СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E17-96-317

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VECTOR AND SCALAR MODEL OF TURBULENT DIFFUSION NEAR TWO DIMENSIONS. THE RENORMALIZATION GROUP ANALYSIS WITH TWO EXPANSION PARAMETERS

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1996

Гнатич М. и др. E17-96-317

Векторная и скалярная модель турбулентной диффузии вблизи размерности два. Ренормгрупповой анализ с двумя параметрами разложения

Методом квантово-полевой ренормализационной группы была изучена стохастическая модель стационарной и сильно развитой магнито-гидродинамической турбулентности и модель, описывающая пассивную примесь в турбулентной среде вблизи размерности пространства, равной двум. В однопетлевом приближении использовалась теория возмущений по двум параметрам — отклонению пространственной размерности и степенного индекса в корреляторе случайной силы от их критических значений. Было проведено систематическое исследование всех возможных режимов с учетом их физического смысла. Для обеих моделей было подтверждено существование кинетической фиксированной точки, ассоциированной с универсальным режимом Колмогорова. В этом режиме было вычислено универсальное значение ($u^* = 1{,}562$) обратных чисел Прандтля. Используемый метод двойного разложения сравнивался с ε-разложением, обычно применяемым в статистической турбулентности.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 1996

Hnatich M. et al. E17-96-317

Vector and Scalar Model of Turbulent Diffusion near Two Dimensions. The Renormalization Group Analysis with Two Expansion Parameters

The model of randomly forced stationary and strongly developed magnetohydrodynamical turbulence and the model of passive scalar in turbulent environment have been studied by means of the quantum field-theoretical renormalization group near two dimensions. A perturbative, two-parameter expansion scheme, the parameters of which are the deviation from the spatial dimension and the deviation of the exponent of the powerlike correlation function of the random force from their critical values, has been used in one-loop approximation. Systematic investigation of all possible critical regimes, corresponding to the fixed points of the renormalization group, has been performed taking into account the criterion of their physical relevance. For both models the analysis has confirmed the presence of a kinetic fixed point associated with the Kolmogorov universal regime. In this regime the universal value ($u^* = 1.562$) of inverse Prandtl numbers has been calculated. A comparison of the double-expansion method and the ε-expansion scheme usually applied in the statistical turbulence has also been carried out.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 1996

1 Introduction

It is known that systems with large number of degrees of freedom display similar behaviour in certain asymptotic regimes independently of numerous microscopic details of the system. In the theory of strongly developed turbulence this *universality* is connected with long-distance asymptotics of velocity correlation functions. The main indication of the universality in the turbulence comes from the well-known Kolmogorov scaling theory [1] describing the large-scale behaviour of equal-time velocity pair correlation function.

In the present paper we discuss the problem of universality for two models: (i) the model of randomly forced stationary and strongly developed magnetohydrodynamical turbulence (MHD) and (ii) the model of turbulent diffusion of passive scalar admixture (PA). After a brief description of the statistical methods used accompanied by the functional-integral approach we focus our attention to specific problems which are typical of the application of the renormalization group (RG).

The RG method represents a powerful, general and systematic technique in the theory of critical dynamics [2, 3], quantum-field theory or e.g. in the models of polymers. Since 1977 several authors [4]–[7] have applied the RG method to the model of randomly stirred fluid. This model describing hydrodynamic systems randomly stirred up by forces active at large spatial scales is considered to be close to the statistical behaviour of turbulent velocity modes at very high Reynolds numbers [8, 9]. Here the application of the RG method allows an investigation with minimal empirical input and adjustable parameters.

In hydrodynamics stochastic evolution of the local velocity field $\vec{v} \equiv \vec{v}(x)$, $x \equiv (\vec{x}, t)$ of incompressible fluid is described by the Navier-Stokes equation

$$\vec{N}(\{\vec{v}\}; [\nu_0]) \equiv \partial_t \vec{v} + (\vec{v}.\vec{\nabla})\vec{v} - \nu_0 \nabla^2 \vec{v} = \vec{f}^v \,, \tag{1}$$

and the incompressibility conditions

$$\vec{\nabla}.\vec{v} = 0, \qquad \vec{\nabla}.\vec{f}^v = 0. \tag{2}$$

The statistics of $\vec{v}$ is completely determined by the non-linear equations (1), (2) and certain assumptions about the statistics of the external large-scale random force $\vec{f}^v$. The dissipation term $\nu_0 \nabla^2 \vec{v}$ contains the parameter of kinematic viscosity ν_0.

As in refs. [4]–[7], [10] the random forces are chosen Gaussian with the probability distribution having the weight

$$\exp\left[-\frac{1}{2} \int \mathrm{d}x_1 \int \mathrm{d}x_2 \, f_j^v(x_1) \, [D^{-1}]_{js}(x_1 - x_2; [g_{v0}]) \, f_s^v(x_2) \right] , \tag{3}$$

with the notation: $x_j \equiv (\vec{x}_j, t)$, $\int \mathrm{d}x_j \equiv \int \mathrm{d}\vec{x}_j \int \mathrm{d}t$ for $j = 1, 2$; d is the space dimensionality. The superscript "v" denotes that $\vec{f}^v$ corresponds to the forcing of the velocity field. Complete determination of the forcing statistics requires definition of the averages

$$\langle f_j^v(x) \rangle = 0 \,, \qquad \langle \, f_j^v(x_1) f_s^v(x_2) \, \rangle = D_{js}(x_1 - x_2; [g_{v0}]) \,. \tag{4}$$

For positive values of the coupling parameter $g_{v_0} > 0$ the distribution function kernel

$$D_{js}(x;[g_{v_0}]) = g_{v_0}\,\delta(t) \int \frac{\mathrm{d}^d\vec{k}}{(2\pi)^d}\, k^{4-d-2\epsilon} \exp\left[i\vec{k}.\vec{x}\right] , \tag{5}$$

is a positive-definite $(d \times d)$-square matrix of the forcing correlation functions, where $P_{js}(\vec{k}) = \delta_{js} - k_j k_s/k^2$ is a solenoidal second-rank projector, and g_{v_0} is the coupling parameter. From (5) we see that time correlations of $\vec{f}^v$ have the character of white noise, while the spatial falloff of the correlations is controlled by the parameter ϵ. The matrix (5) shows translational invariance and for the value $\epsilon = 2$ becomes scale invariant. The value $\epsilon = 2$ is physically most acceptable, since it represents the assumption that random forces act at very large scales, which substitutes the effect of boundary conditions. The central problem of the ϵ-expansion, i.e the final continuation from $\epsilon = 0$ to sufficiently large $\epsilon = 2$ has been discussed in [11].

In the studies of statistics of the turbulence the most important measurable quantities of interest are N-th order correlation functions

$$\langle\, v_{j_1}(x_1) v_{j_2}(x_2) v_{j_3}(x_3) \cdots v_{j_N}(x_N)\,\rangle, \qquad 1 \le j_r \le d, \qquad r = 1, 2, \cdots N\,. \tag{6}$$

In this paper we have applied the RG method along the way initiated in [10]. The nature of the method adapted from critical dynamics (e.g. see [3]) is a formal mapping of the stochastic model (5) to a quantum field formulation, which is determined by an effective "action" $S\{\vec{v}, \vec{v}^{\cdot}\}$ constructed on the basis of the original stochastic problem. This action is a functional of the stochastic velocity field $\vec{v}$ and an independent transverse auxiliary field $\vec{v}^{\cdot}$. Then the correlation functions (6) can be expressed as variational derivatives

$$\frac{\delta^N G\left(\vec{A}\right)}{\delta A^v_{i_1}(x_1)\delta A^v_{i_2}(x_2)\delta A^v_{i_3}(x_3)\cdots\delta A^v_{i_N}(x_N)}\Big|_{\vec{A}^v=\vec{A}^{v'}=0}\,, \tag{7}$$

where the generating functional G is

$$G(\vec{A}^v, \vec{A}^{v^{\cdot}}) = \int \mathcal{D}\vec{v}\mathcal{D}\vec{v}^{\cdot}\ \exp\left[S\{\vec{v}, \vec{v}^{\cdot}\} + \int \mathrm{d}x\ \left(\vec{A}^v.\vec{v} + \vec{A}^{v'}.\vec{v}'\right)\right], \tag{8}$$

with the effective action

$$\begin{aligned} S\{\vec{v}, \vec{v}^{\cdot}\} &= \frac{1}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \left\{\nu_0^3\ v_j^{\cdot}(x_1) D_{js}(x_1 - x_2; [g_{v_0}])\, v_s^{\cdot}(x_2)\right\} \\ &+ \int \mathrm{d}x\ \vec{v}^{\cdot}.\left(-\vec{N}(\{\vec{v}\};[\nu_0])\right) \end{aligned} \tag{9}$$

and $\vec{A}^v, \vec{A}^{v^{\cdot}}$ are so called source fields, which are equivalent to the regular external forces; $\mathcal{D}\vec{v}\mathcal{D}\vec{v}^{\cdot}$ is the measure of functional integration over the fields $\vec{v}$ and $\vec{v}^{\cdot}$. Then the turbulence problem is transferred to calculation of functional integrals. They can

be in principle calculated perturbatively by means of the Feynman diagrammatic technique, which has been first applied to hydrodynamics by Wyld [12].

In the present paper we have studied the universal statistical features of two models. The *first* is the model of stochastic MHD described by the system of equations for the fluctuating velocity and magnetic fields [13, 14]

$$\begin{aligned} \vec{N}(\{\vec{v}\};[\nu_0]) - (\vec{b}.\vec{\nabla})\vec{b} &= \vec{f}^v\,, \\ \partial_t\vec{b} + (\vec{v}.\vec{\nabla})\vec{b} - (\vec{b}.\vec{\nabla})\vec{v} - \nu_0 u_0 \nabla^2\vec{b} &= \vec{f}^b\,. \end{aligned} \tag{10}$$

The *second* model is the statistical model of diffusion of passive admixture (PA) characterized by the concentration $c(x)$ in the turbulent environment (see e.g.[18],[24])

$$\begin{aligned} \vec{N}(\{\vec{v}\};[\nu_0]) &= \vec{f}^v\,, \\ \partial_t c + (\vec{v}.\vec{\nabla})c - \nu_0 u_0 \nabla^2 c &= f^c\,. \end{aligned} \tag{11}$$

The statistics of the random forces $\vec{f}^b, f^c$ introduced here will be specified later. It is a well-known fact that the diffusion of the magnetic field and the concentration has a different physical nature, therefore in this paper we do not distingush between magnetic and passive scalar inverse Prandtl numbers u_0 as it can be seen from (10) and (11).

It has been shown[14] that the behaviour of invariant variables corresponding to (10) and (11) leads to some common features in both problems in the universal Kolmogorov regime [17]. This is mainly due to that in the suitably reparametrized stochastic MHD theory the prefactor of the Lorentzian term $(\vec{b}.\vec{\nabla})\vec{b}$ vanishes at large spatial scales, i.e. the Lorentz force in the universal regime "dies out". We have compared vector and scalar stochastic models (10) and (11) in the framework of the RG double-expansion scheme, which will be specified later.

For the purposes associated with the application of the RG we introduce *one-particle irreducible* (1PI) Green functions [16]. These functions provide a useful tool for the study of RG problems in the theory of the turbulence [10]. For our purposes we introduce two types of 1PI Green functions Γ which correspond schematically to connected Feynman diagrams with two or three external legs.

For 3-d stochastic MHD within the one parameter ϵ-expansion scheme the following one-particle irreducible Green functions

$$\Gamma^{vv'}, \qquad \Gamma^{bb'}, \qquad \Gamma^{v'bb}, \tag{12}$$

possess superficial ultraviolet (UV) divergences [14].

A detailed analysis [18] of the renormalization of the passive scalar problem has shown the presence of superficial divergences in graphs corresponding to the 1PI Green function

$$\Gamma^{c'c}, \tag{13}$$

in the case of the ϵ-expansion scheme.

Ronis [19] has proposed a double-expansion approach with a simultaneous deviation $2\delta = d - 2$ from the spatial dimension d and also a deviation ϵ from the k^2 form of the forcing pair correlation function proportional to $k^{2-2\delta-2\epsilon}$. This work deals purely with an incompressible Navier-Stokes system and is based on the fact that at $d = 2$ a new divergent 1PI Green function $\Gamma^{v'v'}$ occurs. The main conclusion of the paper cited is that final prediction for the averaged energy dissipation composite operator is not trivial.

Of special relevance to the our investigation is the paper of Honkonen and Nalimov [20], which is in particular a critical response on the work of Ronis [19]. The essence of the argument used in [20] is that a non-local term

$$\int d^d\vec{x}_1 \int d^d\vec{x}_2\, \vec{v}'(\vec{x}_1,t)\vec{v}'(\vec{x}_2,t) \int \frac{d^d\vec{k}}{(2\pi)^d}\, k^{2-2\delta-2\epsilon} \exp\left[i\vec{k}.(\vec{x}_1-\vec{x}_2)\right]$$

is not renormalized and counter terms removing the divergences must always be local in space and time. The paper [20] contains not only the criticism of incorrectly applied renormalization, but puts forward a constructive proposal how to remove correctly the divergences by means of the *local* counter term $\vec{v}'\nabla^2\vec{v}'$.

Inspired by the study [20], we present here a generalized treatment of the models (10) and (11). In these theories the nonlocal terms of the action

$$\int d^d\vec{x}_1 \int d^d\vec{x}_2\, \vec{b}'(\vec{x}_1,t)\vec{b}'(\vec{x}_2,t) \int \frac{d^d\vec{k}}{(2\pi)^d}\, k^{2-2\delta-2a\epsilon} \exp\left[i\vec{k}.(\vec{x}_1-\vec{x}_2)\right] \qquad (14)$$

$$\int d^d\vec{x}_1 \int d^d\vec{x}_2\, c'(\vec{x}_1,t)c'(\vec{x}_2,t) \int \frac{d^d\vec{k}}{(2\pi)^d}\, k^{2-2\delta-2a\epsilon} \exp\left[i\vec{k}.(\vec{x}_1-\vec{x}_2)\right] \qquad (15)$$

(where a free parameter a is used to control the power form of magnetic and passive scalar forcings) requires the presence of analytic counter terms proportional to

$$\vec{b}'\nabla^2\vec{b}', \qquad c'\nabla^2 c'. \qquad (16)$$

The next section starts from the quantum field functional formulation of the problem of the turbulence for MHD. Such a formulation is suitable for the analysis based on the RG, which is described in detail in the third section. In the fourth section the RG analysis has been applied to the statistical theory of diffusion of PA in a turbulent environment. Finally the conclusions are presented, and the models are compared. The systematic classification of the RG fixed points (FP) and stability regions of different scaling regimes are presented in Appendices I and II. A useful methodological guide the reader can find in Appendix III, where the extraction of superficial divergences from the function $\Gamma^{c'c'}$ is presented.

2 Functional model of stochastic MHD

Let us consider the model (10) supplemented by the forcing statistics

$$\langle f_j^v(x_1) f_s^b(x_2)\rangle = 0,$$

$$\begin{aligned}\langle f_j^v(x_1) f_s^v(x_2)\rangle &= u_0\nu_0^3 D_{js}(x_1-x_2;[1,g_{v10},g_{v20}])\\ \langle f_j^b(x_1) f_s^b(x_2)\rangle &= u_0^2\nu_0^3 D_{js}(x_1-x_2;[a,g_{b10},g_{b20}])\,,\end{aligned} \qquad (17)$$

where the correlation matrix

$$\begin{aligned}D_{js}(x;\{A,B,C\}) &= \delta(t_1-t_2)\int \frac{d^d\vec{k}}{(2\pi)^d} P_{js}(\vec{k})\exp\left[i\vec{k}.\vec{x}\right]\\ &\times\left[B\,k^{2-2\delta-2A\epsilon} + C\,k^2\right]\end{aligned} \qquad (18)$$

is determined by the parameters A, B and C. Transversality of $f_j^v(x), f_s^b(x)$, is a consequence of the equations $\vec{\nabla}.\vec{v} = \vec{\nabla}.\vec{b} = 0$); The matrix D_{js} reflects a detailed intrinsic statistical definition of forcing, whose consequences are deeply discussed in [22]. The necessity to introduce a combined forcing, and also to include the additional couplings (g_{v2}, g_{b2}) in order to obtain multiplicatively renormalizable 2-d stochastic MHD is absent in the traditional formulation of stochastic hydrodynamics. The definition (18) includes two principal – low- and high-wave number – scale kinetic forcings separated by a transition region at the vicinity of the characteristic wavenumber of order $O([g_{v10}/g_{v20}]^{\frac{1}{\delta}})$. In the language of classical hydrodynamics the forcing contribution $\propto k^2$ corresponds to the appearence of large eddies convected by small and active ones. In analogy with (9) the stochastic MHD system can be described by the field-theoretical action

$$\begin{aligned}S^{\rm MHD} &= \frac{1}{2}\int dx_1 \int dx_2\\ &\quad \Big\{u_0\nu_0^3\, v_j'(x_1) D_{js}(x_1-x_2;[1,g_{v10},g_{v20}])\, v_s'(x_2) +\\ &+ u_0^2\nu_0{}^3\, b_j'(x_1)\, D_{js}(x_1-x_2;[a,g_{b10},g_{b20}])\, b_s'(x_2)\Big\} +\\ &+ \int dx\, \vec{v}'.\left(-\vec{N}(\{\vec{v}\};[\nu_0]) + (\vec{b}.\vec{\nabla})\vec{b}\right)\\ &+ \vec{b}'.\left(-\partial_t\vec{b} + u_0\nu_0\nabla^2\vec{b} + (\vec{b}.\vec{\nabla})\vec{v} - (\vec{v}.\vec{\nabla})\vec{b}\right).\end{aligned} \qquad (19)$$

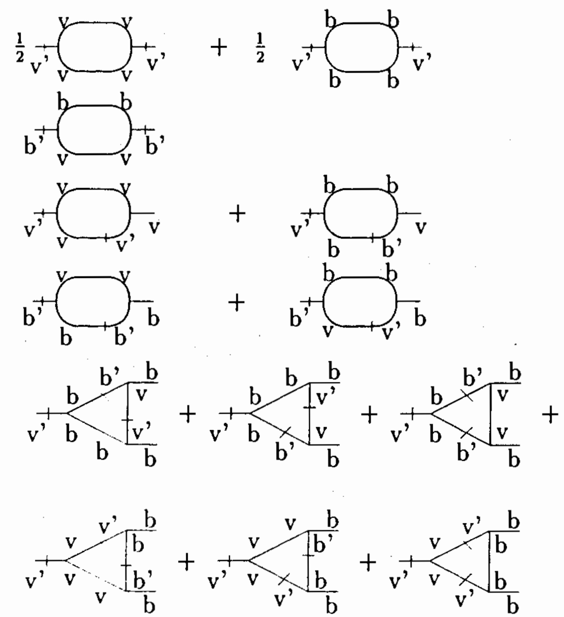
From previous work we know that this action containing non-analytic terms (here proportional to constants g_{v10}, g_{b10}) needs the introduction of two new terms (proportional to g_{v20} and g_{b20}). All dimensional constants g_{v10}, g_{b10}, g_{v20} and g_{b20}, which control the amount of randomly injected energy given by (17), (18), play the role of coupling constants of the perturbative expansion. Their universal values have been determined after the parameters ϵ, δ have been choosen to give the desired power form of forcing and desired dimension. In addition to this for $\delta = \frac{1}{2}$ one can obtain the action at $3d$.

For the convenience of further calculations the factors $\nu_0^3 u_0$ and $\nu_0^3 u_0^2$ including the "bare" (molecular) viscosity ν_0 and the "bare" (molecular or microscopic) magnetic inverse Prandtl number u_0 have been extracted.

3 Calculation of the fixed points

Let us summarize the main points of the RG procedure, which we have used. The general properties of applied methods and perturbative techniques are described elsewhere [16].

The model (19) is renormalizable by the standard power-counting rules, and for $\delta \to 0, \epsilon \to 0$, $d \to 2$ possesses the divergences (12) and also additional UV divergences $\Gamma^{v'v'}$, $\Gamma^{b'b'}$ typical of $d = 2$. In the diagrammatic representation the one-loop contribution to these functions looks as follows



In the framework of the quantum-field RG the computing of these Feynman diagrams includes:

1. the association of cubic terms of (19) with the vertices $v' \!\!-\!\!<^{v}_{v}$, $b' \!\!-\!\!<^{v}_{b}$, $v' \!\!-\!\!<^{b}_{b}$ of the diagrams;
2. the association of the elements of the free propagator $\hat{\Delta}$ with the internal diagrammatic lines $\underline{v' \quad v}$, $\underline{b' \quad b}$, $\underline{v \quad v}$, $\underline{b \quad b}$.

The free propagator $\hat{\Delta}$ calculated from the quadratic part of (19) has the matrix

structure

$$\hat{\Delta}_{js} = \begin{pmatrix} 0 & 0 & \Delta^{vv'}_{js} & 0 \\ 0 & 0 & 0 & \Delta^{bb'}_{js} \\ \Delta^{v'v}_{js} & 0 & \Delta^{vv}_{js} & 0 \\ 0 & \Delta^{b'b}_{js} & 0 & \Delta^{bb}_{js} \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & \underline{v_j \quad v_s'} & 0 \\ 0 & 0 & 0 & \underline{b_j \quad b_s'} \\ \underline{v_j' \quad v_s} & 0 & \underline{v_j \quad v_s} & 0 \\ 0 & \underline{b_j' \quad b_s} & 0 & \underline{b_j \quad b_s} \end{pmatrix} \qquad (20)$$

with the elements

$$\begin{aligned} \Delta^{vv'}_{js}(\vec{k},t) &= \Delta^{v'v}_{js}(-\vec{k},-t) = \theta(t)\, P_{js}(\vec{k})\, \exp[-\nu_0\, k^2 t]\,, &(21)\\ \Delta^{bb'}_{js}(\vec{k},t) &= \Delta^{b'b}_{js}(-\vec{k},-t) = \theta(t)\, P_{js}(\vec{k})\, \exp\left[-u_0 \nu_0 k^2 t\right]\,, \\ \Delta^{vv}_{js}(\vec{k},t) &= \frac{1}{2} u_0\, \nu_0^2\, P_{js}(\vec{k})\, \left(g_{v10}\, k^{-2\epsilon-2\delta} + g_{v20} \right) \exp\left[-\nu k^2 |t|\right]\,, \\ \Delta^{bb}_{js}(\vec{k},t) &= \frac{1}{2} u_0\, \nu_0^2\, P_{js}(\vec{k})\, \left(g_{b10}\, k^{-2a\epsilon-2\delta} + g_{b20} \right) \exp\left[-u\nu k^2 |t|\right]\,. \end{aligned}$$

The superscripts vv', bb', $v'v$, $b'b$, bb, vv in (21) correspond to notation of the lines of diagrams; $\theta(t)$ is the usual step function.

Having in mind that we need to calculate only divergences of diagrams and we are not interested in their finite parts, we performed (i) the integration over the internal time of diagrams (see Appendix III); (ii) the integration over the internal wave-numbers of diagrams with noninteger dimensionality $2 + 2\delta$ (dimensional regularization); (iii) the extraction of the poles

$$\text{Poles}_{\,\delta,\epsilon} \equiv \text{linear combination of terms} \left\{ \frac{1}{\delta}, \frac{1}{\epsilon}, \frac{1}{2\epsilon + \delta}, \frac{1}{(a+1)\epsilon + \delta} \right\}, \qquad (22)$$

(for $\delta \to 0, \epsilon \to 0$) in agreement with the general minimal subtraction scheme [23].

After the ultraviolet divergences have been removed, the continuation back to the original or "physical" values $\delta \to \frac{1}{2}$, $\epsilon \to 2$, $(d \to 3)$ is possible.

The UV divergences can be removed by adding suitable counter terms to the basic action $S^{\rm MHD}_B$, obtained from (19) by the substitution: $g_{v10} \to \mu^{2\epsilon} g_{v1}$, $g_{v20} \to \mu^{-2\delta} g_{v2}$, $g_{b10} \to \mu^{2a\epsilon} g_{b1}$, $g_{b20} \to \mu^{-2\delta} g_{b2}$, $\nu_0 \to \nu$, $u_0 \to u$ where μ is a scale setting parameter having the same canonical dimension as the wave number.

The original form of $S^{\rm MHD}_B$ implies the counter terms

$$\begin{aligned} S^{\rm MHD}_{\rm counter} &= \int \mathrm{d}x \left[\nu\, (1 - Z_1)\, \vec{v}'\nabla^2 \vec{v} + u\nu\, (1 - Z_2)\, \vec{b}'\nabla^2 \vec{b} + \right. \\ &+ \frac{1}{2} (Z_4 - 1) u\nu^3 g_{v2}\, \mu^{-2\delta}\, \vec{v}'\nabla^2 \vec{v}' + \frac{1}{2} (Z_5 - 1)\, u^2 \nu^3 g_{b2}\, \mu^{-2\delta}\, \vec{b}'\nabla^2 \vec{b}' + \\ &+ \left. (1 - Z_3)\, \vec{v}'(\vec{b}.\vec{\nabla})\, \vec{b} \right], \end{aligned} \qquad (23)$$

determined to cancel the superficial UV divergences of the Green functions.

Within the ultraviolet renormalization the divergences appearing in the form of a Laurent series in (22) are contained in the constants Z_1, Z_2, Z_4, Z_5 renormalizing the "bare" parameters e_0 and the constant Z_3 renormalizing the fields $\vec{b}, \vec{b}'$. Their general form is

$$Z_j \equiv 1 - \text{Poles}_{\delta,\epsilon}, \qquad j = 1, 2 \cdots 5. \tag{24}$$

Due to the Galilean invariance of action (19) the fields $\vec{v}', \vec{v}$ are not renormalized.

A property essential for application of the RG method is that the counter terms (23) can be chosen in a form containing a finite number of terms of the same algebraic structure as the terms of the action (19). Then all UV divergences of diagrams may be eliminated by a redefinition of the parameters of the original theory.

Renormalized Green functions are expressed in terms of the renormalized parameters

$$\begin{aligned} g_{v1} &= g_{v10}\,\mu^{-2\epsilon}\, Z_1^2 Z_2, \;\; g_{v2} = g_{v20}\,\mu^{2\delta}\, Z_1^2 Z_2 Z_4^{-1}, \\ g_{b1} &= g_{b10}\,\mu^{-2\,a\epsilon}\, Z_1 Z_2^2 Z_3^{-1}, \;\; g_{b2} = g_{b20}\,\mu^{2\delta}\, Z_1 Z_2^2 Z_3^{-1} Z_5^{-1}, \\ \nu &= \nu_0\, Z_1^{-1}, \quad u = u_0\, Z_2^{-1} Z_1 \end{aligned} \tag{25}$$

appearing in the renormalized action $S_R^{\text{MHD}} = S_B^{\text{MHD}} - S_{\text{counter}}^{\text{MHD}}$ connected with the action (19) by the relation of multiplicative renormalization

$$S_R^{\text{MHD}}\{\, \vec{v}, \vec{b}, \vec{v}', \vec{b}'\; e\} = S^{\text{MHD}}\{\, \vec{v}, \vec{b}Z_3^{\frac{1}{2}}, \vec{v}', \vec{b}' Z_3^{-\frac{1}{2}}, e_0\}.$$

The renormalized action S_R, which depends on the renormalized parameters $e(\mu)$, yields renormalized Green functions without UV divergences. The RG is mainly concerned with the prediction of the asymptotic behavior of correlation functions expressed in terms of anomalous dimensions γ_j by the use of β functions, both defined via differential relations

$$\gamma_j = \mu \frac{\partial \ln Z_j}{\partial \mu}\Big|_0, \quad \beta_g = \mu \frac{\partial g}{\partial \mu}\Big|_0, \text{ with } g \equiv \{g_{v1}, g_{v2}, g_{b1}, g_{b2}, u\}, \tag{26}$$

where the subscript "0" refers to partial derivatives taken at fixed bare values of e_0.

Using the RG routine the anomalous dimensions $\gamma_j(g_v, g_b, g_{v2})$ can be extracted from one-loop diagrams. The definitions (26) and expressions (25) for $d = 2 + 2\delta$ yield

$$\begin{aligned} \beta_{gv1} &= g_{v1}\,(-2\epsilon + 2\gamma_1 + \gamma_2), \;\; \beta_{gv2} = g_{v2}\,(2\delta + 2\gamma_1 + \gamma_2 - \gamma_4), \\ \beta_{gb1} &= g_{b1}\,(-2\,a\epsilon + \gamma_1 + 2\gamma_2 - \gamma_3), \;\; \beta_{gb2} = g_{b2}\,(2\delta + \gamma_1 + 2\gamma_2 - \gamma_3 - \gamma_5) \\ \beta_u &= u\,(\gamma_1 - \gamma_2). \end{aligned} \tag{27}$$

The partial derivative with respect to μ in (26) generates δ and $\epsilon -$ dependent terms which cancel some "mixed" poles of the type (22). As a consequence a finite result can be written

$$\begin{aligned} \gamma_1 &= \frac{1}{32\pi}(u\, g_v + g_b), \quad \gamma_2 = \frac{1}{8\pi}\frac{g_v - g_b}{u+1}, \\ \gamma_3 &= \frac{1}{16\pi}(g_b - g_v), \quad \gamma_4 = \frac{1}{32\pi}\frac{u\, g_v^2 + g_b^2}{g_{v2}}, \end{aligned} \tag{28}$$

where the notation $g_v = g_{v1} + g_{v2}$, $g_b = g_{b1} + g_{b2}$ has been used. Calculations also show that there are no divergences in $\Gamma^{b'b'}$ in one loop approximation and therefore

$$\gamma_5 = 0, \qquad Z_5 = 1, \tag{29}$$

which is a specific property of two-dimensional MHD.

Correlation functions of the fields are expressed in terms of scaling functions of the variable $s = \frac{k}{\mu}$, $s \in \langle 0, 1 \rangle$. Then the asymptotic behaviour and the universality of MHD statistics stem from the existence of a stable FP. The continuous RG transformation is an operation linking a sequence of invariant parameters $\overline{g}(s)$ determined by the Gell-Mann Law equation

$$\frac{d\overline{g}(s)}{d \ln s} = \beta_g\,(\overline{g}(s)) \;\; \text{with the abbrevation } \overline{g} \equiv \{\overline{g}_{v1}, \overline{g}_{v2}, \overline{g}_{b1}, \overline{g}_{b2}, \overline{u}\}, \tag{30}$$

where the scaling variable s parametrizes the RG flow with initial conditions $\overline{g}|_{s=1} \equiv g$ (the critical behaviour corresponds to infrared limit $s \to 0$). The expression of the $\beta(\overline{g}(s))$ function is known in the framework of the δ, ϵ expansion (see Eqs.(28) and also (27)). The FP $g^*(s \to 0)$ satisfies a system of equations $\beta_g(g^*) = 0$, while a stable FP, weakly dependent on initial conditions, is defined by positive definiteness of the real part of the matrix $\Omega = (\partial\beta_g/\partial g)|_{g^*}$. In other words, a FP is stable if all the trajectories $g(s)$ in its vicinity approach the FP.

The initial conditions $\overline{g}|_{s \to 1} = g$ of the equations (30), dictated by a micromodel, are insufficient since our aim is the large-scale limit of statistical theory, where $g^* \equiv \overline{g}|_{s \to 0}$. The RG fixed point is defined by the equation

$$\beta(\, g^*\,) = 0\,. \tag{31}$$

For $\overline{g}(s)$ close to g^* we obtain a system of linearized equations

$$\left(I\, s\frac{\mathrm{d}}{\mathrm{d}s} - \Omega \right)(\overline{g} - g^*) = 0, \tag{32}$$

where I is (5×5) unit matrix. Solutions of this system behave like $\overline{g} = g^* + \mathcal{O}(s^{\lambda_j})$ if $s \to 0$. The exponents λ_j are the elements of the diagonalized matrix $\Omega^{diag} = (\, \lambda_1,\, \lambda_2,\, \lambda_3,\, \lambda_4, \lambda_5\,)$ and can be obtained as roots of the characteristic polynomial $\mathrm{Det}(\Omega - \lambda I)$. The positive defineteness of Ω represented by the conditions $\mathrm{Re}_j(\lambda) \geq 0, j = 1, 2, ...5$ is the test of the infrared asymptotical stability of discussed theory.

Within the approach discussed above we have found a nontrivial stable *kinetic* FP [25] of the RG with the universal inverse Prandtl number

$$u^* = \frac{\sqrt{17} - 1}{2} \simeq 1.562\,,$$

the values of the coupling parameters

$$g_{v1}^* = \frac{64\,\pi}{9\,u^*}\frac{\epsilon\,(2\epsilon + 3\delta)}{\epsilon + \delta}, \quad g_{v2}^* = \frac{64\pi}{9\,u^*}\frac{\epsilon^2}{\delta + \epsilon} \tag{33}$$

and

$$g_{b1}^{*} = g_{b2}^{*} = 0\,.$$

The values of g_{v1}^{*} and g_{v2}^{*} correspond to those found previously in [20].

Detailed numerical calculations have shown that the region of stabilityof the *kinetic* FP is limited by the values of the parameter $a \le 1.42$ independently of the value of $\tilde{\delta} = \delta/\epsilon$.

The results obtained for *magnetic* FP (e.g. for all possible FP's, where $u^{*} = 0$) differ from those obtained in previous work [13, 14] with respect to the stability of the scaling regimes. We have arrived at the conclusion that the double expansion method leads to an*unstable magnetic regime*. All the FP's are listed in Appendix I.

4 Stability of kinetic scaling with passive admixture

Let us suppose that the transport of PA in the turbulent environment is characterized by the system of equations (11). A similar model was studied in [18], but here, as in the previous section, we introduce more complex forcing with the correlation function

$$\langle f^{c}(x_1) f^{c}(x_2)\rangle = \frac{u^2\nu^3}{(d-1)}\, D_{jj}(x_1 - x_2; [a, 1, g_c\mu^{-2(a\epsilon+\delta)}])\,. \tag{34}$$

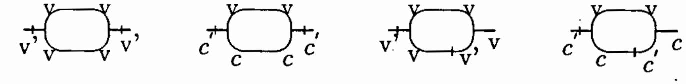
where g_c is a coupling parameter. The PA theory also can by transformed to a field theory with the basic action

$$\begin{aligned} S_B^{PA} &= \frac{1}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \\ &\Big\{ u_0\,\nu_0^3\; v_j'(x_1)\, D_{js}\,(x_1 - x_2; [1, g_{v_1}\mu^{2\epsilon}, g_{v_2}\mu^{-2\delta}])\; v_s'(x_2) \\ &+ \frac{1}{d-1} u^2\nu^3\; c'(x_1)\, D_{jj}(\,x_1 - x_2; [a, 1, g_c\mu^{-2(a\epsilon+\delta)}]\,)\; c'(x_2) \Big\} \\ &+ \int \mathrm{d}x \Big\{ c' \Big[-\partial_t c + u\nu\nabla^2 c - (\vec{v}.\vec{\nabla})c \Big] + \vec{v}'.\Big[-\vec{N}(\{\vec{v}\};[\nu]) \Big] \Big\}, \end{aligned} \tag{35}$$

containing the parameters u, ν, g_{v1}, g_{v2}, g_c, a (the notation is compatible with the previous section). Then the counter terms to the basic action are

$$\begin{aligned} S^{PA}_{\text{counter}} &= \int \mathrm{d}x \Big[\nu\,(1 - Z_1)\,\vec{v}'\nabla^2\vec{v} + u\nu\,(1 - Z_2)\,c'\nabla^2 c \\ &+ \frac{1}{2}(\,Z_4 - 1)u\nu^3 g_{v2}\,\mu^{-2\delta}\,\vec{v}'\nabla^2\vec{v}' \\ &+ \frac{1}{2}(\,Z_5 - 1)\,u^2\nu^3 g_c\,\mu^{-2(a\epsilon+\delta)}\,c'\nabla^2 c' \Big]\,. \end{aligned} \tag{36}$$

All the UV divergences (13) are present in the one-particle irreducible Green functions $\Gamma^{vv'}$, $\Gamma^{cc'}$, $\Gamma^{v'v'}$, $\Gamma^{c'c'}$. Their one loop diagrams are



Having performed the standard calculation of Z_1, Z_2, Z_4, Z_5, we have found the system of one loop β functions corresponding to the model (35)

$$\begin{aligned} \beta_{v1} &= g_{v1}\,(-2\epsilon + \gamma_2 + 2\,\gamma_1)\,, & \beta_{v2} &= g_{v2}\,(2\delta + \gamma_2 + 2\,\gamma_1 - \gamma_4)\,, \\ \beta_c &= g_c\,(2\,a\,\epsilon + 2\,\delta - \gamma_5)\,, & \beta_u &= u\,(\gamma_1 - \gamma_2)\,, \end{aligned} \tag{37}$$

where the $\gamma_1, \gamma_2, \gamma_3$ functions are

$$\gamma_1 = \frac{u g_v}{32\pi}\,, \qquad \gamma_2 = \frac{g_v}{8\pi(u+1)}, \qquad \gamma_4 = \frac{u g_v^2}{32\pi g_{v2}} \tag{38}$$

and $g_v = g_{v1} + g_{v2}$. In contrast with (29) we have found

$$Z_5 = 1 - A_5, \qquad A_5 = \frac{u}{16 g_c \pi (u+1)} \left[\frac{g_{v1}}{\delta + (1+a)\epsilon} + \frac{g_c g_{v1}}{\epsilon} + \frac{g_{v2}}{a\epsilon} - \frac{g_c g_{v2}}{\delta} \right] \tag{39}$$

with non-zero

$$\gamma_5 = \frac{g_v\,(\,g_c + 1\,)}{8\pi g_c (u+1)}\,. \tag{40}$$

The solution of the system of equations $\beta_{v_1} = \beta_{v_2} = \beta_c = \beta_u = 0$ with the aid of (37), (38) yields for the kinetic FP the same $g_{v1}^{*}, g_{v2}^{*}, u^{*}$ as in (33) and

$$g_c^{*} = (\,3a + 3\tilde{\delta} - 1\,)^{-1}\,. \tag{41}$$

We remind again that $\tilde{\delta} = \delta/\epsilon$. The calculation of the Ω matrix (up to the $\mathcal{O}(\epsilon)$ order) at this fixed point yields

$$\Omega^{diag} = \epsilon \left(\frac{2}{3}(-1 + 3a + 3\tilde{\delta}), \frac{17 - \sqrt{17}}{12}, \frac{4 + 3\tilde{\delta} \pm \sqrt{-8 - 12\tilde{\delta} + 9\tilde{\delta}^2}}{3} \right). \tag{42}$$

From this expression we see that the region of stability of the kinetic FP is $a \ge \frac{1}{3} - \tilde{\delta}$ for $\tilde{\delta} \ge 0$.

A classification of remaining fixed points of the PA problem is presented in Appendix II.

5 Conclusions

In the present work we have investigated and compared properties of two models, which display similar asymptotic behaviour: stochastic magnetohydrodynamics and diffusion of passive admixture in a turbulent environment. We have constructed field theoretical actions corresponding to the original stochastic problems. The actions are shown to be multiplicatively renormalizable for $d \geq 2$. We have constructed an expansion of both models in two dimensionless parameters: the deviation of the space from two and the deviation of the exponent of the powerlike correlation functions of stochastic forcing from the logarithmic value.

The principal consequences of the renormalizability conditions are (i) more complex form of the action [20] or alternatively (ii) more complex forcing correlation function [22] including large and small scale terms (18).

The results emphasize the sensitivity on the expansion strategy in many aspects. The double-expansion scheme with very complex structure of FP's has revealed some essential differences between transport of vector (MHD) and scalar (PA) species in turbulent velocity field. In the framework of the MHD model we have found only one stable kinetic FP corresponding to the known Kolmogorov scaling. The expected magnetic regime is unstable. At $d = 2$ diagrams of $\Gamma^{b'b'}$ are superficially divergent by power counting. However, calculations have shown that the one-loop contribution to $\Gamma^{b'b'}$ vanishes. Contrary to this, the model of PA yields non-zero function $\Gamma^{c'c'}$ even in the one-loop order. An open question is, whether the higher order corrections would generate essential changes to the picture of stability of the MHD model.

An important question is the physical relevance of the models discussed. A perhaps oversimplified idea would be to consider the results of the double-expansion scheme physically suitable to describe the quasi two dimensional ($2 < d < 3$) turbulent flows of the atmospheric and oceanografic systems and the more complex action (35) should describe a random spreading of pollutants in the turbulent atmosphere.

The most relevant physical picture of the turbulence is related to the kinetic FP. The next short table summarizes the information about the regions of stability of the kinetic FP obtained using different expansion schemes.

theory	Regions of stability	
	double expansion	ϵ-expansion
MHD	$a < 1.42$	$a < 1.15$ [14]
PA	$a > 1/3 - \tilde{\delta}$	$a > 2/3$ [24]

From this table we see that the character of inequality (i.e $<$ or $>$) remains the same despite the small differences of the boundary parameters. The universal values of the inverse Prandtl numbers (the same for both the models)

common	inverse Prandtl number	
	double expansion	ϵ-expansion
MHD and SPC	$u^* = 1.561$ [25]	$u^* = 1.393$ [18, 14]

exhibit an agreement with the general formula [26] obtained using $\epsilon-$ expansion scheme

$$u^*(d) = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8\,(d+2)}{d}}\right],$$

After the substitution $d = 2$, $d = 3$ we obtain the numerical values

$$u^*(2) = 1.561, \qquad u^*(3) = 1.393$$

which are consistent with the previous table. A remarkable feature of the results corresponding to expansion schemes applied and also a common feature of the models investigated is that the asymptotical stability of the kinetic FP is qualitatively not very sensitive to the way of the energy injection. We hope that all possibilities of perturbative RG are not drained off from afar and our work will have a stimulating influence on the development more eludicated expansion procedures.

M.Hnatich is grateful to D.I.Kazakov and to director D.V.Shirkov for hospitality at the Laboratory of Theoretical Physics, JINR, Dubna. This work was supported by Slovak Grant agency for science (grant 2/550/96).

Appendix I: Classification of fixed points of stochastic MHD .

In this Appendix we review all the FP corresponding to MHD. For unstable fixed points we use an abbreviation (**U.1–U.12**), whereas the stable ones (but unphysical, i.e. when at least one fixed parameter is negative) we denote as (**S.1–S.2**). Our investigation was made for the region of the parametric space, where $a \geq 0$, $\tilde{\delta} = \delta/\epsilon \geq 0$ and $\epsilon \geq 0$. We have fully excluded the complex fixed points from our consideration. When possible, we have tried to express the FP and the full structure of the diagonalized Ω-matrix in the most general form. In some cases, however, due to very complicated algebraic structure of the expressions obtained, we were forced to investigate the asymptotic stability only numerically.

U.1:

$$\begin{aligned} g^*_{v1} &= 0,\ g^*_{v2} = 0,\ g^*_{b1} = 0,\ g^*_{b2} = 0,\ u^* \text{is free parameter}, \\ \Omega^{diag} &= 2\epsilon(0,\, \tilde{\delta},\, \tilde{\delta},\, -a, -2). \end{aligned}$$

U.2:

$$\begin{aligned} g^*_{v1} &= 0,\ g^*_{v2} = -4\pi(1 \pm \sqrt{17})\epsilon\,\tilde{\delta},\ g^*_{b1} = 0,\ g^*_{b2} = 0, \\ u^* &= \frac{-1 \pm \sqrt{17}}{2}. \end{aligned}$$

U.3:

$$\begin{aligned} g^*_{v1} &= 16\pi\,\epsilon,\ g^*_{v2} = 0,\ g^*_{b1} = 0,\ g^*_{b2} = 0,\ u^* = 0, \\ \Omega^{diag} &= \epsilon(\pm 2, 5 + 2\tilde{\delta},\, 2(1 + \tilde{\delta}), 5 - 2a). \end{aligned}$$

U.4:

$$\begin{aligned} g^*_{v1} &= 0,\ g^*_{v2} = -16\pi\,\tilde{\delta}\,\epsilon,\ g^*_{b1} = 0,\ g^*_{b2} = 0,\ u^* = 0, \\ \Omega^{diag} &= \epsilon(\pm 2\tilde{\delta}, -3\tilde{\delta}, -(2a + 5\tilde{\delta}),\, -2(1 + \tilde{\delta})). \end{aligned}$$

U.5:

$$g^*_{v1} = 0,\ g^*_{v2} = \frac{16\pi\,\tilde\delta\,\epsilon}{11},\ g^*_{b1} = 0,\ g^*_{b2} = \frac{96\pi\,\tilde\delta\,\epsilon}{11},\ u^* = 0,$$
$$\Omega^{diag} = \epsilon(-2\tilde\delta, -2(a+\tilde\delta), -\frac{2}{11}(11+2\tilde\delta), \frac{15}{11}\tilde\delta, \frac{13}{11}\tilde\delta).$$

U.6:

$$g^*_{v1} = 0,\ g^*_{v2} = -64\pi\,\tilde\delta\,\epsilon,\ g^*_{b1} = 0,\ g^*_{b2} = -64\pi\,\tilde\delta\,\epsilon,\ u^* = 0,$$
$$\Omega^{diag} = 2(-\tilde\delta, -\tilde\delta, 5\tilde\delta, -(1+2\tilde\delta), -(a+\tilde\delta)).$$

U.7:

$$g^*_{v1} = 0,\ g^*_{v2} = -32\pi\tilde\delta\epsilon\frac{u^*+5}{(u^*)^2+8u^*+7},$$
$$g^*_{b1} = 0,\ g^*_{b2} = 32\pi\tilde\delta\epsilon\frac{(u^*)^2+u^*-4}{(u^*)^2+8u^*+7},\ u^* \simeq -2.72816.$$

U.8:

$$g^*_{v1} = \frac{\pi(11+24\tilde\delta+4\tilde\delta^2)\epsilon}{1+\tilde\delta},\ g^*_{v2} = \frac{\pi(5+2\tilde\delta^2)\epsilon}{1+\tilde\delta},$$
$$g^*_{b1} = 0,\ g^*_{b2} = 8\pi(5+2\tilde\delta)\,\epsilon,\ u^* = 0.$$

U.9:

$$g^*_{v1} = \frac{-8\pi(53+156\tilde\delta+153\tilde\delta^2+54\tilde\delta^3)\epsilon}{27(1+\tilde\delta)^2},\quad g^*_{v2} = \frac{8\pi(59+180\tilde\delta+171\tilde\delta^2+54\tilde\delta^3)\epsilon}{27(1+\tilde\delta)^2},$$
$$g^*_{b1} = 0,\ g^*_{b2} = \frac{16\pi(19+39\tilde\delta+18\tilde\delta^2)\epsilon}{9(1+\tilde\delta)},\ u^* = 0.$$

U.10:

$$g^*_{v1} = 0,\ g^*_{v2} = -\frac{8\pi}{11}(38a+81\tilde\delta\pm 9\sqrt{20a^2+76a\tilde\delta+81\tilde\delta^2})\epsilon,$$
$$g^*_{b1} = -\frac{16\pi}{11}(26a+45\tilde\delta\pm 5\sqrt{20a^2+76a\tilde\delta+81\tilde\delta^2})\epsilon,$$
$$g^*_{b2} = \frac{16\pi(19+39\tilde\delta+18\tilde\delta^2)\epsilon}{9(1+\tilde\delta)},\quad u^* = 0.$$

U.11:

$$g^*_{v1} = 0,\ g^*_{v2} = 32\pi a\epsilon\frac{u^*+5}{(u^*)^2+8u^*+7},\ g^*_{b1} = -32\pi a\epsilon\frac{(u^*)^2+u^*-4}{(u^*)^2+8u^*+7},\ g^*_{b2} = 0,$$

where u^* is the real and positive root of equation

$$(u^*)^3a + 2(\tilde\delta - a)(u^*)^2 - (11a+24\tilde\delta)u^* - 70\tilde\delta - 44a = 0\,.$$

U.12:

$$g^*_{v1} = \frac{\epsilon\pi(11+12a-4a^2+36\tilde\delta-8a\tilde\delta)}{1+\tilde\delta},\ g^*_{v2} = \frac{\epsilon\pi(2a-5)^2}{1+\tilde\delta},\ g^*_{b1} = 8\epsilon\pi(5-2a),$$
$$g^*_{b2} = 0,\ u^* = 0.$$

S.1:

$$g^*_{v1} = \frac{8\epsilon\pi(2+3\tilde\delta)(1-\sqrt{17})}{9(1+\tilde\delta)},\ g^*_{v2} = \frac{8\epsilon\pi(1-\sqrt{17})}{9(1+\tilde\delta)},\ g^*_{b1} = 0,\ g^*_{b2} = 0,$$
$$u^* = \frac{-1-\sqrt{17}}{2}.$$

The point is stable in the region $a < 0.74$.

S.2:

$$g^*_{v1} = \frac{8\epsilon\pi(53-162a+171a^2-54a^3-6\tilde\delta+18a\tilde\delta)}{27(1+\tilde\delta)(a-1)},$$
$$g^*_{v2} = \frac{8\epsilon\pi(-59+180a-171a^2+54a^3)}{27(1+\tilde\delta)(a-1)},$$
$$g^*_{b1} = \frac{16\epsilon\pi(-19+39a-18a^2)}{9(a-1)},\ g^*_{b2} = 0,\ u^* = -7+6a.$$

For this fixed point we have found that the interval of stability of the critical regime is $0.5 \le a \le 0.73$.

Appendix II: Classification of stochastic fixed points for PA.

These are FP of PA (we use the notation of the previous Appendix):

U.1:

$$g^*_{v1} = 0,\ g^*_{v2} = 0,\ g^*_c = 0,\ u^* = \text{free parameter},$$
$$\Omega^{diag} = \epsilon(0,\ 2\tilde\delta,\ 2(a+\tilde\delta),\ -2).$$

U.2:

$$g^*_{v1} = 0,\ g^*_{v2} = -4\pi(1\pm\sqrt{17})\epsilon\,\tilde\delta,\ g^*_c = -\frac{\tilde\delta}{2a+3\tilde\delta},\ u^* = \frac{-1\pm\sqrt{17}}{2},$$
$$\Omega^{diag} = \epsilon(-2\tilde\delta,\ \frac{\mp 2\sqrt{17}\,\tilde\delta}{1\pm\sqrt{17}},\ -(2\mp 3\tilde\delta),\ 2a+3\tilde\delta).$$

U.3:

$$\begin{aligned} g^*_{v1} &= 0,\ g^*_{v2} = -16\pi\tilde{\delta}\,\epsilon,\ g^*_c = -\frac{\tilde{\delta}}{a+2\tilde{\delta}},\ u^* = 0, \\ \Omega^{diag} &= \epsilon(\pm 2\tilde{\delta}, -2(1+\tilde{\delta}), 2(a+2\tilde{\delta})). \end{aligned}$$

U.4:

$$\begin{aligned} g^*_{v1} &= 16\pi\,\epsilon,\ g^*_{v2} = 0,\ g^*_c = \frac{1}{-1+a+\tilde{\delta}},\ u^* = 0, \\ \Omega^{diag} &= \epsilon(\pm 2, 2(1+\tilde{\delta}), 2(-1+a+\tilde{\delta})). \end{aligned}$$

An alternation of the signes of the kinetic FP (41) gives a non-physical analog which is stable:

S.1:

$$\begin{aligned} g^*_{v1} &= \frac{8\pi\,\epsilon(1-\sqrt{17})(2+3\tilde{\delta})}{9(1+\tilde{\delta})},\ g^*_{v2} = \frac{8\pi\,\epsilon(1-\sqrt{17})}{9(1+\tilde{\delta})},\ g^*_c = \frac{1}{(-1+3a+3\tilde{\delta})}, \\ u^* &= -\frac{1+\sqrt{17}}{2}, \\ \Omega^{diag} &= \epsilon(\frac{2(-1+3a+3\tilde{\delta})}{3}, \frac{4+3\tilde{\delta}\pm\sqrt{-8-12\tilde{\delta}+9\tilde{\delta}^2}}{3}, \frac{17+\sqrt{17}}{12}). \end{aligned}$$

and the interval of Kolmogorov scaling stability is $a \geq \frac{1}{3} - \tilde{\delta}$.

Appendix III: Superficial UV divergences of the function $\Gamma^{c'c'}$.

In this appendix we present some details of calculation of the renormalization constant Z_5 from (39). We start from the expression for the one loop contribution to the function $\Gamma^{c'c'}$, which is given by the Feynman integral

$$\int \frac{d^d\vec{k}}{(2\pi)^d} \int_{-\infty}^{\infty} dt\ \Delta^{cc}(\vec{q}-\vec{k},t)\Delta^{vv}_{js}(\vec{k},t)\,(iq_j)(iq_s)\,, \tag{43}$$

where Δ^{vv}_{js} is the same as in the (21) and $\Delta^{cc}(\vec{k},t)$ propagator is

$$\Delta^{cc}(\vec{k},t) = \frac{1}{2}u\nu^2\,\left(\mu^{2a\epsilon}k^{-2\epsilon a-2\delta} + \mu^{-2\delta}g_c\right)\,\exp\left[-u\nu k^2|t|\right]\,. \tag{44}$$

The time integration and the trace over the indices j, s yields

$$\begin{aligned} \Gamma^{c'c'}_{\text{1-loop}}(q) = \frac{u^2\nu^3}{2} \int \frac{d^d\vec{k}}{(2\pi)^d} \frac{(\vec{k}.\vec{q})^2 - q^2k^2}{k^2\left[k^2+u(\vec{k}-\vec{q})^2\right]} \\ \times\ \left(\mu^{2\epsilon}k^{-2\epsilon-2\delta}g_{v1} + \mu^{-2\delta}g_{v2}\right)\left(\mu^{2a\epsilon}|\vec{k}-\vec{q}|^{-2a\epsilon-2\delta} + \mu^{-2\delta}g_c\right)\,. \end{aligned} \tag{45}$$

The transformation of (45) into the spherical (k,φ) coordinates

$$\vec{k}.\vec{q} = k\,q\cos\varphi\,,\quad \varphi \in \langle 0, 2\pi),\quad k \in <0,\infty>,\quad d^d\vec{k} = k^{1+2\delta}dk\,d\varphi \tag{46}$$

and the separation of the quadratic in q part leads to

$$\begin{aligned} \Gamma_5 \equiv \frac{1}{2}\frac{\partial^2}{\partial q^2}\Gamma^{c'c'}_{\text{1-loop}}(q)\Big|_{q\to 0} = -\frac{u^2\nu^3}{8\pi(u+1)} \int_{\mu}^{\infty} \frac{dk}{k}\left(\mu^{-4\delta}k^{2\delta}g_{v2}g_c\right. \\ +\ \left.\mu^{2\epsilon(1+a)}k^{-2[\delta+\epsilon(1+a)]}g_{v1} + \mu^{2(\epsilon-\delta)}k^{-2\epsilon}g_c + \mu^{2(a\epsilon-\delta)}k^{-2a\epsilon}g_{v2}\right)\,. \end{aligned} \tag{47}$$

This integral diverges logarithmically as $\delta \to 0$ and $\epsilon \to 0$. The limits of integration $<0,\mu>$ have been omitted in (47), since integration does not lead to divergences for small k. Further, in agreement with the general rules of the minimal subtraction method [23], we use the convention

$$\mu^{\alpha} \int_{\mu}^{\infty} \frac{dk}{k^{1+\alpha}} = \frac{1}{\alpha}, \qquad \text{as} \qquad \alpha \to 0 \tag{48}$$

and find

$$\text{Poles}_{\,\delta,\epsilon}\ [\Gamma_5] = u^2\nu^3 g_c A_5, \tag{49}$$

with A_5 given by (39). Consequently, since the action (35) includes the term $c'\nabla^2 c'$ multiplied by the prefactor $u^2\nu^3 g_c$, the expression for Z_5 can be expressed as (39)

$$Z_5 = 1 - A_5\,. \tag{50}$$

A differentiation with respect to μ with fixed bare parameters gives the recurrent relation for $\gamma_5 = \partial \ln Z_5/\partial \ln \mu|_0$ (see (40)).

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Received by Publishing Department
on August 29, 1996.