(\mathbf{k}) &= \frac{1}{2}[v_i(\mathbf{k})(v_s\partial_s v_j)(-\mathbf{k}) + (v_s\partial_s v_i)(\mathbf{k}) v_j(-\mathbf{k})]. \end{aligned} \tag{4}$$

The equation (3) is the first equation of an infinity set of equations for correlation functions $< v_i v_j ... v_s >$ following from the Navier-Stokes equation for random velocity field $\mathbf{v}(\mathbf{k}) = \int d\mathbf{x}\, exp(-i\mathbf{k}\mathbf{x})\mathbf{v}(\mathbf{x})$ in the incompressible medium ($\partial_i v_i = 0$). (Here the traces over the repeated vector indexes are implied.) All tensors are proportional to the transverse projector $\mathcal{P}_{jl} \equiv \delta_{jl} - k_j k_l k^{-2}$ (incompressibility) in ordinary turbulence. In the case of parity violation, all tensors are proportional to a projector, which consist of an antisymmetric part $\sim iQ_{jl} \equiv i\varepsilon_{jls}k_s/k$ (ε_{jls} - the full antisymmetric tensor) besides the symmetric part $\sim \mathcal{P}_{jl}$. Here the effect of spontaneous generation of helicity is investigated that is why we use the external source d_{ij} of turbulent energy without the helical term.

The source energy is localized on the small wave numbers. Therefore, $d_{ij}(\mathbf{k}) = \mathcal{P}_{ij}(\mathbf{k})d(k)$ where the spectral density of the source energy localized on the momenta $k \sim r_c^{-1}$ is of the form $d(\mathbf{k}) = (2\pi)^3 W\delta(\mathbf{k})/2$ in the inertial interval $kr_c \gg 1$.

The maximal randomness principle is applied to the distribution function $\rho(\mathbf{v}(\mathbf{x}))$, which is constructed from the demand of the maximum of information entropy $\sigma = -< ln\rho >= -\int D\mathbf{v}\rho(\mathbf{v})ln\rho(\mathbf{v})$ ($D\mathbf{v}$ signifies the functional integration), scrutinizing Eq.(3) as an equation of condition. Of course, it is also meant that the condition of normalization $\int D\mathbf{v}\rho(\mathbf{v}) = 1$. Introducing Lagrange multipliers $\lambda_{ij}(\mathbf{k})$ and passing to the problem of the unconditional extreme, it yields

$$\rho(\mathbf{v}) = Q^{-1}\exp[-(2\pi)^{-3}\int d\mathbf{k}[\nu k^2\lambda_{ij}(\mathbf{k})v_j(\mathbf{k})v_i(-\mathbf{k}) + \lambda_{ij}(\mathbf{k})v_j(\mathbf{k})(v_s\partial_s v_i(-\mathbf{k})], \quad (5)$$

where the normalization factor Q is independent on $\mathbf{v}$. The multiplier λ_{ij} must be obtained from the equation of condition (3).

To obtain the self-consistent equations, the action of Eq.(5) may be rewritten in a more convenient form with local interaction. For this reason, unit operation $\int \Pi D u_i \delta(u_i(\mathbf{k}) - \lambda_{ij}(\mathbf{k})v_j(\mathbf{k}))$ at the calculation of statistical averages is introduced. The functional δ-function $\delta(u - \lambda v)$ can be written in Fourier representation by using an auxiliary field $\mathbf{w}$ (and then $i\mathbf{w} \longrightarrow \mathbf{w}$). That allows one to introduce the 'distribution function' of three transversal fields: the basic field $\mathbf{v}$ and auxiliary field $\mathbf{u}$ and $\mathbf{w}$ by relationship $\rho(\mathbf{v},\mathbf{u},\mathbf{w}) = Q^{-1}expS$, with quantum field action:

$$S = -\nu(\partial_j u_i)(\partial_j v_i) + (\partial_j u_i)v_i v_j - w_i\hat{\lambda}_{ij}v_j + w_i u_i. \quad (6)$$

The action is written in the coordinate representation; there is meant the integration over $\mathbf{x}$, $\hat{\lambda}$ is the operator with kernel corresponding to $\lambda(\mathbf{k})$ in the momentum representation.

From the Schwinger equation of the theory of Eq.(6), $< v_i\delta S/\delta u_j >= 0$, one obtains

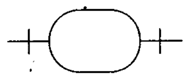
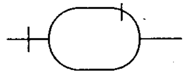
$$< \nu k^2 v_i(\mathbf{k})v_j(-\mathbf{k}) - v_i(\mathbf{k})(v_s\partial_s v_j)(-\mathbf{k}) + v_i(\mathbf{k})w_j(-\mathbf{k}) >= 0. \quad (7)$$

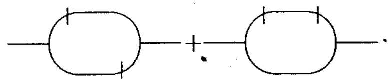
Comparison of equations (3) and (7) yields

$$< v_i(\mathbf{k})w_j(-\mathbf{k}) > = d_{ij}(\mathbf{k}) . \quad (8)$$

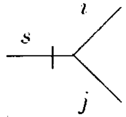
It means that the 'cross' propagator $G^{vw}ij \equiv < v_i w_j >$ is fixed by the relation (8) and has the meaning of energy injection [9].

The distribution function obtained has the form of standard quantum field theory of three transversal vector fields $\phi \equiv \{v, u, w\}$ with quasilocal action (6). There is only one nonlinear (interaction) term, which can be written in the form of $-(\partial_\beta u_\alpha)v_\alpha v_\beta$ and it does not contain the field $\mathbf{w}$. Therefore, the only self-energy functions $\Sigma^{vv}, \Sigma^{vu}, \Sigma^{uv}$ and Σ^{uu} are necessary for the theory construction. In one-loop approximation these are expressed by the following Feynman diagrams:

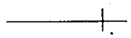
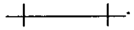
$\Sigma^{uu} = \frac{1}{2}$  , $\Sigma^{uv} =$  ,

$\Sigma^{vv} = \Sigma_1^{vv} + \Sigma_2^{vv} =$  . (9)

where following graphic representations are introduced for the symmetrized vertex of interaction:



$= \frac{1}{2}v_i v_j(\partial_i\delta_{js} + \partial_j\delta_{is})u_s$, $=< vv >\equiv G^{vv}$.

 $=< vu >\equiv G^{vu}$,  $=< uu >\equiv G^{uu}$.

The full matrix K_{ij} $(i, j = v, u, w)$, which follows from symmetrical form of quadratic part of the action S, $S = \frac{1}{2}\phi_i Kij\phi_j$, acquires the shape:

$$\hat{K} = \hat{G}^{-1} = \begin{pmatrix} -\Sigma^{vv} & \nu k^2 - \Sigma^{vu} & -\lambda \\ \nu k^2 - \Sigma^{uv} & -\Sigma^{uu} & 1 \\ -\lambda & 1 & 0 \end{pmatrix}, \quad (10)$$

from which one can easily obtains the matrix $\hat{G}$ of propagators:

$$\hat{G} = G^{vv}\begin{pmatrix} 1 & \lambda & \nu k^2 - \Sigma^{vu} - \\ & & -\lambda\Sigma^{uu} \\ \lambda & \lambda^2 & \lambda(-\nu k^2 + \Sigma^{uv}) - \\ & & \Sigma^{vv} \\ \nu k^2 - \Sigma^{uv} - & \lambda(-\nu k^2 + \Sigma^{vu}) - & |-\nu k^2 + \Sigma^{uv}|^2 - \\ -\lambda\Sigma^{uu} & -\Sigma^{vv} & -\Sigma^{uu}\Sigma^{vv} \end{pmatrix}. \quad (11)$$

Here

$$(G^{vv})^{-1} = -Det(\hat{G})^{-1} = -\Sigma^{vv} - \lambda^2\Sigma^{uu} + \lambda(2\nu k^2 - \Sigma^{uv} - \Sigma^{vu}). \quad (12)$$

In Eq.(11) the equality $G^{vv} = -(\mathrm{Det}\hat{K})^{-1} \cdot 1$ was used.

The propagators G^{vv}, G^{vu}, G^{uu} are coupled to each other by obvious relations such as

$$G^{vu} = \lambda G^{vv}, \qquad G^{uu} = \lambda^2 G^{vv}, \quad (13)$$

which follow from definition of the field $\mathbf{u} = \hat{\lambda}\mathbf{v}$. Consequently, it is sufficient to obtain a closed set of equations for the propagator G^{vv} and function $\lambda(\mathbf{k})$. The relation (12) becomes the first equation of that set. Substituting $< vw >$ from Eq.(11) into Eq.(8) one obtains the second equation:

$$G^{vv}(\nu k^2 - \Sigma^{uv} - \hat{\lambda}\Sigma^{uu}) = d(k). \tag{14}$$

The viscosity and injected energy may be neglected in the inertial interval. Therefore, the closed set of self-consistent equations is in the form:

$$\begin{aligned} (G^{vv})^{-1} &= -\Sigma^{vv} - \lambda\Sigma^{uv}, & (15)\\ G^{vv}(\Sigma^{uv} + \lambda\Sigma^{uu}) &= 0. & (16) \end{aligned}$$

It is very important that all infrared and ultraviolet divergences, which arise in Feynman diagrams of perturbation theory for Σ, can be eliminated. Therefore, the self-consistent equations are free of singularities [9].

3 First order expansion

For calculation of the energy spectrum (2) the solution of Eqs.(15,16) for the propagator $G^{vv}(\mathbf{k})$ and function $\lambda(\mathbf{k})$ was searched in the following form:

$$G^{vv} = G_0^{vv} + \delta G^{vv}, \qquad \lambda = \lambda_0 + \delta\lambda. \tag{17}$$

Let $\delta G^{vv}, \delta\lambda$ are neglected in the zero order expansion and solution is searched in the form

$$\begin{aligned} G_0^{vv}(\mathbf{k}) &= bk^{-2\gamma}(\mathcal{P} + i\theta\mathcal{Q}),\\ \lambda_0(\mathbf{k}) &= ak^{2\sigma}(\mathcal{P} + i\chi\mathcal{Q}). & (18) \end{aligned}$$

Substituting these into diagrams (9) one can obtain the self-energy functions

$$\begin{aligned} \Sigma_0^{vv} &= (4\pi)^{-3/2}a^2b^2k^{5-4\gamma+4\sigma}(A^{vv}\mathcal{P} + iB^{vv}\mathcal{Q}),\\ \Sigma_0^{uv} &= (4\pi)^{-3/2}ab^2k^{5-4\gamma+2\sigma}(A^{uv}\mathcal{P} + iB^{uv}\mathcal{Q}), & (19)\\ \Sigma_0^{uu}) &= (4\pi)^{-3/2}b^2k^{5-4\gamma}(A^{uu}\mathcal{P} + iB^{uu}\mathcal{Q}), \end{aligned}$$

where A, B are functions of θ and $z = \chi/\theta$:

$$\begin{aligned} A^{vv} &= (T_4 - 3T_5/2)(1 + 2\theta^2 z + \theta^2 z^2)b + 2T_{13}\,\theta^2(1 + 2z + \theta^2 z^2)\\ &\quad + (T_6 - T_7)(1 + \theta^2 z)^2 - T_{16}\,\theta^2(1 + z)^2,\\ B^{vv} &= -T_{14}\,\theta(1 + 2z + \theta^2 z^2) - T_{15}\,\theta(1 + 2\theta^2 z + \theta^2 z^2)\\ &\quad + 2T_{17}\,\theta(1 + z)(1 + \theta^2 z),\\ A^{uv} &= (T_2 - T_3)(1 + \theta^2 z) + T_{10}\,\theta^2(1 + z), & (20)\\ B^{uv} &= T_{11}\,\theta(1 + \theta^2 z) - T_{12}\,\theta(1 + z),\\ A^{uu} &= T_1 - T_8\,\theta^2/2,\\ B^{uu} &= T_9\,\theta/2. \end{aligned}$$

All T_i are written in Appendix I. Using functions (19) the self-consistent equations can be solved. They yield the solution with the Kolmogorov exponent $2\gamma = 11/3$ and $2\sigma = 3$, which has a non-zero helical part with $\chi = \mp 1.68$ and θ close to unit ($\theta = \pm 1.03$) [9]. Instead of θ and χ the numerical solution yields very well defined quantity $z \equiv \chi/\theta = -1.628$ as well as $M \equiv (4\pi)^{3/2}/(a^2b^3(1-\theta^2)) = 2.50$, $b > 0$.

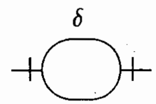
Now we will search small deviations from G_0^{vv}, λ_0 in the form

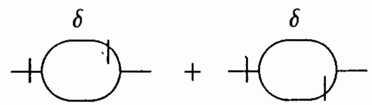
$$\begin{aligned} \delta G^{vv}(\mathbf{k}) &= bk^{-2\gamma+2\xi}(\beta\mathcal{P} + i\tilde{\theta}\mathcal{Q}), & (21)\\ \delta\lambda(\mathbf{k}) &= ak^{3+2\xi}(\alpha\mathcal{P} + i\tilde{\chi}\mathcal{Q}). & (22) \end{aligned}$$

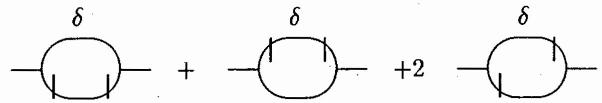
Here the exponent ξ and parameters $\{\alpha, \beta, \tilde{\theta}, \tilde{\chi}\}$ are unknown quantities to be determined the calculation of exponent ξ being the main purpose of the paper. After substitution of expression (17) into Eqs.(15), (16), and after linearization with respect to small deviations, it yields

$$\begin{aligned} G_0^{vv}[(\delta\Sigma^{uv} + (\delta\lambda)\Sigma_0^{uu})] + \lambda_0(\delta\Sigma^{uu}) &= 0, & (23)\\ \delta\Sigma^{vv} + (\delta\lambda)(\Sigma_0^{uv} + \lambda_0(\delta\Sigma^{uv}) &= -\delta(G^{vv})^{-1}. & (24) \end{aligned}$$

Now the calculation of the self-energy functions $\delta\Sigma$ is necessary. If the expansion of the propagators G^{vv}, G^{uv} and G^{uu}, as in expression (17), are substituted into the diagrams (9), they will contain parts of the zeroth, of the first and also of the second order. The last can be neglected in our approximation of the first order and parts of the zeroth order are given in expressions (19). Therefore, in one-loop aproximation the first order terms $\delta\Sigma$ can be expressed by the diagrams

$\delta\Sigma^{uu} =$  ,

$\delta\Sigma^{uv} =$  ,

$\delta\Sigma^{vv} =$  .

(25)

Here notation δ means the variation of corresponding propagator. Expressions for the zeroth order propagators G_0^{uv}, G_0^{uu} follows from equations (13) and from the expressions for the zeroth order propagator (18), i.e.,

$$G_0^{uv} = abk^{3-2\gamma}[Y\mathcal{P} + iZ\mathcal{Q}], \quad (26)$$
$$G_0^{uu} = a^2bk^{6-2\gamma}[L\mathcal{P} + iN\mathcal{Q}], \quad (27)$$

where

$$\begin{aligned} Y &= 1+z\theta^2, & Z &= \theta(1+z), \\ L &= 1+2z\theta^2+z^2\theta^2, & N &= \theta(1+2z+z^2\theta^2). \end{aligned} \quad (28)$$

One easily obtains their variations:

$$\delta G^{uv}(\mathrm{k}) = abk^{3-2\gamma+2\xi}(C\mathcal{P} + iD\mathcal{Q}), \quad (29)$$
$$\delta G^{uu}(\mathrm{k}) = a^2bk^{6-2\gamma+2\xi}(E\mathcal{P} + iF\mathcal{Q}), \quad (30)$$

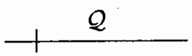
where

$$\begin{aligned} C &= \alpha+\beta+\tilde{\chi}\theta+\tilde{\theta}\theta z, \\ D &= \tilde{\chi}+\tilde{\theta}+\alpha\theta+\beta\theta z, \\ E &= 2\alpha(1+\theta^2 z)+\beta(1+\theta^2 z^2)+2\tilde{\chi}\theta(1+z)+2\tilde{\theta}\theta z, \\ F &= 2\alpha\theta(1+z)+2\beta\theta z+2\tilde{\chi}(1+\theta^2 z)+\tilde{\theta}(1+\theta^2 z^2). \end{aligned} \quad (31)$$

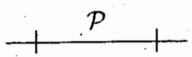
Variation of the inverse propagator is equal to

$$\delta(G^{vv})^{-1} = \frac{k^{2\gamma+2\xi}}{b(1-\theta^2)^2}\Big[(2\theta\tilde{\theta}-\beta(1+\theta^2))\mathcal{P} + i(2\theta\beta-\tilde{\theta}(1+\theta^2))\mathcal{Q}\Big]. \quad (32)$$

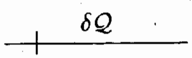
Below both the tensor structure of propagators and the power of k is obvious from the marks $\mathcal{P}$, $\mathcal{Q}$, $\delta\mathcal{P}$ and $\delta\mathcal{Q}$ above the corresponding line in diagrams, for instance,



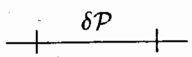
$\equiv iQ_{ij}k^{3-2\gamma}$,



$\equiv \mathcal{P}_{ij}k^{6-2\gamma}$.

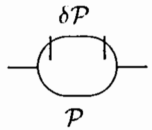


$\equiv iQ_{ij}k^{3-2\gamma+2\xi}$,

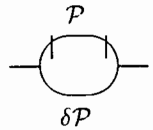


$\equiv \mathcal{P}_{ij}k^{6-2\gamma+2\xi}$.

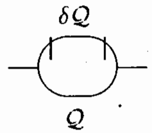
In these notations, the calculated elementary contributions into the diagrams (25) for $\delta\Sigma$ are



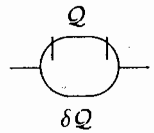
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} ES_{30}\mathcal{P}$,



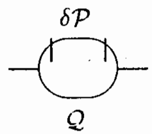
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} \beta LR_{30}\mathcal{P}$,



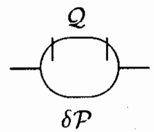
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} F(S_{22}-S_{23})\mathcal{P}$,



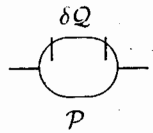
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} \tilde{\theta}N(R_{22}-R_{23})\mathcal{P}$,



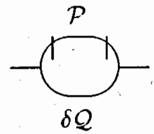
$\equiv -\frac{a^2b^2}{(4\pi)^{3/2}} ES_{15}\mathcal{Q}$,



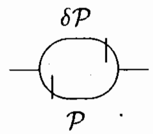
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} \beta N(R_{24}-R_{25})\mathcal{Q}$,



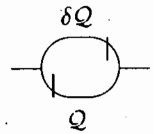
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} F(S_{24}-S_{25})\mathcal{Q}$,



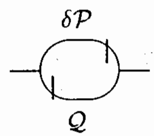
$\equiv -\frac{a^2b^2}{(4\pi)^{3/2}} \tilde{\theta}LR_{15}\mathcal{Q}$,



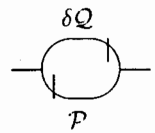
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} CY(S_6-S_7)\mathcal{P}$,



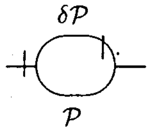
$\equiv -\frac{a^2b^2}{(4\pi)^{3/2}} DZS_{16}\mathcal{P}$,



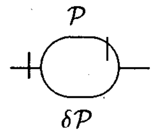
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} CZS_{17}\mathcal{Q}$,



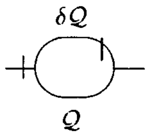
$\equiv \frac{a^2b^2}{(4\pi)^{3/2}} DYR_{17}\mathcal{Q}$,



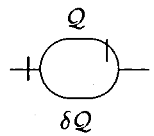
$\equiv \frac{ab^2}{(4\pi)^{3/2}} C(S_2 - S_3)\mathcal{P},$



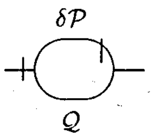
$\equiv \frac{ab^2}{(4\pi)^{3/2}} \beta Y(R_2 - R_3)\mathcal{P},$



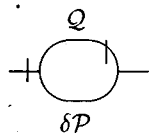
$\equiv \frac{ab^2}{(4\pi)^{3/2}} \theta D S_{10}\mathcal{P},$



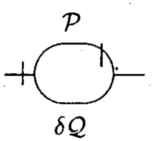
$\equiv \frac{ab^2}{(4\pi)^{3/2}} \tilde{\theta} Z R_{10}\mathcal{P},$



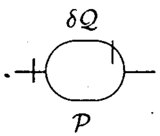
$\equiv \frac{ab^2}{(4\pi)^{3/2}} \theta C S_{11}\mathcal{Q},$



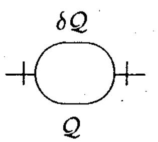
$\equiv -\frac{ab^2}{(4\pi)^{3/2}} \beta Z R_{12}\mathcal{Q},$



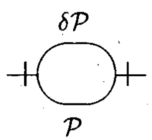
$\equiv \frac{ab^2}{(4\pi)^{3/2}} \tilde{\theta} Y R_{11}\mathcal{Q},$



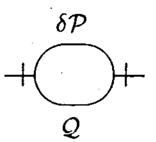
$\equiv -\frac{ab^2}{(4\pi)^{3/2}} D S_{12}\mathcal{Q},$



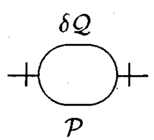
$\equiv -\frac{b^2}{(4\pi)^{3/2}} \tilde{\theta}\theta S_8\mathcal{P},$



$\equiv \frac{b^2}{(4\pi)^{3/2}} \beta S_{31}\mathcal{P},$



$\equiv \frac{b^2}{(4\pi)^{3/2}} \beta\theta(S_{26} - S_{27})\mathcal{Q},$



$\equiv \frac{b^2}{(4\pi)^{3/2}} \tilde{\theta}(R_{26} - R_{27})\mathcal{Q},$

The combinations of Γ-functions denoted by R_i and S_i are written in Appendix II. Summarizing all similar contributions one obtains the variations $\delta\Sigma$.

4 Corrections exponents

After substitution of the calculated variations $\delta\Sigma$ of functions (25) into Eqs.(23) and (24), the equal power of wave number k can be canceled in all of the terms. Each of these matrix equations determines two scallar equations in reference to $\mathcal{P}$ and $\mathcal{Q}$ contributions. The result is a set of four homogeneous linear equations with respect to α, β, $\tilde{\theta}$ and $\tilde{\chi}$, yielding:

$$\begin{aligned} C_{11}\beta + C_{12}\tilde{\theta} + C_{13}\alpha + C_{14}\tilde{\chi} &= 0, \\ C_{21}\beta + C_{22}\tilde{\theta} + C_{23}\alpha + C_{24}\tilde{\chi} &= 0, \end{aligned}$$

$$\begin{aligned} (-\frac{M(1+\theta^2)}{(1-\theta^2)} + C_{31})\beta + (\frac{2M\theta}{(1-\theta^2)} + C_{32})\tilde{\theta} + C_{33}\alpha + C_{34}\tilde{\chi} &= 0, \\ (\frac{2M\theta}{(1-\theta^2)} + C_{41})\beta + (-\frac{M(1+\theta^2)}{(1-\theta^2)} + C_{42})\tilde{\theta} + C_{43}\alpha + C_{44}\tilde{\chi} &= 0. \end{aligned} \qquad (33)$$

The coefficients C_{ij} are functions of ξ and θ, and they compose the matrix

$$\hat{C} = \begin{bmatrix} \theta^2 H_1 + H_2 & \theta H_3 & \theta^2 H_4 + H_5 & \theta H_6 \\ \theta H_7 & \theta^2 H_8 + H_9 & \theta H_{10} & \theta^2 H_{11} + H_{12} \\ \theta^2 H_{13} + H_{14} & \theta(\theta^2 H_{15} + H_{16}) & \theta^2 H_{17} + H_{18} & \theta(\theta^2 H_{19} + H_{20}) \\ \theta(\theta^2 H_{21} + H_{22}) & \theta^2 H_{23} + H_{24} & \theta(\theta^2 H_{25} + H_{26}) & \theta H_{27} + H_{28} \end{bmatrix}.$$

The functions H_i depend on z and ξ, and they are written in Appendix III. Nontrivial solution of the set of Eqs.(33) exists if its determinant is equal to zero. By equaling this determinant to zero, we obtain the equation for ξ. Many roots of such equation exist; we are interested in minimal roots in their absolute values, of course.

The direct numerical solution of this set of equations gives the minimal negative root $\xi^- \simeq -0.30$, which is practically insensitive to a change of θ close to unit. The minimal positive root ξ^+ monotonically changes from 0.13 up to 0.15 for the change of θ from 0.96 up to 0.9999 . The range of change of θ is chosen for the corresponding value $\mathbf{C}_k$ not to be far from the experimental estimations, $1.4 < \mathbf{C}_k < 2$.

One can easily convinced about the existence of 'limit' solutions ξ_0^+ and ξ_0^- for $\theta \to 1$. The determinant of the set can be written in the form of a power series of $\epsilon \equiv (1-\theta^2)$, which begins from the term proportional to ϵ^{-2} . But the calculation shows that the leading pole proportional to ϵ^{-2} vanishes at the Kolmogorov index $2\gamma = 11/3$. Therefore, the leading term is proportional to ϵ^{-1} . one easily obtains the equation

$$\left\{ \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ -2 & 2 & 0 & 0 \\ C_{41} & C_{42} & C_{43} & C_{44} \end{vmatrix} + \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ 2 & -2 & 0 & 0 \end{vmatrix} \right\}_{\theta=1} = 0, \qquad (34)$$

where the common multiplicator (M/ϵ) was omitted. Numerical solutions of Eq.(34) gives the roots $\xi_0^- = -0.303$ and $\xi_0^+ = 0.157$; i.e. they are close to the preceding values.

5 Corrections of a higher order

The investigation of error of the obtained roots in dependence on the value θ seems to be an important question, because the coefficients of the set of Eqs.(33) depend sensitively on θ. To be sure that the error is in reality relative small, the next correction to the value of θ_0 in a power expansion of $\epsilon = (1 - \theta^2)$ at $\epsilon = 0$ has been investigated.

The expansion of components of the matrix $\hat{C}$ in a series of ϵ gives

$$C_{ij} \equiv C_{ij}(\xi_0 + \epsilon\xi_1, \theta) = C_{ij}(\xi_0, 1) + \epsilon\, \delta C_{ij}\,, \tag{35}$$

where $\theta \doteq (1 - \epsilon/2)$ and

$$\delta C_{ij} \equiv \left(\frac{\partial C_{ij}(\xi_0 + \epsilon\xi_1, \theta)}{\partial \epsilon}\right)\bigg|_{\epsilon=0}\,.$$

For example,

$$\begin{aligned} C_{11} &= C_{11}(\xi_0) + \epsilon \left[(-1)\cdot H_1(\xi_0) + (1-0)\frac{\partial H_1}{\epsilon} + \frac{\partial H_2}{\epsilon}\right] \\ &\doteq \Big(H_1(\xi_0) + H_2(\xi_0)\Big) + \epsilon\Big[-H_1(\xi_0) + \xi_1(D_1 + D_2)\Big]\,, \end{aligned}$$

where $D_i \equiv (\partial H_i/\partial \xi)\,|_{\xi_0}$. After calculation of all D_i and δC_{ij}, one can obtain the linear equation for ξ_1 in the form

$$\begin{aligned} &\mathrm{Det}\ \hat{C}(\xi_0)/2M - (C_{11} + C_{12})_0 + L_2 - (C_{31} + C_{32} + C_{41} + C_{42})_0 L_3 \\ &\qquad + (C_{21} + C_{22})_0 L_4 + L_5 = 0\,. \end{aligned} \tag{36}$$

Here we denote $(...)_0 \equiv (...)|_{\xi_0}$. The functions $L_i, \delta C_{ij}$ and D_i are completed in Appendix IV. Solving the equation (36) one obtain the next correction exponent value $\xi_1^+ = -0.0044$ and $\xi_1^- = 0.0133$ at the point $\xi_0^+ = 0.15$ and $\xi_0^- = -0.30$, respectively. Therefore, the corrections to corresponding values of ξ_0 are negligible even for non small value of ϵ.

6 Conclusion

In the present paper the set of the second order self-consistent equations has been used for calculation of additional corrections to the Kolmogorov spectrum in the model [9] of developed turbulence with parity violation. The calculated values of both the ultraviolet correction index, $\xi^+ = 0.15$, and infrared one, $\xi^- = -0.30$, have been also examined by their possible corrections of a higher order. However, the high order corrections $\xi_1^\pm$ become be negligible.

The rather small indexes $\xi^\pm$, especially the ultraviolet one, can make the corresponding terms in the general expression (2) to be relevant in the inertial interval for realistic Reynolds numbers and they lead to deviation from the Kolmogorov exponent if experimental data are fitted by a single exponent.

Acknowledgement

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Appendix I

All T_i are expressed by the standard combination of Γ - functions:

$$(x_1, x_2, x_3; x_4, x_5, x_6) \equiv \frac{\Gamma(x_1)\Gamma(x_2)\Gamma(x_3)}{\Gamma(x_4)\Gamma(x_5)\Gamma(x_6)}\,:$$

$$\begin{aligned}
T_1 &= (2\gamma - 3/2, 7/2 - \gamma, 3/2 - \gamma; 5 - 2\gamma, \gamma + 1, \gamma), \\
T_2 &= (5/2 - \gamma, 5/4 + \gamma/2, \gamma/2 - 1/4; \gamma, 9/4 - \gamma/2, 15/4 - \gamma/2), \\
T_3 &= (3/2 - \gamma, 5/4 + \gamma/2, \gamma/2 - 1/4; \gamma + 1, 5/4 - \gamma/2, 11/4 - \gamma/2), \\
T_4 &= (3/2 - \gamma, 2\gamma + 1, 2 - \gamma; 7/2 - 2\gamma, \gamma + 1, 5/2 + \gamma), \\
T_5 &= (5/2 - \gamma, 2\gamma, 1 - \gamma; 7/2 - 2\gamma, \gamma + 1, 5/2 + \gamma), \\
T_6 &= (1 - \gamma, 5/4 + \gamma/2, 5/4 + \gamma/2; 3/2 + \gamma, 9/4 - \gamma/2, 9/4 - \gamma/2), \\
T_7 &= (-\gamma, 5/4 + \gamma/2, 5/4 + \gamma/2; 5/2 + \gamma, 5/4 - \gamma/2, 5/4 - \gamma/2). \\
T_8 &= (2\gamma - 3/2, 2 - \gamma, 2 - \gamma; 4 - 2\gamma, \gamma + 1/2, \gamma + 1/2), \\
T_9 &= (2\gamma - 1, 3/2 - \gamma, 2 - \gamma; 9/2 - 2\gamma, \gamma + 1, \gamma + 1/2), \\
T_{10} &= (7/4 + \gamma/2, 2 - \gamma, \gamma/2 - 1/4; 7/4 - \gamma/2, \gamma + 1/2, 15/4 - \gamma/2), \\
T_{11} &= (5/4 + \gamma/2, 2 - \gamma, 1/4 + \gamma/2; 9/4 - \gamma/2, \gamma + 1/2, 13/4 - \gamma/2), \\
T_{12} &= (7/4 + \gamma/2, 3/2 - \gamma, 1/4 + \gamma/2; 7/4 - \gamma/2, \gamma + 1, 13/4 - \gamma/2), \\
T_{13} &= (2\gamma + 1/2, 2 - \gamma, -\gamma; 3 - 2\gamma, \gamma + 1/2, \gamma + 5/2), \\
T_{14} &= (2\gamma + 1/2, 3/2 - \gamma, 1/2 - \gamma; 3 - 2\gamma, \gamma + 1, \gamma + 2), \\
T_{15} &= (2\gamma, 2 - \gamma, 1/2 - \gamma; 5/2 - 2\gamma, \gamma + 1/2, \gamma + 2), \\
T_{16} &= (7/4 + \gamma/2, 7/4 + \gamma/2, -\gamma; 7/4 - \gamma/2, 7/4 - \gamma/2, 5/2 + \gamma), \\
T_{17} &= (7/4 + \gamma/2, 5/4 + \gamma/2, 1/2 - \gamma; 9/4 - \gamma/2, 7/4 - \gamma/2, 2 + \gamma).
\end{aligned}$$

Appendix II

$$\begin{aligned}
S_2 &= (5/2 - \gamma, 5/4 + \gamma/2 + \xi, \gamma/2 - 1/4 - \xi; \gamma, 9/4 - \gamma/2 - \xi, 15/4 - \gamma/2 + \xi) \\
S_3 &= (3/2 - \gamma, 5/4 + \gamma/2 + \xi, \gamma/2 - 1/4 - \xi; \gamma + 1, 5/4 - \gamma/2 - \xi, 11/4 - \gamma/2 + \xi) \\
S_6 &= (1 - \gamma - \xi, 5/4 + \gamma/2, 5/4 + \gamma/2 + \xi; 3/2 + \gamma + \xi, 9/4 - \gamma/2, 9/4 - \gamma/2 - \xi) \\
S_7 &= (-\gamma - \xi, 5/4 + \gamma/2 + \xi, 5/4 + \gamma/2; 5/2 + \gamma + \xi, 5/4 - \gamma/2 - \xi, 5/4 - \gamma/2) \\
S_8 &= (2\gamma - 3/2 - \xi, 2 - \gamma + \xi, 2 - \gamma; 4 - 2\gamma + \xi, \gamma + 1/2 - \xi, \gamma + 1/2) \\
S_9 &= 2\ (S_{26} - S_{27}) \\
S_{10} &= (7/4 + \gamma/2 + \xi, 2 - \gamma, \gamma/2 - 1/4 - \xi; 7/4 - \gamma/2 - \xi, \gamma + 1/2, 15/4 - \gamma/2 + \xi) \\
S_{11} &= (5/4 + \gamma/2 + \xi, 2 - \gamma, 1/4 + \gamma/2 - \xi; 9/4 - \gamma/2 - \xi, \gamma + 1/2, 13/4 - \gamma/2 + \xi) \\
S_{12} &= (7/4 + \gamma/2 + \xi, 3/2 - \gamma, 1/4 + \gamma/2 - \xi; 7/4 - \gamma/2 - \xi, \gamma + 1, 13/4 - \gamma/2 + \xi) \\
S_{13} &= (S_{22} - S_{23})/2
\end{aligned}$$

$$
\begin{aligned}
S_{14} &= S_{25} - S_{24} \\
S_{15} &= (2\gamma + \xi, 2 - \gamma, 1/2 - \gamma - \xi; 5/2 - 2\gamma - \xi, \gamma + 1/2, \gamma + 2 + \xi) \\
S_{16} &= (7/4 + \gamma/2, 7/4 + \gamma/2 + \xi, -\gamma - \xi; 7/4 - \gamma/2, 7/4 - \gamma/2 - \xi, 5/2 + \gamma + \xi) \\
S_{17} &= (7/4 + \gamma/2, 5/4 + \gamma/2 + \xi, 1/2 - \gamma - \xi; 9/4 - \gamma/2 - \xi, 7/4 - \gamma/2, 2 + \gamma + \xi) \\
S_{18} &= (2\gamma - 3/2 - \xi, 5/2 - \gamma + \xi, 3/2 - \gamma; 4 - 2\gamma + \xi, \gamma + 1, \gamma - \xi) \\
S_{19} &= (2\gamma - 3/2 - \xi, 5/2 - \gamma, 3/2 - \gamma + \xi; 4 - 2\gamma + \xi, \gamma + 1 - \xi, \gamma) \\
S_{20} &= (2\gamma - 3/2 - \xi, 5/2 - \gamma + \xi, 5/2 + \gamma; 5 - 2\gamma + \xi, \gamma + 1, \gamma - \xi) \\
S_{21} &= (2\gamma - 3/2 - \xi, 5/2 - \gamma + \xi, 5/2 - \gamma; 5 - 2\gamma + \xi, \gamma + 1 - \xi, \gamma) \\
S_{22} &= (2\gamma + 1/2 + \xi, 3 - \gamma, -\gamma - \xi; 3 - 2\gamma - \xi, \gamma + 1/2, \gamma + 5/2 + \xi) \\
S_{23} &= (2\gamma + 1/2 + \xi, 2 - \gamma, 1 - \gamma - \xi; 3 - 2\gamma - \xi, \gamma + 1/2, \gamma + 5/2 + \xi) \\
S_{24} &= (2\gamma + 1/2 + \xi, 3/2 - \gamma, 3/2 - \gamma - \xi; 3 - 2\gamma - \xi, \gamma + 1, \gamma + 2 + \xi) \\
S_{25} &= (2\gamma + 1/2 + \xi, 5/2 - \gamma, 1/2 - \gamma - \xi; 3 - 2\gamma - \xi, \gamma + 1, \gamma + 2 + \xi) \\
S_{26} &= (2\gamma - 1 - \xi, 3/2 - \gamma + \xi, 2 - \gamma; 9/2 - 2\gamma + \xi, \gamma - \xi, \gamma + 1/2) \\
S_{27} &= (2\gamma - 1 - \xi, 3/2 - \gamma + \xi, 2 - \gamma; 9/2 - 2\gamma + \xi, \gamma + 1 - \xi, \gamma - 1/2) \\
S_{28} &= (3/2 - \gamma, 2\gamma + \xi, -\gamma - \xi; 7/2 - 2\gamma - \xi, \gamma + 1, 3/2 + \gamma + \xi) \\
S_{29} &= (5/2 - \gamma, 2\gamma + \xi, -\gamma - \xi; 7/2 - 2\gamma - \xi, \gamma + 1, 5/2 + \gamma + \xi) \\
S_{30} &= \alpha_1 S_{28} + \beta_1 S_{29} \\
S_{31} &= S_{18} + S_{19} - S_{20} - S_{21} \\
R_2 &= (5/2 - \gamma + \xi, 5/4 + \gamma/2, \gamma/2 - 1/4 - \xi; \gamma + \xi, 9/4 - \gamma/2, 15/4 - \gamma/2 + \xi) \\
R_3 &= (3/2 - \gamma + \xi, 5/4 + \gamma/2, \gamma/2 - 1/4 - \xi; \gamma + 1 - \xi, 5/4 - \gamma/2, 11/4 - \gamma/2 + \xi) \\
R_9 &= 2\ (R_{26} - R_{27}) \\
R_{10} &= (7/4 + \gamma/2, 2 - \gamma + \xi, \gamma/2 - 1/4 - \xi; 7/4 - \gamma/2, \gamma + 1/2 - \xi, 15/4 - \gamma/2 + \xi) \\
R_{11} &= (5/4 + \gamma/2, 2 - \gamma + \xi, 1/4 + \gamma/2 - \xi; 9/4 - \gamma/2, \gamma + 1/2 - \xi, 13/4 - \gamma/2 + \xi) \\
R_{12} &= (7/4 + \gamma/2, 3/2 - \gamma + \xi, 1/4 + \gamma/2 - \xi; 7/4 - \gamma/2, \gamma + 1 - \xi, 13/4 - \gamma/2 + \xi) \\
R_{13} &= (R_{22} - R_{23})/2 \\
R_{14} &= R_{25} - R_{24} \\
R_{15} &= (2\gamma, 2 - \gamma + \xi, 1/2 - \gamma - \xi; 5/2 - 2\gamma, \gamma + 1/2 - \xi, \gamma + 2 + \xi) \\
R_{17} &= (7/4 + \gamma/2 + \xi, 5/4 + \gamma/2, 1/2 - \gamma - \xi; 9/4 - \gamma/2, 7/4 - \gamma/2 - \xi, 2 + \gamma + \xi) \\
R_{22} &= (2\gamma + 1/2, 3 - \gamma + \xi, -\gamma - \xi; 3 - 2\gamma, \gamma + 1/2 - \xi, \gamma + 5/2 + \xi) \\
R_{23} &= (2\gamma + 1/2, 2 - \gamma + \xi, 1 - \gamma - \xi; 3 - 2\gamma, \gamma + 1/2 - \xi, \gamma + 5/2 + \xi) \\
R_{24} &= (2\gamma + 1/2, 3/2 - \gamma + \xi, 3/2 - \gamma - \xi; 3 - 2\gamma, \gamma + 1 - \xi, \gamma + 2 + \xi) \\
R_{25} &= (2\gamma + 1/2, 5/2 - \gamma + \xi, 1/2 - \gamma - \xi; 3 - 2\gamma, \gamma + 1 - \xi, \gamma + 2 + \xi) \\
R_{26} &= (2\gamma - 1 - \xi, 3/2 - \gamma, 2 - \gamma + \xi; 9/2 - 2\gamma + \xi, \gamma, \gamma + 1/2 - \xi) \\
R_{27} &= (2\gamma - 1 - \xi, 3/2 - \gamma, 2 - \gamma + \xi; 9/2 - 2\gamma + \xi, \gamma + 1, \gamma - 1/2 - \xi) \\
R_{28} &= (3/2 - \gamma + \xi, 2\gamma, -\gamma - \xi; 7/2 - 2\gamma, \gamma + 1 - \xi, 3/2 + \gamma + \xi) \\
R_{29} &= (5/2 - \gamma + \xi, 2\gamma, -\gamma - \xi; 7/2 - 2\gamma, \gamma + 1 - \xi, 5/2 + \gamma + \xi)
\end{aligned}
$$

$$
R_{30} = \alpha_2 R_{28} + \beta_2 R_{29}
$$

$$
\begin{aligned}
\alpha_1 &= 2 + (1/2 - 2\gamma - \xi)(\gamma - 1/2) - (5/2 - 2\gamma - \xi)(1 + \gamma + \xi) \\
\beta_1 &= (5/2 - 2\gamma - \xi)(\gamma - 1/2) + 2 - 2\gamma \\
\alpha_2 &= 2 + (1/2 - 2\gamma)(\gamma - 1/2 - \xi) - (5/2 - 2\gamma)(1 + \gamma + \xi) \\
\beta_2 &= (5/2 - 2\gamma)(\gamma - 1/2 - \xi) + 2 - 2\gamma + 2\xi
\end{aligned}
$$

Apendix III

$$
\begin{aligned}
H_1 &= z(R_2 - R_3 + S_9/2 + S_{10})\,, \\
H_2 &= S_2 - S_3 + R_2 - R_3 + S_{18} + S_{19} - S_{20} - S_{21}\,, \\
H_3 &= z(S_2 - S_3 + R_9/2 + R_{10}) + S_{10} + R_{10} - S_8\,, \\
H_4 &= S_{10} - T_8/2\,, \\
H_5 &= T_1 + S_2 - S_3\,, \\
H_6 &= T_9/2 + S_{10} + S_2 - S_3\,, \\
H_7 &= z(-S_{12} - R_{12} + S_{18} + S_{19} - S_{20} - S_{21}) + S_{11} - R_{12} + S_9/2\,, \\
H_8 &= z(R_{11} + S_{11} - S_8)\,, \\
H_9 &= R_{11} + R_9/2 - S_{12}\,, \\
H_{10} &= T_9/2 + S_{11} - S_{12}\,, \\
H_{11} &= S_{11} - T_8/2\,, \\
H_{12} &= T_1 - S_{12}\,, \\
H_{13} &= z^2(-R_{12} + \alpha_2 R_{28} + \beta_2 R_{29} - S_{12} - 2S_{16} + \alpha_1 S_{28} + \beta_1 S_{29}) + z(-R_{12} \\
&\quad + R_2 - R_3 + 2\alpha_2 R_{28} + 2\beta_2 R_{29} + S_{10} + S_{11} + 4S_{13} - 2S_{16} + 2S_6 - 2S_7)\,, \\
H_{14} &= R_2 - R_3 + \alpha_2 R_{28} + \beta_2 R_{29} + S_2 - S_3 \alpha_1 S_{28} + \beta_1 S_{29} + 2S_6 - 2S_7\,, \\
H_{15} &= z^2(R_{11} + R_{13} + S_{11} + 2S_{13} + 2T_6 - 2S_7)\,, \\
H_{16} &= z(R_{10} + R_{11} + 4R_{13} - S_{12} - 2S_{16} + S_2 - S_3 + 2\alpha_1 S_{28} + 2\beta_1 S_{29} \\
&\quad + 2S_6 - 2S_7) + R_{10} + S_{10} + 2R_{13} + 2S_{13} - 2S_{16}\,, \\
H_{17} &= z(T_{10} + T_2 - T_3 + S_{11} - S_{12} + 4S_{13} - 2S_{16} + 2\alpha_1 S_{28} + 2\beta_1 S_{29} \\
&\quad + 2S_6 - 2S_7) + T_{10} + S_{10} + 4S_{13} - 2s_{16}\,, \\
H_{18} &= T_2 - T_3 + S_2 - S_3 + 2\alpha_1 S_{28} + 2\beta_1 S_{29} + 2S_6 - 2S_7\,, \\
H_{19} &= z(T_{11} + S_{11} + 4S_{13} + 2S_6 - 2S_7)\,, \\
H_{20} &= z(-T_{12} - S_{12} - 2S_{16} + 2\alpha_1 S_{28} + 2\beta_1 S_{29}) + T_{11} - T_{12} + S_{10} + 4S_{13} \\
&\quad - 2S_{16} + S_2 - S_3 + 2\alpha_1 S_{28} + 2\beta_1 S_{29} + 2S_6 - 2S_7\,, \\
H_{21} &= z^2(-R_{14} + 2R_{17} + R_2 - R_3 + S_{10} - S_{15})\,, \\
H_{22} &= z(-R_{12} - 2R_{14} + 2R_{17} + R_2 - R_3 - S_{12} - 2S_{14} + 2S_{17} + S_2 - S_3)
\end{aligned}
$$

$$
\begin{aligned}
& -R_{12} - R_{14} + S_{11} - S_{15} + 2S_{17}\,, \\
H_{23} &= z^2(R_{10} - R_{15} - S_{14} + 2S_{17} + S_2 - S_3) + z(R_{10} + R_{11} - 2R_{15} + 2R_{17} \\
& \quad + S_{10} + S_{11} - 2S_{15} + 2S_{17})\,, \\
H_{24} &= R_{11} - R_{15} + 2R_{17} - S_{12} - S_{14}\,, \\
H_{25} &= z(T_{11} + S_{10} + 2R_{17} - 2S_{15})\,, \\
H_{26} &= z(-T_{12} - 2S_{14} + 2S_{17} + S_2 - S_3) + T_{11} - T_{12} + S_{11} - S_{12} + 2R_{17} \\
& \quad - 2S_{14} - 2S_{15} + 2S_{17}\,, \\
H_{27} &= z(T_2 - T_3 + 2R_{17} + S_{10} - 2S_{14} - 2S_{15} + 2S_{17} + S_2 - S_3 + T_{10}) \\
& \quad + S_{11} - 2S_{15} + 2S_{17} + T_{10}\,, \\
H_{28} &= T_2 - T_3 - S_{12} + 2R_{17} - 2S_{14}\,.
\end{aligned}
$$

Appendix IV

The functions L_i :

$$
\begin{aligned}
L_1 &= C_{23}(\delta C_{34} + \delta C_{44}) - C_{24}(\delta C_{33}) + \delta C_{43}) + \delta C_{23}(C_{34} + C_{44}) - \delta C_{24}(C_{33} + C_{43})\,, \\
L_2 &= (M - \delta C_{31} - \delta C_{32} - \delta C_{41} - \delta C_{42})(C_{13}C_{24} - C_{14}C_{23})\,, \\
L_3 &= (C_{13}\,\delta C_{24} - C_{14}\,\delta C_{23}) + (\delta C_{13}\,C_{24} - \delta C_{14}\,C_{23})\,, \\
L_4 &= C_{13}(\delta C_{34} + \delta C_{44}) - C_{14}(\delta C_{33} + \delta C_{43}) + \delta C_{13}(C_{34} + C_{44}) - \delta C_{14}(C_{33} + C_{43})\,, \\
L_5 &= -(\delta C_{11} + \delta C_{12})\Big(C_{23}(C_{34} + C_{44}) - C_{24}(C_{33} + C_{43})\Big) \\
& \quad + (\delta C_{21} + \delta C_{22})\Big(C_{13}(C_{34} + C_{44}) - C_{14}(C_{33} + C_{43})\Big)\,.
\end{aligned}
$$

The derivatives δC_{ij} :

$$
\begin{aligned}
\delta C_{11} &= -H_1(\xi_0) + \xi_1(D_1 + D_2) \\
\delta C_{12} &= -H_3(\xi_0)/2 + \xi_1 D_3\,, \\
\delta C_{13} &= -H_4(\xi_0) + \xi_1(D_4 + D_5)\,, \\
\delta C_{14} &= -H_6(\xi_0)/2 + \xi_1 D_6\,, \\
\delta C_{21} &= -H_7(\xi_0)/2 + \xi_1 D_7\,, \\
\delta C_{22} &= -H_8(\xi_0) + \xi_1(D_8 + D_9)\,, \\
\delta C_{23} &= -H_{10}(\xi_0)/2 + \xi_1 D_{10}\,, \\
\delta C_{24} &= -H_{11}(\xi_0) + \xi_1(D_{11} + D_{12})\,, \\
\delta C_{31} &= -H_{13}(\xi_0) + \xi_1(D_{13} + D_{14})\,, \\
\delta C_{32} &= -3H_{15}(\xi_0)/2 - H_{16}(\xi_0) + \xi_1(D_{15} + D_{16})\,, \\
\delta C_{33} &= -H_{17}(\xi_0) + \xi_1(D_{17} + D_{18})\,, \\
\delta C_{34} &= -3H_{19}(\xi_0)/2 - H_{20}(\xi_0) + \xi_1(D_{19} + D_{20})\,, \\
\delta C_{41} &= -3H_{21}(\xi_0)/2 - H_{22}(\xi_0) + \xi_1(D_{21} + D_{22})\,,
\end{aligned}
$$

$$
\begin{aligned}
\delta C_{42} &= -H_{23}(\xi_0) + \xi_1(D_{23} + D_{24})\,, \\
\delta C_{43} &= -3H_{25}(\xi_0)/2 - H_{26}(\xi_0) + \xi_1(D_{25} + D_{26})\,, \\
\delta C_{44} &= -H_{27}(\xi_0) + \xi_1(D_{27} + D_{28})\,.
\end{aligned}
$$

The required derivatives D_i :

$$
\begin{aligned}
D_1 &= z(W_2 - W_3 + V_9/2 + V_{10})\,, \\
D_2 &= V_2 - V_3 + W_2 - W_3 + V_{18} + V_{19} - V_{20} - V_{21}\,, \\
D_3 &= z(V_2 - V_3 + W_9/2 + W_{10}) + V_{10} + W_{10} - V_8\,, \\
D_4 &= V_{10}\,, \qquad D_5 = V_2 - V_3\,, \qquad D_6 = V_{10} + V_2 - V_3\,, \\
D_7 &= z(-W_{12} - V_{12} + V_{18} + V_{19} - V_{20} - V_{21}) + V_{11} - W_{12} + V_9/2\,, \\
D_8 &= z(W_{11} + V_{11} - V_8)\,, \qquad D_9 = W_{11} + W_9/2 - V_{12}\,, \\
D_{10} &= V_{11} - V_{12}\,, \qquad D_{11} = V_{11}\,, \qquad D_{12} = -V_{12}\,, \\
D_{13} &= z^2(-W_{12} + W_{30} - V_{12} - 2V_{16} + V_{30}) + z(-W_{12} + W_2 - W_3 + 2W_{30} \\
& \quad + V_{10} + V_{11} + 4V_{13} - 2V_{16} + 2(V_6 - V_7))\,, \\
D_{14} &= W_2 - W_3 + W_{30} + V_2 - V_3 + V_{30} + 2(V_6 - V_7)\,, \\
D_{15} &= z^2(W_{11} + 2W_{13} + v_{11} + 2(V_{13} + V_6 - V_7))\,, \\
D_{16} &= z(W_{10} + W_{11} + 4W_{13} - V_{12} - 2V_{16} + V_2 - V_3 + 2(V_{30} + V_6 - V_7)) \\
& \quad + W_{10} + V_{10} + 2(W_{13} + V_{13} - V_{16})\,, \\
D_{17} &= z(V_{11} - V_{12} + 2(V_6 - V_7 + 2V_{13} - V_{16} + V_{30})) + V_{10} + 4V_{13} - 2V_{16}\,, \\
D_{18} &= V_2 - V_3 + 2(V_6 - V_7 + V_{30})\,, \\
D_{19} &= z(V_{11} + 2(V_6 - V_7 + 2V_{13}))\,, \\
D_{20} &= z(-V_{12} - 2(V_{16} - V_{30})) + V_2 - V_3 + V_{10} + 2(V_6 - V_7 + 2V_{13} - V_{16} + V_{30})\,, \\
D_{21} &= z^2(W_2 - W_3 + V_{10} - W_{14} - V_{15} + 2W_{17})\,, \\
D_{22} &= z(W_2 - W_3 + V_2 - V_3 - W_{12} - V_{12} - 2(W_{14} - W_{17} + V_{14} - V_{17})) + V_{11} - \\
& \quad W_{12} - W_{14} - V_{15} + 2V_{17}\,, \\
D_{23} &= z^2(V_2 - V_3 + W_{10} - W_{15} - V_{14} + 2V_{17}) + z(W_{10} + W_{11} + V_{10} + V_{11} - \\
& \quad 2(W_{15} - W_{17} + V_{15} - V_{17}))\,, \\
D_{24} &= W_{11} - V_{12} - V_{14} - W_{15} + 2W_{17}\,, \\
D_{25} &= z(V_{10} - 2V_{15} + 2W_{17})\,, \\
D_{26} &= z(V_2 - V_3 - 2V_{14} + V_{17}) + V_{11} - V_{12} + 2(-V_{14} - V_{15} + W_{17} + V_{17})\,, \\
D_{27} &= z(V_2 - V_3 + V_{10} - 2(V_{14} + V_{15} - W_{17} - V_{17})) + V_{11} - 2(V_{15} - V_{17})\,, \\
D_{28} &= -V_{12} - 2V_{14} + 2W_{17}\,.
\end{aligned}
$$

Here we denote $W_i \equiv (\partial R_i/\partial\xi)_{\xi_0}$ and $V_i \equiv (\partial S_i/\partial\xi)_{\xi_0}$; we have, for example,

$$
V_2 = \frac{\partial}{\partial\xi}\left(\frac{5}{2} - \gamma, \frac{5}{4} + \frac{\gamma}{2} + \xi, \frac{\gamma}{2} - \frac{1}{4} - \xi; \gamma, \frac{9}{4} - \frac{\gamma}{2} - \xi, \frac{15}{4} - \frac{\gamma}{2} + \xi\right)_{\xi_0} =
$$

$$= S_2(\xi_0)\left\{\psi\left(\frac{5}{4}+\frac{\gamma}{2}+\xi_0\right)+\psi\left(\frac{\gamma}{2}-\frac{1}{4}-\xi_0\right)\right.$$
$$\left.-\psi\left(\frac{9}{4}-\frac{\gamma}{2}-\xi_0\right)-\psi\left(\frac{15}{4}-\frac{\gamma}{2}+\xi_0\right)\right\},$$

where the ψ-function is logarithmic derivative of the Γ-function.

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