ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E17-95-348

J.Buša[1], M.Hnatich[2], J.Honkonen[3], D.Horvath[2]

THE INSTABILITIES OF ANISOTROPICALLY DRIVEN DEVELOPED TURBULENCE IN *d*-DIMENSIONAL CASE

Submitted to International Conference MAHYD-95, Riga

[1]Technical University, Košice, Slovakia
[2]Slovak Academy of Sciences, Košice, Slovakia
[3]University of Helsinky, Helsinky, Finland

1995

Буша Я. и др. E17-95-348

Неустойчивости в развитой турбулентности с анизотропной накачкой d-мерном случае

В настоящей работе рассматривается стохастическая модель развитой однородной турбулентности с сильной анизотропией. Была вычислена константа Колмогорова и найдена ее связь с амплитудами спектральных проекций (параллельные и перпендикулярные потоку компоненты) в случае когда конкурирующее взаимодействие между анизотропией внешней накачки и динамикой, описываемой уравнением Навье Стокса, может оказывать влияние на стабильность колмогоровского режима. Исследуется также более сложная стохастическая магнитная гидродинамика.

Работа выполнена в Лаборатории теоретической физики им.Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1995

Busa J. et al. E17-95-348

The Instabilities of Anisotropically Driven Developed Turbulence in d-dimensional Case

In the present paper the stochastic model of the homogeneous developed turbulence with strong anizotropy is investigated. The Kolmogorov constant has been calculated and its relation with amplitudes of spectral projections (streamwise and cross-streamwise components) has been established in the situations, where the competing mechanism between anisotropy of external forcing and dynamics governed by Navier-Stokes equation can have influence on the stability of the Kolmogorov regime. An extension to more complex stochastic magnetic hydrodynamics is also considered.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research, Dubna, 1995

1 Introduction

The strong nonlinearity of the Navier–Stokes equation for high Reynolds numbers makes the theoretical description of developed turbulence very difficult. From the 70-ties there has been a significant increase of interest in the theoretical investigation of the statistical hydrodynamic models, where the phenomenological statistics of external random forcing has been used for the modelling of very complex flow instabilities. In the present paper we have discussed the stochastic variant of Navier–Stokes and MHD equations in the case when the axial symmetry of external random forcing has been taken into account. Let us note, that stochastic differential equations can be solved directly by using numerical schemes [1]. Instead, in this paper we have applied the analytical approach leading directly to the approximate expressions for correlation functions.

The possible deviation of the fully developed turbulence from the isotropic statistics of velocity fluctuations was confirmed by a number of experiments and computer simulations [2]. The contribution of dominantly oriented fluctuations developed as a consequence of large scale anisotropy is a permanently discussing aspect of the turbulence physics.

This deviation may be induced by a presence of specific initial or boundary conditions, interactions of fluctuating fields with a mean flow gradients or external fields. In the context of measurement it is known, that the turbulence behind grid tends to be anisotropic with streamwise component slightly larger than the cross-stream components [3] and anisotropy contributions could be relevant for the experimental errors in determination of the turbulent energy dissipation [4].

The fundamental theoretical study of the kinetic energy spectra splitting induced by the presence of axial symmetry has been given in the comprehensive paper of Herring [5], where applied direct interaction approximation could be considered as an important starting point for development of this particular scientific region.

Many authors incline to the opinion, that the Kolmogorov local isotropy postulate, with the assumption that developed turbulence of the inertial range is independent of the viscous cutoff (also shape and integral length characterizing the boundary conditions), creates only the rough basis for the satisfactory understanding of turbulence complexity. Therefore, the advanced statistical hydrodynamics tries to give more accurate answer on the validity of the classical Kolmogorov phenomenology, which ignores the effect of anisotropy inside the inertial subscales. The results of renormalization group (RNG) analysis, which are presented in this paper hopefully sheds some more light on this problem.

An initial important motivation of the present work comes from the study of weakly anisotropic stochastic magnetohydrodynamics (MHD) [6]. It shows that even small driving anisotropy of the forcing leads to an asymptotical increase of large scale effective Lorentzian terms. It was found that the presence of the latter is crucial for suppression of the Kolmogorov scaling invariance (called the instability of critical regime in this paper). The present work, which sufficiently complements the calculations of Rubinstein and Barton [7], tries to give a proposal how to modify the stability of critical regimes of stochastic axisymmetric MHD.

2 The stability of anisotropic hydrodynamic theory

We start with the description of the random velocity flow $\vec{v}(\vec{x},t)$ ($\vec{v}$ and $\vec{x}$ are d-dimensional vectors) the evolution of which is governed by the randomly forced Navier–Stokes equation (modified from [8])

$$\begin{aligned} &\partial_t \vec{v} + (\vec{v}\vec{\nabla})\vec{v} - \nu\vec{\nabla}^2\vec{v} - \vec{f}^{\text{A-dis-v}} = \vec{f}^v ,\\ &\vec{\nabla}.\vec{v} = \vec{\nabla}.\vec{f}^v = 0 . \end{aligned} \tag{1}$$

The explicit form of the anisotropic dissipation force $\vec{f}^{\text{A-dis-v}}$ (as a consequence of symmetry of $\vec{f}^v$) will be specified later. Here we continue the generalization of the traditional turbulence models based on the assumption of the presence of large scale random forcing $\vec{f}^v(\vec{x},t)$ with the Gaussian statistics completely defined by the averages

$$\langle f_j^v\rangle = 0, \quad \langle f_j^v(\vec{x}_1,t_1)\, f_s^v(\vec{x}_2,t_2)\rangle = D_{js}\,(\vec{x}_1 - \vec{x}_2, t_1 - t_2) . \tag{2}$$

The two-point correlation matrix

$$D_{js}(\vec{x},t) = \delta(t)\int \frac{d^d\vec{k}}{(2\pi)^d}\, D_{js}^{st}(\vec{k})\exp\left[i\,\vec{k}\vec{x}\right] , \tag{3}$$

can be parametrized as [7, 9]

$$D_{js}^{st}(\vec{k}) = g_v\nu^3\,\Lambda^{2\epsilon}\,k^{4-d-2\epsilon}\left(\left[1+\alpha_1\xi_k^2\right]P_{js}(\vec{k}) + \alpha_2 R_{js}(\vec{k})\right) \tag{4}$$

in d-dimensional and anisotropic case. In expression (3) vector $\vec{k}$ denotes the wave number and vector $\vec{n}(|\vec{n}| = 1)$ determines the anisotropy axis. The solenoidal P, R and longitudinal $P^{\|}$ operators are defined by relations

$$\begin{aligned} P_{js}^{\|}(\vec{k}) &= \frac{k_j\,k_s}{k^2}, \quad P_{js}(\vec{k}) = \delta_{js} - P_{js}^{\|}(\vec{k}),\\ R_{js}(\vec{k}) &= \left(n_j - \xi_k\frac{k_j}{k}\right)\left(n_s - \xi_k\frac{k_s}{k}\right), \quad \xi_k = \frac{\vec{k}.\vec{n}}{k}. \end{aligned}$$

The most general parametrization of the non-helical D_{js} tensor (dependent on the undimensional free parameters α_1 and α_2 ($\alpha_1 > -1, \alpha_2 > 0$) is constructed to maintain the requirement of the incompressibility. We emphasize that in contrast with the stimulating paper [7], these parameters are not considered as strictly small in the presented work. The parameter Λ with the dimension of wave number is the

scale setting parameter having the size from micro-eddy region; g_v is the positive undimensional constant. The usual assumption of stochastic theories exhibiting the Kolmogorov scaling behavior is that the forcing to be localized around the zero of wave number, and physical value of ϵ parameter is $\epsilon = 2$.

As usual, the measurable quantities in the study of the strong turbulence are considered the statistical objects represented by N-th order velocity correlations functions defined by means of functional integral

$$\langle\prod_{j=1}^{N} v_m(\vec{x}_j,t_j)\rangle = \prod_{j=1}^{N}\int \left[d^d\vec{v}\right]\left[d^d\tilde{\vec{v}}\right]\; v_m(\vec{x}_j,t_j)\,\mathcal{P}(\vec{v},\tilde{\vec{v}}), \;\; m \in 1,2...d. \tag{5}$$

The conclusions of [10] enable us to write the probability distribution in form $\mathcal{P}$ =norm$\times$exp$[S]$ with the action

$$\begin{aligned} S \;=\; & \int d^d\vec{x}_1 dt_1\, d^d\vec{x}_2 dt_2 \left[\frac{1}{2}\,\tilde{v}_j(\vec{x}_1,t_1)\,D_{js}(\vec{x}_1-\vec{x}_2,t_1-t_2)\,\tilde{v}_s(\vec{x}_2,t_2)\right]\\ & + \int d^d\vec{x}dt\left\{\tilde{\vec{v}}(\vec{x},t)\left[-\partial_t\vec{v} - (\vec{v}\vec{\nabla})\vec{v} + \nu\vec{\nabla}^2\vec{v} + \vec{f}^{\text{A-dis-v}}\right]_{(\vec{x},t)}\right\}. \end{aligned} \tag{6}$$

where auxiliary incompressible field $\tilde{\vec{v}}$ was introduced along the way of transformation of (1) into the functional form. The RNG method here provides the correct description of the large scale behaviour of the correlation functions (5).

In order to obtain power - counting renormalizable theory [10, 7] we need to introduce terms proportional to nonzero parameters (they will be denoted here χ_1, χ_2 and χ_3). These new parameters, which are required to bring the cancellation of diagram divergences are viscous tensor components representing the orientational redistribution of energy dissipation. The force of anisotropic viscous dissipation has the explicit form

$$\vec{f}^{\text{A-dis-v}} = \nu\left[\chi_1(\vec{n}.\vec{\nabla})^2\vec{v} + \chi_2\vec{n}\vec{\nabla}^2(\vec{n}.\vec{v}) + \chi_3\vec{n}(\vec{n}.\vec{\nabla})^2(\vec{n}.\vec{v})\right] . \tag{7}$$

From the methodological point of view at some stage of RNG procedure the correlations (5) are expressed in terms of scaling function containing the continuos effective variables $\overline{g}(s), \overline{\chi}(s)$, which are the functions of rescaled wave number $s = k/\Lambda$. The scale dependent effective variables are governed by the set of differential equations [11]:

$$\begin{aligned} s\frac{d\overline{g}_v}{ds} &= \beta_{g_v}\left(\overline{g}_v, \overline{\chi}_j;\, \alpha_{1,2}, d\right), \quad j = 1,2,3,\\ s\frac{d\overline{\chi}_j}{ds} &= \beta_{\chi_j}\left(\overline{g}_v, \overline{\chi}_j;\, \alpha_{1,2}, d\right), \quad s \in [0,1] \end{aligned} \tag{8}$$

with the initial conditions

$$\overline{g}_v|_{(s=1)} = g_v, \quad \overline{\chi}_j|_{(s=1)} = \chi_j . \tag{9}$$

d	$\alpha_2\vert_{\text{for }\alpha_1=0}$	$\alpha_1\vert_{\text{for }\alpha_2=0}$
2.7	$\langle 0; 0.00022\rangle$	$\langle -0.17; 0.05\rangle$
2.8	$\langle 0; 0.0034\rangle$	$\langle -0.51; 0.18\rangle$
2.9	$\langle 0; 0.0096\rangle$	$\langle -0.72; 0.33\rangle$
3	$\langle 0; 0.018\rangle$	$\langle -0.85; 0.49\rangle$

Table 1: The limits on the selection of free parameters $\alpha_{1,2}$ and d necessary to create the conditions for formation of the stable kinetic fixed point. The investigation of stability was made in two cases $\alpha_1 = 0$ and $\alpha_2 = 0$. We found that the size of stable regions of $\alpha - d$ parametric space increases with the increase of spatial dimension for $3 > d > d_c$. For $d = d_c$ the destabilization occurs in the zero anisotropy limit.

The large scale limit of statistical theory is described by the fixed point parameters of RNG

$$\begin{aligned} \overline{g}_{v(s\to 0)} &= g_v^*, \quad \overline{\chi}_{j(s\to 0)} = \chi_j^* \\ \beta_{g_v}(g_v^*, \chi_j^*, \alpha_{1,2}, d) &= 0, \quad \beta_{\chi_j}(g_v^*, \chi_j^*, \alpha_{1,2}, d) = 0. \end{aligned} \tag{10}$$

The explicit form of complicated β_g and β_{χ_j} functions obtained from one loop diagram is presented in Appendix II. Because of complexity of (8) only the numerical study of RNG flows can clarify for us the selected aspects of strong anisotropy problem.

The numerical investigation confirms the existence of the universal kinetic scaling regime corresponding to *stable fixed point of RNG*. The Kolmogorov spectral index of this fixed point remains fixed and insensible to d value.

The various aspects of axisymmetrically driven turbulence were discussed by Rubinstein and Barton [7]. The presented study reflects a preliminary effort at analyzing of improved version of their model. A key step of a new formulation is recovering of the relevance of the term $\chi_3 \nu \vec{n}(\vec{n}.\vec{\nabla})^2(\vec{n}.\vec{v})$ missed in the consideration [7]. According to our investigation of (1) in small anisotropy limit, the existence of nonzero χ_3 parameter is irrelevant for the stability of theory at $d = 3$. But, as it will be shown later, to achieve the correct theory near some dimension $d = d_c < 3$ (where even a small driving anisotropy destabilizes the inertial scaling) the incorporation of χ_3 term turns out to be necessary.

Even a preliminary calculations have suggested us that qualitative changes of anisotropicaly driven turbulence can take place at dimensions somewhere between two and three. We found the value of critical dimension

$$d_c = \frac{3\sqrt{17} - 7}{2} \simeq 2.6846584384, \tag{11}$$

below of which the inertial range (kinetic regime) cannot exist under the conditions of nonzero anisotropy. The numerical studies of equations show that for $d > d_c$, the values of parameters $\alpha_{1,2}$ must be limited in the stable scaling regime. The numerical studies focussed on selected dimensions $d = 2.7, 2.8, 2.9, 3.0$ show that the range of kinetic regime stability in $\alpha-$ parametric space increases with dimension [see table (1)]. For $d = 3$ the equations (8) enable to find a stable solution when $(\alpha_1)_{min} < \alpha_1 \le (\alpha_1)_{max}$ and $0 < \alpha_2 < (\alpha_2)_{max}$, for extremal values

$$(\alpha_1)_{max} \simeq 0.49, \quad (\alpha_1)_{min} \simeq -0.85, \quad (\alpha_2)_{max} \simeq 0.18. \tag{12}$$

3 The effect of the induced anisotropy on the effective Kolmogorov constant

The basic parameter characterizing the properties of the inertial range energy cascade is the Kolmogorov constant C_k. Here we discuss the anisotropic case, where the effective C_k depends on the parameters α_1, α_2.

In the anisotropic case the equal–time two point velocity pair correlation function can be expressed in the form

$$\langle v_j(\vec{x}_1, t_1)\, v_s(\vec{x}_2, t_2)\rangle|_{t_1 \equiv t_2} = \int \frac{\mathrm{d}^d \vec{k}}{(2\pi)^d} G^R_{js}(\vec{k}) \exp\left[i\vec{k}.(\vec{x}_1 - \vec{x}_2)\right] \tag{13}$$

with Fourier transformation

$$G^R_{js}(\vec{k}) = c_a\, \bar{\varepsilon}^{\frac{2}{3}}\, k^{2-d-\frac{4\epsilon}{3}} \left[P_{js}(\vec{k}) w_1^*(\xi_k, 0) + R_{js}(\vec{k}) w_2^*(\xi_k, 0) \right], \tag{14}$$

containing desired large scale asymptotic $k^{2-d-\frac{4\epsilon}{3}}$ and satisfying constraints based on RNG symmetry. The parameters c_a, S_d are

$$c_a = \left(\frac{2(2\pi)^d \sqrt{2g_v^*}}{S_d(d-1)\left(1 + \frac{\alpha_1 + \alpha_2}{d}\right)} \right)^{\frac{2}{3}}, \quad S_d = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})}, \tag{15}$$

where $\Gamma(x)$ is the standard gamma function. The explicit form of w_1^* and w_2^* functions taken in this section at kinetic fixed point $g_v^*, \chi_j^*, j = 1, 2, 3$ is presented in Appendix I. We expressed parameters g_v, ν via measurable dissipation rate $\bar{\varepsilon}$ by the relation

$$\bar{\varepsilon} = \frac{1}{2(2\pi)^d} \int_{k<\Lambda} \mathrm{d}^d \vec{k}\, D^{st}_{jj}(\vec{k}) = g_v\, \nu^3 \frac{S_d}{2(2\pi)^d} \frac{\Lambda^4}{4 - 2\epsilon} \left(1 + \frac{\alpha_1 + \alpha_2}{d} \right), \tag{16}$$

which follows from the energy balance equation.

In the following text one can find how the Kolmogorov constant may be reached within the RNG approach using the numerically determined fixed point parameters. To better understand the consequences of anisotropy presence we suggest to separate the longitudinal and transversal part of radial spectral tensor $E_{js}(k)$

$$E_{js}(k) = \frac{1}{2(2\pi)^d} k^{d-1} \int \mathrm{d}\Omega_{\vec{k}}\, G^R_{js}(\vec{k}), \tag{17}$$

where $d\Omega_{\vec{k}}$ denotes the integration over the angular coordinates of $\vec{k}$ space for the fixed amplitude $|\vec{k}|$, in agreement with formula

$$E_{js}(k) = \frac{1}{d}\left(C_\perp P_{js}(\vec{n}) + C_\parallel P^{\parallel}_{js}(\vec{n})\right)\bar{\varepsilon}^{\frac{2}{3}}\,k^{1-\frac{4\epsilon}{3}}. \tag{18}$$

Using $P(\vec{n})$ and $P^{\parallel}(\vec{n})$ operators and expressions (17), (18) the scalar amplitudes $C_\parallel$ and $C_\perp$ can be expressed after some algebraic calculations as

$$\begin{aligned} C_\parallel &= c_b\int_0^1 \mathrm{d}\xi\,(1-\xi^2)^{\frac{d-1}{2}}\left[w_1^*(\xi,0) + (1-\xi^2)w_2^*(\xi,0)\right] \\ C_\perp &= \frac{c_b}{d-1}\int_0^1 \mathrm{d}\xi\,(1-\xi^2)^{\frac{d-3}{2}}\left[(d-2+\xi^2)w_1^*(\xi,0) + \xi^2(1-\xi^2)\,w_2^*(\xi,0)\right] \end{aligned} \tag{19}$$

where $c_b = c_a S_{d-1} d/(2\pi)^d$. The comparison of traced (18) with the energy spectrum definition

$$\mathrm{tr}(E_{jj}(k)) = C_k\,\bar{\varepsilon}^{\frac{2}{3}} k^{1-\frac{4\epsilon}{3}} \tag{20}$$

gives

$$C_k = \frac{(d-1)\,C_\perp + C_\parallel}{d}. \tag{21}$$

4 The implications for MHD stochastic dynamics, generalized anisotropic resistive and Lorentzian forces

For completeness we should mention that there are many mechanisms throughout which the MHD turbulent media becomes anisotropic. The anisotropy can be arise from the presence of an uniform background magnetic field [13, 14], macroscopic polarization of the turbulent media or may be induced by specific random forcing [9]. The last example of the paper deals with problem how the anisotropic forcing affects the inertial properties of MHD with high magnetic Reynolds number.

The micromodel of stochastic anisotropic hydrodynamics is used to construct the renormalizable statistical theory of randomly driven anisotropic magnetohydrodynamics in this section. The governing equations of the anisotropically driven MHD fluid [9] may be written as

$$\begin{aligned} \partial_t\vec{v} &+ (\vec{v}\vec{\nabla})\vec{v} - \nu\vec{\nabla}^2\vec{v} - \vec{f}^{\,\text{A-dis-v}} + (\vec{b}.\vec{\nabla})\vec{b} + \vec{f}^{\,\text{A-Lorentz}} = \vec{f}^{v}, \\ \partial_t\vec{b} &+ (\vec{v}.\vec{\nabla})\vec{b} - (\vec{b}.\vec{\nabla})\vec{v} - u\nu\nabla^2\vec{b} - \vec{f}^{\,\text{A-dis-b}} = \vec{f}^{b} \\ \vec{\nabla}.\vec{v} &= \vec{\nabla}.\vec{b} = \vec{\nabla}.\vec{f}^{v} = \vec{\nabla}.\vec{f}^{b} = 0, \end{aligned} \tag{22}$$

α_1	α_2	g_v^*	χ_1^*	χ_2^*	χ_3^*	C_k	$\sigma^{\parallel,\perp}$	σ^{isotr}
0	0.01	267	0.027	0.025	−0.184	2.990	0.8	0.8
0	0.015	269	0.045	0.042	−0.309	2.992	1.2	1.5
0	0.018	272	0.066	0.061	−0.460	2.995	1.6	2.5
−0.85	0	406	−0.112	0.319	−0.662	2.966	4.4	−7.2
−0.5	0	327	−0.065	0.132	−0.183	2.982	2.8	−1.9
−0.01	0	264	−0.001	0.001	−0.000072	2.988	0.1	0.1
0,01	0	262	0.001	0.001	0.00035	2.988	−0.1	0.1
0.2	0	246	0.030	−0.027	−0.030	2.987	−1.0	−0.2
0.49	0	230	0.108	−0.029	−0.324	2.987	−1.8	−0.2

Table 2: The examples of numerical values of the parameters associated with the kinetic fixed point of RNG obtained for various α_1, α_2 and $d = 3$. The importance of χ_3^* term as compared with χ_1^* and χ_2^* is an interesting feature of theory with strong anisotropy. The spectral properties of strong turbulence characterized by the amplitudes $C_k, C^{\parallel}$ and $C^{\perp}$ are also deduced from RNG. Consequently, the relative measures $\sigma^{\parallel,\perp}$ and σ^{isotr} of system deviation from isotropic state can be constructed in the form $\sigma^{\parallel,\perp} = 10^2 \times (C^{\parallel} - C^{\perp})/C_k$ and $\sigma^{isotr} = 10^3 \times \left(C_k - C_k^{isotr}\right)/C_k^{isotr}$, where $C_k^{isotr} = \left(\frac{80}{3}\right)^{\frac{1}{3}} \simeq 2.9876$ is the Kolmogorov constant of the isotropic RNG theory. From presented data we find limitations $|\sigma^{\parallel,\perp}| < 4.4$ and $|\sigma^{isotr}| < 7.2$. The principal conclusion is that $C_k(\alpha_1, \alpha_2)$ is not very sensitive to $\alpha_{1,2}$ variations.

where u is dimensionless magnetic Prandtl number, $\vec{b}(\vec{x}, t)$ is realization of fluctuating magnetic field. The magnetic forcing correlations satisfy the standard assumptions

$$\begin{aligned} \langle f_j^b(\vec{x}_1, t_1)\, f_s^v(\vec{x}_2, t_2)\rangle &= 0, \\ \langle f_j^b(\vec{x}_1, t_1)\, f_s^b(\vec{x}_2, t_2)\rangle &= D_{js}(\vec{x}_1 - \vec{x}_2, t_1 - t_2)_{g_v\to g_b,\alpha_1\to\alpha_3,\alpha_2\to\alpha_4,\epsilon\to\epsilon'}, \end{aligned} \tag{23}$$

adapted from (3) and [9]. In expression (23) α_3, α_4 and exponent ϵ' are additional free parameters, g_b is a new coupling parameter. The magnetic forcing anisotropy needs the suitable inclusion of terms with lower symmetry (not appearing in the standard MHD equations). The anisotropic magnetodissipation force analogous to (7) is

$$\vec{f}^{\,\text{A-dis-b}} = u\nu\left[\chi_4(\vec{n}.\vec{\nabla})^2\vec{b} + \chi_5\vec{n}\nabla^2(\vec{n}.\vec{b}) + \chi_6\vec{n}(\vec{n}.\vec{\nabla})^2(\vec{n}.\vec{b})\right], \tag{24}$$

where $\chi_j, j = 4, 5, 6$ are new parameters. Performing the systematic RNG process, which complements the initial results [9], new terms are generated with λ_j parameters coming into definition of modified Lorentzian force

$$\begin{aligned} \vec{f}^{\,\text{A-Lorentz}} &= \lambda_1\vec{b}(\vec{n}.\vec{\nabla})(\vec{n}.\vec{b}) + \lambda_2\vec{n}(\vec{n}.\vec{\nabla})\,b^2 \\ &+ \lambda_3\vec{n}(\vec{b}.\vec{\nabla})(\vec{n}.\vec{b}) + \lambda_4\vec{n}(\vec{n}.\vec{b})(\vec{n}.\vec{\nabla})(\vec{n}.\vec{b}). \end{aligned} \tag{25}$$

In the work [9] the renormalizable variant of the MHD theory with simpler forces $\vec{f}^{\,\text{A-Lorentz}}_{\lambda_4\to 0}$, $\vec{f}^{\,\text{A-dis-v}}|_{\chi_3\to 0,\chi_6\to 0}$ has been studied in detail. This consideration has been limited by the fact that anisotropy (i.e. free parameters α_l, $l = 1,2,3,4$ and consequently fixed $\chi_j^*, j = 1,2,4,5$; $\lambda_i^*, i = 1,2,3$) are small. It was shown that the investigation of MHD in the weak anisotropy limit leads to the Kolmogorov's spectral prediction only (for the kinetic and the magnetic energy spectra) if the forcing exponents satisfy inequality $\epsilon' < 0.65\epsilon$. Now the results achieved in the hydrodynamic theory allow to speculate about the consequences for MHD and MHD possible stabilization by means of symmetry and completeness requirements.

We believe that the presence of χ_6 magneto - dissipative term, analogous to previously discussed viscous χ_3 term will have also an immediate implication for the large scale effect of $\vec{f}^{\text{A-Lorentz}}$ and therefore will play an important role in stabilization (eventually destabilization) of critical scaling in MHD.

5 Conclusion

This paper deals with the RNG study of anisotropically driven hydrodynamic and MHD turbulence. It examines the Kolmogorov scaling regime. The principal conclusion of paper is that the scaling can become unstable throughout the spatial dimension changes or eventually by anisotropy variations. In this context the relevance of χ_3 term has been discussed. These results lead us to hypothetical build up of the improved anisotropically driven MHD theory.

The authors wish to thank Dr. L.Ts.Adzhemyan for useful discussions. M.H. and D.H. are grateful to D.I.Kazakov and to director D.V.Shirkov for hospitality at the Laboratory of Theoretical Physics, Jinr, Dubna.

This work was supported in part by Slovak Grant agency for science (grant 2/550/93).

6 Appendix I: Renormalization of anisotropic viscous dissipation

The renormalization of the $\vec{f}^{\text{A-dis-v}}$ force has been performed after the terms (singular in the limit $\epsilon \to 0$) of the order

$$O\left(k^2\delta_{ab}/\epsilon\right), \quad O\left((\vec{k}.\vec{n})^2\delta_{ab}/\epsilon\right), \quad O\left(k^2 n_a n_b/\epsilon\right), \quad O\left((\vec{k}.\vec{n})^2 n_a n_b/\epsilon\right) \tag{26}$$

were extracted from the one-loop irreducible Feynman integral

$$\Gamma_{ab}^{(2)}(\vec{k}) = \int \frac{d^d\vec{p}}{(2\pi)^d} \int_0^\infty dt\, \Delta_{\alpha\beta}(\vec{p},t)\tilde{\Delta}_{\gamma\delta}(\vec{k}-\vec{p},t)V_{a\alpha\gamma}(\vec{k})V_{\delta b\beta}(\vec{k}-\vec{p}) \tag{27}$$

in agreement with quite standard rules of minimal substraction scheme [15]. From the bilinear terms and triple $\tilde{v}vv$ part of the action (6) the basic elements (propagators and vertex) of the perturbation analysis have been obtained. The propagators $\tilde{\Delta}$, Δ and the symmetrized vertex V are written in k,t (for $t > 0$) representation as

$$\begin{aligned}
\Delta_{\alpha\beta}(\vec{k},t) &= \frac{g_v}{2}\nu^2\Lambda^{2\epsilon}k^{2-d-2\epsilon}\left\{w_1(\xi_k,\tau_k)P_{\alpha\beta}(\vec{k}) + w_2(\xi_k,\tau_k)R_{\alpha\beta}(\vec{k})\right\}, \\
\tilde{\Delta}_{\alpha\beta}(\vec{k},t) &= \theta(t)\left\{w_3(\xi_k,\tau_k)P_{\alpha\beta}(\vec{k}) + w_4(\xi_k,\tau_k)R_{\alpha\beta}(\vec{k})\right\}, \\
V_{\alpha\beta\gamma}(\vec{k}) &= i\left\{P_{\alpha\beta}(\vec{k})k_\gamma + P_{\alpha\gamma}(\vec{k})k_\beta\right\},
\end{aligned}$$

where

$$w_j = e^{-\tau A}w_{(j,1)} + e^{-\tau B}w_{(j,2)}, \text{for} \quad j = 1,2 \tag{28}$$

with coefficients

$$\begin{aligned}
w_{(1,1)} &= \frac{1+\alpha_1\xi^2}{A}, \quad w_{(1,2)} = 0, \\
w_{(2,1)} &= \frac{1}{A}\left(\alpha_2 - \frac{M(1+\alpha_1+(\alpha_2-\alpha_1)(1-\xi^2))}{(B-A)}\right), \\
w_{(2,2)} &= \frac{M}{B(B-A)}\left(1+\alpha_1+(\alpha_2-\alpha_1)(1-\xi^2)\right), \quad \tau = \nu k^2 t, \\
w_3 &= e^{-\tau A}, \\
w_4 &= -\frac{M}{B-A}\left(e^{-\tau B} - e^{-\tau A}\right), \\
A &= 1+\chi_1\xi^2, \quad M = \chi_2 + \chi_3\xi^2, \quad B = A + (1-\xi^2)M.
\end{aligned} \tag{29}$$

7 Appendix II: The derivation of β functions

In order to clarify the calculations and reduce the number of the terms under investigation of (27) through the analytical computational stages as much as possible we have made some preliminary identities and relations. At first, we specify the form, in which the functions $w_3(\xi_{k-p}, \tau_{k-p})$, $w_4(\xi_{k-p}, \tau_{k-p})$ and also operators $P(\vec{k}-\vec{p})$ and $R(\vec{k}-\vec{p})$ will be expanded into the Taylor series in powers of $\vec{k}$ [the external wave number of integral (27)]. Using the fact that $V_{a\alpha\gamma}(\vec{k}) \sim k$, the final extraction of quadratic terms (26) needs the Taylor expansion of product $\tilde{\Delta}_{\gamma\delta}(\vec{k}-\vec{p},t)V_{\delta b\beta}(\vec{k}-\vec{p})$ up to the first order in $\vec{k}$ only.

If we define the coefficients $W_{(j,i)}$, $i=0,1,2,3$ (dependent on scalar ξ_p) as the first order derivatives [at zero of scalar products $\vec{k}.\vec{n}$ and $\vec{k}.\vec{p}$]

$$
\begin{aligned}
W_{(j,0)} &= w_j|_{(\vec{k}.\vec{n};\vec{k}.\vec{p}\to 0)}, \quad W_{(j,2)} = p\frac{\partial w_j}{\partial(\vec{k}.\vec{n})}|_{(\vec{k}.\vec{n};\vec{k}.\vec{p}\to 0)}, \\
W_{(j,1)} &= p^2\frac{\partial w_j}{\partial(\vec{k}.\vec{p})}|_{(\vec{k}.\vec{n};\vec{k}.\vec{p}\to 0)},
\end{aligned}
$$

then the mentioned expansion of w_3, w_4 looks like

$$
w_j = W_{(j,0)} + W_{(j,1)}\left(\frac{\vec{k}.\vec{p}}{p^2}\right) + W_{(j,2)}\left(\frac{\vec{k}.\vec{n}}{p}\right), \quad \text{for} \quad j=3,4. \tag{30}
$$

To perform the differentiation of w_3 and w_4 according to (30) we use the identities

$$
\begin{aligned}
& p^2\frac{\partial w(A(\xi_{k-p}), B(\xi_{k-p}), M(\xi_{k-p}), \tau_{k-p})}{\partial(\vec{k}.\vec{p})}|_{(\vec{k}.\vec{n},\vec{k}.\vec{p}\to 0)} \\
&= \left[2\xi_p^2\hat{\nabla}_{AMB_p} - 2\tau_p\frac{\partial}{\partial\tau_p}\right] w(A_p, B_p, M_p, \tau_p) \\
& p\frac{\partial w(A(\xi_{k-p}), B(\xi_{k-p}), M(\xi_{k-p}))}{\partial(\vec{k}.\vec{n})}|_{(\vec{k}.\vec{n},\vec{k}.\vec{p}\to 0)} = -2\xi_p\,\hat{\nabla}_{AMB_p} w(A_p, B_p, M_p),
\end{aligned} \tag{31}
$$

with operator

$$
\hat{\nabla}_{AMB_p} = \chi_1\frac{\partial}{\partial A_p} + \chi_3\frac{\partial}{\partial M_p} + B_p'\frac{\partial}{\partial B_p}, \quad B_p' = \chi_1 + (1-\xi_p^2)\chi_3 - M_p,
$$

where w is either of function (w_3 or w_4). The further computational purposes motivate us to separate τ dependent terms writing algebraic structure of $W_{(x,y)}$ in the form

$$
\begin{aligned}
W_{(j,s)} &= e^{-\tau A}W_{(j,s,1)} + e^{-\tau B}W_{(j,s,2)} \\
&+ \tau e^{-\tau A}W_{(j,s,3)} + \tau e^{-\tau B}W_{(j,s,4)}.
\end{aligned} \tag{32}
$$

The expressions (29) and (31) allow the explicit derivation of coefficients appearing in (32)

$$
\begin{aligned}
W_{(3,0,1)} &= 1, \quad W_{(3,0,2)} = W_{(3,0,3)} = W_{(3,0,4)} = 0, \\
W_{(3,1,1)} &= W_{(3,1,2)} = W_{(3,1,4)} = 0, \quad W_{(3,1,3)} = 2(A - \xi^2\chi_1), \\
W_{(3,2,1)} &= W_{(3,2,2)} = W_{(3,2,4)} = 0, \quad W_{(3,2,3)} = 2\xi\chi_1, \\
W_{(4,0,1)} &= -W_{(4,0,2)} = M/(B-A), \quad W_{(4,0,3)} = W_{(4,0,4)} = 0, \\
W_{(4,1,1)} &= -W_{(4,1,2)} = -\xi W_{(4,2,1)}, \\
W_{(4,1,3)} &= W_{(4,0,1)}W_{(3,1,3)}, \quad W_{(4,1,4)} = 2W_{(4,0,1)}(B'\xi^2 - B), \\
W_{(4,2,1)} &= -W_{(4,2,2)} = 2\xi(B'M - M\chi_1 + A\chi_3 - B\chi_3)/(B-A)^2, \\
W_{(4,2,3)} &= W_{(4,0,1)}W_{(3,2,3)}, \quad W_{(4,2,4)} = 2W_{(4,0,1)}\xi B'.
\end{aligned} \tag{33}
$$

For the operators P and R at wave number $\vec{k}-\vec{p}$ we find simply the expansion rules

$$
\begin{aligned}
P_{js}\left(\vec{k}-\vec{p}\right) &= P_{js}(\vec{p}) - 2p^{-4}\left(\vec{k}.\vec{p}\right)p_jp_s + p^{-2}\left(k_sp_j + k_jp_s\right), \\
R_{js}(\vec{k}-\vec{p}) &= R_{js}(\vec{p}) \\
&+ (n_j - p^{-1}\xi_p p_j)\left[p^{-1}\xi_p k_s + p^{-2}k\,\xi_k p_s - 2p^{-3}\xi_p\left(\vec{k}.\vec{p}\right)p_s\right] \\
&+ (n_s - p^{-1}\xi_p p_s)\left[p^{-1}\xi_p k_j + p^{-2}k\,\xi_k p_j - 2p^{-3}\xi_p\left(\vec{k}.\vec{p}\right)p_j\right].
\end{aligned} \tag{34}
$$

Using these expressions and (30) we find the quadratic part of kernel (27) in the form

$$
\begin{aligned}
& \Delta_{\alpha\beta}(\vec{p})\tilde{\Delta}_{\gamma\delta}(\vec{k}-\vec{p})V_{a\alpha\gamma}(\vec{k})V_{\delta b\beta}(\vec{k}-\vec{p}) \\
&= \Delta_{\alpha\beta}(\vec{p})\tilde{\Delta}_{\gamma\delta}(\vec{p})V_{a\alpha\gamma}(\vec{k})V_{\delta b\beta}(\vec{k}) + \Delta_{\alpha\beta}(\vec{p})k_l\tilde{\Delta}'_{\gamma\delta l}(\vec{p})V_{a\alpha\gamma}(\vec{k})V_{\delta b\beta}(-\vec{p}) \\
&+ \text{nonquadratic in k part},
\end{aligned} \tag{35}
$$

where we have used decompositions

$$
\begin{aligned}
& \tilde{\Delta}_{\gamma\delta}(\vec{k}-\vec{p}) = \tilde{\Delta}_{\gamma\delta}(\vec{p}) + k_l\tilde{\Delta}'_{\gamma\delta l}(\vec{p}), \\
& \tilde{\Delta}'_{\gamma\delta l}(\vec{p}) = n_l\Delta^{(1)}_{\gamma\delta}(\vec{p}) + \frac{p_l}{p}\Delta^{(2)}_{\gamma\delta}(\vec{p}), \\
& \Delta^{(1)}_{\gamma\delta} = \delta_{\gamma\delta}W_{(3,1)} + n_\gamma n_\delta W_{(4,1)} + p^{-1}(n_\gamma p_\delta + n_\delta p_\gamma)(W_{(4,0)} - \xi_p W_{(4,1)}) \\
+ \;& p^{-2}p_\gamma p_\delta(-W_{(3,1)} - 2\xi_p W_{(4,0)} + \xi_p^2 W_{(4,1)}), \\
& \Delta^{(2)}_{\gamma\delta} = \delta_{\gamma\delta}W_{(3,2)} + n_\gamma n_\delta W_{(4,2)} - \xi_p p^{-1}(n_\gamma p_\delta + n_\delta p_\gamma)(2W_{(4,0)} + W_{(4,2)}) \\
+ \;& p^{-2}p_\gamma p_\delta(-2W_{(3,0)} - W_{(3,2)} + 4\xi_p^2 W_{(4,0)} + \xi_p^2 W_{(4,2)}).
\end{aligned} \tag{36}
$$

In the following we present procedure, which considerably reduces the computational effort and replaces the integration with $d^d\vec{p}$ measure by single integral over the ξ variable. The integral can be calculated numerically. If an arbitrary function of $\vec{p}.\vec{n}/p = \xi_p$ argument is denoted by $\Psi(\cdot)$, then the following rules for the extraction of $1/\epsilon$ poles (right hand side of the following identities) have been derived from the minimal subtraction procedure [15]

$$\begin{aligned}
\int \frac{d^d \vec{p}}{(2\pi)^d}\Psi(\xi_p)\frac{p_i p_j p_s p_m}{p^{4+d+2\epsilon}} &= \frac{1}{2\epsilon}\Big[I_{(4,1)}\{\Psi\}\, n_i n_j n_s n_m \\
&+ I_{(4,2)}\{\Psi\}\,(n_i n_j \delta_{sm} + n_i n_s \delta_{jm} \\
&+ n_i n_m \delta_{js} + n_j n_s \delta_{im} + n_j n_m \delta_{is} + n_s n_m \delta_{ij}) \\
&+ I_{(4,3)}\{\Psi\}\,(\delta_{ij}\delta_{sm} + \delta_{is}\delta_{jm} + \delta_{im}\delta_{sj})\Big] \\
\int \frac{d^d \vec{p}}{(2\pi)^d}\Psi(\xi_p)\frac{p_i p_j p_s}{p^{3+d+2\epsilon}} &= \frac{1}{2\epsilon}\Big[I_{(3,1)}\{\Psi\} n_i n_j n_s \\
&+ I_{(3,2)}\{\Psi\}\,(n_i \delta_{js} + n_j \delta_{is} + n_s \delta_{ij})\Big] \\
\int \frac{d^d \vec{p}}{(2\pi)^d}\Psi(\xi_p)\frac{p_i p_j}{p^{2+d+2\epsilon}} &= \frac{1}{2\epsilon}\Big[I_{(2,1)}\{\Psi\} n_i n_j + I_{(2,2)}\{\Psi\}\delta_{ij}\Big] \\
\int \frac{d^d \vec{p}}{(2\pi)^d}\Psi(\xi_p)\frac{p_i}{p^{1+d+2\epsilon}} &= \frac{1}{2\epsilon} n_i I_{(1,1)}\{\Psi\} \\
\int \frac{d^d \vec{p}}{(2\pi)^d}\Psi(\xi_p)\frac{1}{p^{d+2\epsilon}} &= \frac{1}{2\epsilon} I_0\{\Psi\}.
\end{aligned}$$

With minimal subtraction scheme, there is no contributions from finite (for $\epsilon \to 0$) part of integrals. All linear in Ψ functionals $I_{(X,Y)}\{\Psi\}$ are connected with the basic functional defined as

$$I_0\{\Psi\} \equiv I_{(0,0)}\{\Psi\} = \frac{S_{d-1}}{(2\pi)^d}\int_{-1}^{1} d\xi\ (1-\xi^2)^{\frac{d-3}{2}}\ \Psi(\xi) \tag{37}$$

by means of relations

$$\begin{aligned}
I_{(4,1)}\{\Psi\} &= (d_4 d_2 I_0\{\Psi\xi^4\} - 6 d_2 I_0\{\Psi\xi^2\} + 3 I_0\{\Psi\})(d^2-1)^{-1}, \\
I_{(4,2)}\{\Psi\} &= (-d_2 I_0\{\Psi\xi^4\} + d_3 I_0\{\Psi\xi^2\} - I_0\{\Psi\})(d^2-1)^{-1}, \\
I_{(4,3)}\{\Psi\} &= (I_0\{\Psi\xi^4\} - 2 I_0\{\Psi\xi^2\} + I_0\{\Psi\xi^2\})(d^2-1)^{-1}, \\
I_{(3,1)}\{\Psi\} &= (I_0\{\Psi\xi^3\} d_2 - 3 I_0\{\Psi\xi\})(d_{-1})^{-1}, \\
I_{(3,2)}\{\Psi\} &= (-I_0\{\Psi\xi^3\} + I_0\{\Psi\xi\})(d_{-1})^{-1}, \\
I_{(2,1)}\{\Psi\} &= (I_0\{\Psi\xi^2\} d - I_0\{\Psi\})(d_{-1})^{-1}, \\
I_{(2,2)}\{\Psi\} &= (I_0\{\Psi\} - I_0\{\Psi\xi^2\})(d_{-1})^{-1}, \\
I_{(1,1)}\{\Psi\} &= I_0\{\Psi\ \xi\},
\end{aligned}$$

where we used the notation

$$d_j = d + j, \quad j = 2, 3, 4.$$

The renormalized action

$$\begin{aligned}
S^R &= \int d^d \vec{x}_1 dt_1\, d^d \vec{x}_2 dt_2 \left[\frac{1}{2}\tilde{v}_j(\vec{x}_1,t_1)\, D_{js}(\vec{x}_1-\vec{x}_2, t_1-t_2)\, \tilde{v}_s(\vec{x}_2,t_2)\right] \\
&+ \int d^d\vec{x} dt \Big\{ \tilde{\vec{v}}(\vec{x},t)\Big[-\partial_t \vec{v} - (\vec{v}\vec{\nabla})\vec{v} \\
&+ \nu\big[Z_1 \vec{\nabla}^2 \vec{v} + Z_2 \chi_1 (\vec{n}.\vec{\nabla})^2 \vec{v} + Z_3 \chi_2 \vec{n}\vec{\nabla}^2(\vec{n}.\vec{v}) \\
&+ Z_4 \chi_3 \vec{n}(\vec{n}.\vec{\nabla})^2(\vec{n}.\vec{v})\big]\Big]\Big\}
\end{aligned} \tag{38}$$

corresponding to (6), includes Z_j ($\sim \frac{1}{\epsilon}$) terms [12, 16] providing the cancellation of the divergences in (38). They can be obtained equating of corresponding coefficients of divergent part of integral (27), (35) with expected form

$$\begin{aligned}
\Gamma^{(2)}_{ab}(\vec{k}) &= -\nu k^2 \delta_{ab}(1-Z_1) - \nu(\vec{k}.\vec{n})^2 \delta_{ab}(1-Z_2) \\
&- \nu k^2 n_a n_b (1-Z_3) - \nu(\vec{k}.\vec{n})^2 n_a n_b (1-Z_4) \\
&= V_{a\alpha\gamma}(\vec{k}) V_{\delta b\beta}(\vec{k}) \int \frac{d^d\vec{p}}{(2\pi)^d}\int_0^\infty dt\ \Delta_{\alpha\beta}(\vec{p},t)\tilde{\Delta}_{\gamma\delta}(\vec{p},t) \\
&+ k_l V_{a\alpha\gamma}(\vec{k}) \int \frac{d^d\vec{p}}{(2\pi)^d}\int_0^\infty dt\ \Delta_{\alpha\beta}(\vec{p},t)\tilde{\Delta}'_{\gamma\delta l}(\vec{p},t) V_{\delta b\beta}(-\vec{p}).
\end{aligned} \tag{39}$$

In order to keep the number of terms in Z_j as small as possible it is usefull to define the following $J_{a_1 a_2 b_1 b_2 c_1 c_2}$ tensor

$$J_{a_1 a_2 b_1 b_2 c_1 c_2} = \frac{g_v}{4\epsilon} I_{(a_1,a_2)}\left\{ \xi^{b_1} \int_0^\infty d\tau\, w_{b_2}(\xi,\tau) W_{(c_1,c_2)}(\xi,\tau) \right\}. \tag{40}$$

After the substitution of (32) into (40) it is simple to show that integration over the τ gives

$$\begin{aligned}
J_{a_1 a_2 b_1 b_2 c_1 c_2} &= \frac{g_v}{\epsilon} I_{(a_1,a_2)}\Big\{ \frac{\xi^{b_1}}{4}\sum_{s=1,2}\Big[(2A)^{-s} W_{(c_1,c_2,2s-1)} w_{(b_2,1)} \\
&+ (A+B)^{-s}\big[W_{(c_1,c_2,2s-1)} w_{(b_2,2)} + W_{(c_1,c_2,2s)} w_{(b_2,1)}\big] \\
&+ (2B)^{-s} W_{(c_1,c_2,2s)} w_{(b_2,2)}\Big]\Big\}.
\end{aligned} \tag{41}$$

In this expression only the single integration over the ξ variable remains (note that $A, B, W_{(X,Y,Z)}$ are understood as the functions of ξ variable here) and this problem is possible to solve numerically.

The expressions (30) and notation (40) have been used to brought the coefficients $Z_1, ...Z_4$ into the compact forms

$$\begin{aligned} Z_1 - 1 &= -J_{222140} - J_{222240} + J_{224240} - 2J_{430130} - J_{430132} \\ &+ 4J_{432140} + J_{432142} + 2J_{432230} + J_{432232} + 2J_{432240} + J_{432242} \\ &- 4J_{434240} - J_{434242}, \end{aligned}$$

$$\begin{aligned} Z_2 - 1 &= -J_{221141} - J_{221231} - J_{221241} - 2J_{222240} + J_{223241} \\ &- J_{320131} - 3J_{321140} - J_{321142} - 2J_{321230} - J_{321232} - J_{321240} \\ &- J_{321242} + J_{322141} + J_{322231} + J_{322241} + 5J_{323240} + J_{323242} \\ &- J_{324241} - 2J_{420130} - J_{420132} + 4J_{422140} + J_{422142} + 2J_{422230} \\ &+ J_{422232} + 2J_{422240} + J_{422242} - 4J_{424240} - J_{424242}, \end{aligned}$$

$$\begin{aligned} Z_3 - 1 &= J_{112140} + J_{112240} - J_{114240} - J_{212140} - J_{212240} \\ &+ J_{214240} - J_{222240} - 3J_{321140} - J_{321142} - 2J_{321230} - J_{321232} \\ &- 2J_{321240} - J_{321242} + 5J_{323240} + J_{323242} - 2J_{420130} - J_{420132} \\ &+ 4J_{422140} + J_{422142} + 2J_{422230} + J_{422232} + 2J_{422240} + J_{422242} \\ &- 4J_{424240} - J_{424242}, \end{aligned}$$

$$\begin{aligned} Z_4 - 1 &= -J_{000240} + J_{111141} + J_{111231} + J_{111241} + 5J_{112240} \\ &- J_{113241} + 2J_{210140} + J_{210142} + 2J_{210230} + J_{210232} + J_{210240} \\ &+ J_{210242} - 2J_{211141} - 2J_{211231} - 2J_{211241} - 10J_{212240} - J_{212242} \\ &+ 2J_{213241} - J_{310131} - 6J_{311140} - 2J_{311142} - 4J_{311230} - 2J_{311232} \\ &- 3J_{311240} - 2J_{311242} + J_{312141} + J_{312231} + J_{312241} + 10J_{313240} \\ &+ 2J_{313242} - J_{314241} - 2J_{410130} - J_{410132} + 4J_{412140} + J_{412142} \\ &+ 2J_{412230} + J_{412232} + 2J_{412240} + J_{412242} - 4J_{414240} \\ &- J_{414242}. \end{aligned}$$

In the construction of beta functions appearing in (8) (for details see [10, 16]) the one loop connection of Z and β functions can be easily derived

$$\beta_{g_v} = 2\, g_v \epsilon\, (-2 + 3Z_1), \quad \beta_{\chi_j} = 2\, \chi_j \epsilon\, (Z_{j+1} - Z_1), \quad j = 1, 2, 3. \qquad (42)$$

References

[1] C.Török, *Computers Math. Applic.*. **27**, No.4, (1993), 1.

[2] P.K. Yeung and J.G. Brasseur, *Phys.Fluids A*, **3**, No.5, (1991), 884.

[3] W.K. George, *Phys.Fluids A*, **4**, No.7, (1992), 1492.

[4] L.W. Browne, R.A. Antonia and D.A. Shah, *J.Fluid Mech.*, **179**, (1987). 307.

[5] J.R. Herring, *Phys.Fluids*. **17**, No.5, (1974), 859.

[6] L.Ts. Adzhemyan, A.N. Vasil'ev and M. Hnatich, *TMF*, **64**, (1985), 196; (in russian).

[7] R. Rubinstein and J.M. Barton, *J.Phys.Fluids*, **30**, (1987), 2987.

[8] C. DeDominicis and P.C. Martin, *Phys.Rev. A*, **19**, (1979), 419;

[9] L.Ts. Adzhemyan, M.Hnatich, D. Horvath and M. Stehlik , *Proceedings: Energy transfer in MHD flows*, (1991) Aussois, France p. 541-550.

[10] L.Ts. Adzhemyan, A.N. Vasil'ev and Yu.M. Pismak, *TMF*, **57**, (1983), 268; (in russian).

[11] N. N. Bogoljubov and D.V. Shirkov, *Doklady AN SSSR*, **103**, (1955), 203; *Nuovo Cim.*, **3**, (1956), 845.

[12] L.Ts. Adzhemyan, N.V.Antonov and A.N. Vasilyev, *JETP*, **95**, (1989), 1272; (in russian).

[13] M. Garnier, A.Alemany, P.L.Sulem and A.Pouquet, *Journal de Mecanique*, **20**. No.2, (1981), 233.

[14] W.H. Matthaeus and Ch. Smith, *Phys.Rev.A*, **24**, No.4, (1981), 2135.

[15] G.'t Hoft, *Nucl.Phys.*, **B 61**, (1973), 455.

[16] J.Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, (Oxford Univ. Press, 1989).

Received by Publishing Department
on July 27, 1995.