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TM-SURFACE WAVES ALONG THE BOUNDARY BETWEEN TWO NONLINEAR ANISOTROPIC DIELECTRICS

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ТМ-поверхностные волны
на границе двух нелинейных неизотропных диэлектриков

Показывается, что уравнения Максвелла для поверхностных электромагнитных ТМ-волн, распространяющихся вдоль плоской границы раздела двух нелинейных диэлектриков с произвольным диагональным тензором диэлектрической проницаемости, зависящим от $|E|$, могут быть проинтегрированы в квадратурах.

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TM-Surface Waves Along
the Boundary Between Two Nonlinear Anisotropic Dielectrics

It is shown that the Maxwell's equations for surface electromagnetic TM waves, propagating along the plane boundary between two nonlinear dielectrics with arbitrary diagonal tensor of dielectric permittivity, depending on $|E|$, can be integrated in quadratures.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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Surface electromagnetic waves, propagating along the boundary between two media with different optical properties were widely studied [1-7]. Nonetheless the integration of the corresponding equations presents difficulties because of their complicated nonlinear form. We will consider surface TM waves for the case of two dielectric media with the diagonal dielectric permittivity tensor $\varepsilon_{ik} = \mathrm{diag}(\varepsilon_x, \varepsilon_y, \varepsilon_z)$, therewith ε_x, ε_y, ε_z are considered as arbitrary real functions of electric field strength $\mathbf{E}$.

Suppose the plane $x = 0$ to be that of separation and plane electromagnetic wave with frequency ω and wave vector $\mathbf{k}$ to propagate along z-axis. We will consider TM wave, assuming

$$\mathbf{H} = (0, H, 0); \quad \mathbf{E} = (E_x, 0, E_z).$$

Putting $\partial_t = -v\,\partial_z$, where $v = \omega/k = \beta c$ is the wave velocity, we write the system of Maxwell's equations in each medium:

$$\partial_x (\varepsilon_x E_x) + \partial_z (\varepsilon_z E_z) = 0, \tag{1}$$

$$\partial_x (H - \beta\,\varepsilon_x E_x) = 0, \tag{2}$$

$$\partial_z E_x - \partial_x E_z = \beta\,\partial_z H. \tag{3}$$

Inasmuch at $x = \pm\infty$ the field is supposed to vanish, from (2) we will find

$$H = \beta\,\varepsilon_x E_x. \tag{4}$$

Assuming that $\partial_z = ik$, $E_x = iA$, $E_z = B$, where A and B are real functions, from (1), (3) and (4) we deduce

$$(\varepsilon_x A)' + k\,\varepsilon_z B = 0, \tag{5}$$

$$B' = k\,A(\varepsilon_x \beta^2 - 1), \tag{6}$$

where prime denotes differentiation with respect to x. Taking the functions $\varepsilon_x(A^2 + B^2)$ and $\varepsilon_z(A^2 + B^2)$ as given ones, we will admit that $|\mathbf{E}|^2$ and ε_z can be expressed as some functions of ε_x, i.e.

$$|\mathbf{E}|^2 = A^2 + B^2 = I(\varepsilon_x), \qquad \varepsilon_z(|\mathbf{E}|^2) = K(\varepsilon_x). \tag{7}$$

Denoting $\varepsilon_x A = f$, from (5) and (7) we will find

$$I(\varepsilon_x) = \frac{f^2}{\varepsilon_x^2} + \frac{f'^2}{k^2 K^2(\varepsilon_x)}, \tag{8}$$

where, according to (5)

$$B = -\frac{f'}{k\,K(\varepsilon_x)}. \tag{9}$$

Differentiating the equation (8) in view of (6), one gets

$$\frac{dI}{d\varepsilon_x}\varepsilon_x' = (f^2)'\left(\frac{1}{\varepsilon_x^2} - \frac{\beta^2}{K} + \frac{1}{\varepsilon_x K}\right) - \frac{2f^2}{\varepsilon_x^3}\varepsilon_x'. \tag{10}$$

The equation (10) admits the integrating factor $Y(\varepsilon_x)$ (see Appendix 1):

$$\ln Y(\varepsilon_x) = \int^{\varepsilon_x} \frac{dK}{X K^2}\left(\frac{1}{\varepsilon_x} - \beta^2\right) + \int^{\varepsilon_x} \frac{d\varepsilon_x}{\varepsilon_x^2 K X}, \tag{11}$$

where

$$X(\varepsilon_x) = \frac{1}{\varepsilon_x^2} - \frac{\beta^2}{K} + \frac{1}{\varepsilon_x K}. \tag{12}$$

Given $\varepsilon_x(0)$, the value of permittivity ε_x at $\mathbf{E} = 0$, i.e. at $x = \pm\infty$, from (10) we deduce the quadrature

$$\int_{\varepsilon_x(0)}^{\varepsilon_x} Y(\varepsilon_x)\,dI(\varepsilon_x) = f^2 X(\varepsilon_x) Y(\varepsilon_x). \tag{13}$$

The equation (13), thus, defines the function $f = F(\varepsilon_x)$, and finally one gets $A = \varepsilon_x^{-1} F(\varepsilon_x)$.

Solving the equation (8) with respect to f' and taking into account (13), we find the equation

$$f' = \frac{dF}{d\varepsilon_x}\varepsilon_x' = \pm k\,K(\varepsilon_x)\left[I(\varepsilon_x) - \frac{F^2(\varepsilon_x)}{\varepsilon_x^2}\right]^{1/2},$$

which is integrated in quadratures:

$$k\,x = \pm\int \frac{dF(\varepsilon_x)}{K(\varepsilon_x)\left[I(\varepsilon_x) - \frac{F^2(\varepsilon_x)}{\varepsilon_x^2}\right]^{1/2}}. \tag{14}$$

Here the sign $\pm$ is to be chosen in accordance with the domain $x > 0$ or $x < 0$. The integration constants in (14) are to be found from the boundary conditions, which are equivalent to the continuity conditions for the functions f and f' at $x = 0$. The solution of the boundary conditions, mentioned, is defined by the form of the functions $\varepsilon_x(|\mathbf{E}|^2)$ and $\varepsilon_z(|\mathbf{E}|^2)$.

Let us consider a simple example assuming that

$$\varepsilon_x = a_x + b_x|\mathbf{E}|^2, \quad \varepsilon_y = a_y + b_y|\mathbf{E}|^2, \quad \varepsilon_z = a_z + b_z|\mathbf{E}|^2, \tag{15}$$

with a_x, a_y, a_z, b_x, b_y, b_z being constants. Then from (7) one gets the functions:

$$I(\varepsilon_x) = \frac{\varepsilon_x - a_x}{b_x}, \quad K(\varepsilon_x) = q + \frac{b_z}{b_x}\varepsilon_x, \tag{16}$$

with $q = a_z - (b_z/b_x)\,a_x$. Putting them into (11) we get (see Appendix 2)

$$Y(\varepsilon_x) = \left(q + \frac{b_z}{b_x}\varepsilon_x\right)\left[\frac{2\beta^2\varepsilon_x + d - \sqrt{D}}{2\beta^2\varepsilon_x + d + \sqrt{D}}\right]^{d/b_x\sqrt{D}}, \tag{17}$$

where $d = b_z/b_x - 1$ and $D = (1 + b_z/b_x)^2 + 4\beta^2 q$. Here we assume that $D > 0$. For simplicity we will now consider the case when the dielectric is isotropic, i.e.

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon = a + b|\mathbf{E}|^2. \tag{18}$$

In this case one gets the integrating factor $Y(\varepsilon) = \varepsilon$. Putting it into (13) and taking into account that at $x \to \infty$, $f \to 0$ and $\varepsilon \to a$, we find

$$f = F(\varepsilon) = \sqrt{\frac{\varepsilon(\varepsilon^2 - a^2)}{2b(2 - \beta^2\varepsilon)}}. \tag{19}$$

The equation (14) in this case reads:

$$k\,x = \pm\int \frac{3\varepsilon^2 - \beta^2\varepsilon^3 - a^2}{\varepsilon(\varepsilon - a)(2 - \beta^2\varepsilon)\sqrt{P(\varepsilon)}}\,d\varepsilon, \tag{20}$$

where $P(\varepsilon) = (\varepsilon + a)(3\varepsilon - 2\beta^2\varepsilon^2 - a)$. One can rewrite this equality in the form:

$$k\,x = \pm\left[\int \frac{d\varepsilon}{\sqrt{P(\varepsilon)}} + a\int \frac{d\varepsilon}{(\varepsilon - a)\sqrt{P(\varepsilon)}} + \right.$$
$$\left. + \frac{a}{2}\int \frac{d\varepsilon}{\varepsilon\sqrt{P(\varepsilon)}} + \left(1 + \frac{a\beta^2}{2}\right)\int \frac{d\varepsilon}{(2 - \beta^2\varepsilon)\sqrt{P(\varepsilon)}}\right]. \tag{21}$$

Using the substitution

$$\varepsilon = \frac{3}{4\beta^2} + \frac{\mu}{4\beta^2}\cos 2\varphi, \quad \mu = \sqrt{9-8\nu}, \quad \nu = a\beta^2, \qquad (22)$$

one can rewrite the last equality by means of elliptical integrals (see Appendix 3):

$$k\,x = \mp[C_0 F(\varphi, s) + \sum_{j=1}^{3} C_j \Pi_j(\varphi; n_j, s)], \qquad (23)$$

where

$$C_0 = \frac{2\sqrt{2}}{\xi}, \qquad s^2 = \frac{2\mu}{\xi^2},$$
$$C_1 = \frac{8\sqrt{2}\nu}{\xi(3-4\nu+\mu)}, \quad n_1 = -\frac{2\mu}{3-4\nu+\mu},$$
$$C_2 = \frac{4\sqrt{2}\nu}{\xi(3+\mu)}, \quad n_2 = -\frac{2\mu}{3+\mu},$$
$$C_3 = \frac{8\sqrt{2}(1+\nu/2)}{\xi(5+\mu)}, \quad n_3 = -\frac{2\mu}{5+\mu},$$

with $\xi = \sqrt{3+4\nu+\mu}$.

The boundary conditions at $x = 0$

$$\varepsilon_1 E_{x1} = \varepsilon_2 E_{x2}, \qquad (24)$$

and

$$E_{z1} = E_{z2} \qquad (25)$$

for the case considered can be represented as follows:

$$\frac{b_2}{b_1}\frac{\varepsilon_1(\varepsilon_1 - a_1)(\varepsilon_1 + a_1)}{\beta_1^2\varepsilon_1 - 2} = \frac{\varepsilon_2(\varepsilon_2 - a_2)(\varepsilon_2 + a_2)}{\beta_2^2\varepsilon_2 - 2}, \qquad (26)$$

$$\frac{(\varepsilon_1 - q_1 + \tau_1)(\varepsilon_1 - q_1 - \tau_1)}{\varepsilon_1^2(\varepsilon_1 + a_1)} = \frac{(\varepsilon_2 - q_2 + \tau_2)(\varepsilon_2 - q_2 - \tau_2)}{\varepsilon_2^2(\varepsilon_2 + a_2)}, \qquad (27)$$

where $q_i = 3/4\beta_i^2$ and $\tau_i = \sqrt{9 - 8a_i\beta_i^2}/4\beta_i^2, \quad i = 1, 2$.

The explicit form of relations (26) and (27) makes evident the existence of such parameters b_2/b_1 or β for which the equations (26) and (27) are satisfied.

Appendix 1:

Multiplying the equation (10) by Y one gets:

$$Y\frac{dI}{d\varepsilon_x}\varepsilon_x' = (f^2)'\left(\frac{1}{\varepsilon_x^2} - \frac{\beta^2}{K} + \frac{1}{\varepsilon_x K}\right)Y - \frac{2f^2}{\varepsilon_x^3}\varepsilon_x' Y.$$

The righthand side of the last equation can be written as:

$$[f^2 X Y]' - \frac{2f^2}{\varepsilon_x^3}\varepsilon_x' Y - f^2 X Y' - f^2 X' Y.$$

Here $X = 1/\varepsilon_x^2 - \beta^2/K + 1/\varepsilon_x K$. Equating the last three terms of the R.H.S. to zero we come to the equation

$$\frac{Y'}{Y} = \frac{K'}{XK^2}(\frac{1}{\varepsilon_x} - \beta^2) + \frac{\varepsilon'}{\varepsilon^2 K X},$$

which leads to the equation (11).

Appendix 2:

Putting $K(\varepsilon_x)$ and X into (11) one gets

$$\ln Y(\varepsilon_x) = (b_z/b_x)\int\frac{d\varepsilon_x}{K} - d\int\frac{d\varepsilon_x}{K + \Omega},$$

with $\Omega = \varepsilon_x(1 - \beta^2\varepsilon_x)$. Assuming that $D = (1 + b_z/b_x)^2 + 4\beta^2 q > 0$ after integrating one finds:

$$\ln Y(\varepsilon_x) = \ln|\,q + \frac{b_z}{b_x}\varepsilon_x\,| + [d/b_x\sqrt{D}]\ln|\,\frac{2\beta^2\varepsilon_x + d - \sqrt{D}}{2\beta^2\varepsilon_x + d + \sqrt{D}}\,|.$$

Appendix 3:

For $\varepsilon = \frac{3}{4\beta^2} + \frac{\mu}{4\beta^2}\cos 2\varphi$, one gets $d\varepsilon = -\frac{\mu}{2\beta^2}\sin 2\varphi\, d\varphi$, and

$$P(\varepsilon) = \frac{\mu^2}{8\beta^2}\sin 2\varphi\left[\frac{(3 + 4\beta^2) + \mu\cos 2\varphi}{4\beta^2}\right].$$

In view of these expressions we get

$$\int\frac{d\varepsilon}{\sqrt{P(\varepsilon)}} = -C_0\int\frac{d\varphi}{\sqrt{1 - s^2\sin^2\varphi}} = -C_0 F(\varphi, s).$$

Analogously one can find the other terms.

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