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THE CALCULATION OF THE RADIAL VELOCITY PAIR CORRELATION FUNCTION IN THE MODEL OF DECAYING TURBULENCE

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Гнатич М., Горватг Д., Торок Ч. E17-95-347
Расчет радиальной парной корреляционной функции скорости в модели затухающей турбулентности

Рассмотренные в данном сообщении развитые турбулентные потоки предполагаются изотропными, трехмерными, распадными и несжимаемыми. Мы дополняем результаты Аджемяна и сотрудников [Л.Ц.Аджемян и др., препринт E17-94-319, Дубна, 1994; Czech.J.Phys. Vol.45, (6), (1995), 517], полученные для статистической несжимаемой гидродинамики с замыканием, основанным на ренормировке, где замкнутые формы второго статистического момента скорости являются базисом для формулировки теории универсального затухания турбулентности. В настоящем сообщении процесс затухания описывается парной корреляционной функцией скорости в координатном представлении. С помощью оптимализационных вычислительных алгоритмов была найдена её параметрическая автомодельная форма в энергосодержащей области.

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Hnatich M., Horvath D., Török C. E17-95-347
The Calculation of the Radial Velocity Pair Correlation Function in the Model of Decaying Turbulence

The developed turbulent flows considered in this report are assumed to be isotropic, three dimensional, decaying and incompressible. We extend the results of Adzhemyan and collaborators [L.Ts.Adzhemyan et al., Preprint P17-94-319, Dubna, 1994; Czech.J.Phys. Vol.45, (1995), No.6, 517] obtained for renormalization based closure of statistical incompressible hydrodynamics, where the closed forms of second order velocity statistical moments provide the basis that is needed to formulate the theory of universal turbulence decay. In the presented report the decay process has been characterized by means of pair velocity correlation function transformed into the coordinate representation. The parametrization of its scaling form (in the energy containing range) has been found using numerical optimization.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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The experience in use of powerful analytical methods and renormalization group techniques applied to stationary statistical hydrodynamics [1, 2] and their recent extension to description of associated problem of turbulence decay [3] has demonstrated the ability of statistical theories to explain the results of past grid experiments summarized and interpreted by George [4].

The analysis details and physical background of the derivation of closed set of equations for second order velocity moments have been given in [3], here we briefly summarize some details of their derivation exploiting the assumptions about the scaling invariance of the velocity field correlations, energy transfer and power laws of time evolution of certain integral decay characteristics in the energy containing range of wave numbers k.

The governing equations for scaling function can be derived from the assumption that the non-stationary energy spectrum $E(k,t)$ [5] (which is one of the possible measurable quantity available from of the past grid experiments) at high Reynolds numbers can be written in universal form

$$E(k,t) = C_k\,\bar{\epsilon}^{\frac{2}{3}}(t)\,\chi^{\frac{11}{3}}\,\Phi(\chi)\,k^{-\frac{5}{3}} = \frac{1}{3}\,C_k\,u^2(t)\,l(t)\,\chi^2\,\Phi(\chi), \quad \chi = kl(t), \tag{1}$$

where C_k is the Kolmogorov constant, $\Phi(\cdot)$ is the scaling function of single variable with normalization $\Phi(\infty) = 1$. The calculations are carried out using standard variables [5] - the characteristic length of the energy containing eddies (Von Karman scale) $l(t)$, mean square root velocity $u(t)$ and the energy dissipation per unit mass $\bar{\epsilon}(t)$ undergo the power like time dependencies at intermediate time

$$l(t) \sim t^{\frac{2}{5}}, \quad u(t) \sim t^{-\frac{3}{5}}, \quad \bar{\epsilon}(t) \sim t^{-\frac{11}{5}}. \tag{2}$$

The model of universal mean energy transfer is obtained by using definition of the quadratic in $\Phi(\cdot)$, non-local functional

$$\begin{aligned} I[\chi;\Phi] &= \chi^{\frac{19}{3}} \int_0^\infty \mathrm{d}q \int_0^\pi \mathrm{d}\theta\, \mathcal{K}^{(0)}(q,\theta) \\ &\times \Big\{ \mathcal{K}^{(1)}(q,\theta)\,\Phi(\chi q)\,\Phi(\chi p(q,\theta)) + \mathcal{K}^{(2)}(q,\theta)\,\Phi(\chi q)\,\Phi(\chi) \\ &+ \mathcal{K}^{(3)}(q,\theta)\,\Phi(\chi p(q,\theta))\,\Phi(\chi) \Big\}, \\ p(q,\theta) &= \sqrt{1 - 2q\cos\theta + q^2}. \end{aligned} \tag{3}$$

The non-linear integral kernels

$$\begin{aligned} \mathcal{K}^{(0)}(q,\theta) &= \frac{2\sin^3\theta}{1 + q^{\frac{2}{3}} + p^{\frac{2}{3}}} \left(\frac{q}{p}\right)^2, \\ \mathcal{K}^{(1)}(q,\theta) &= 1 - 3q\cos\theta + q^2 + 2q^2\cos^2\theta, \\ \mathcal{K}^{(2)}(q,\theta) &= -p^2 + q^2(1 - q\cos\theta), \\ \mathcal{K}^{(3)}(q,\theta) &= q\,(p^2\cos\theta - q) \end{aligned} \tag{4}$$

stem from the terms describing the interaction of hydrodynamic modes of the Navier-Stokes equation.

The time derivative of $E(k,t)$ can be obtained from expression (1) using (2)

$$\frac{\partial E}{\partial t} = \frac{2}{27\sqrt{3}}\,C_k\,u^3(t)\,\chi^3\,\frac{\mathrm{d}\Phi}{\mathrm{d}\chi}, \tag{5}$$

in addition the mean energy transfer is

$$T(k,t) = \frac{\sqrt{5}}{18}\,C_k^{\frac{3}{2}}\,u^3(t)\,I[\chi;\Phi]. \tag{6}$$

The scale invariant forms (5) and (6), both mutually connected by means of the mean energy balance equation [5] (in the wave number - time representation)

$$\frac{\partial E(k,t)}{\partial t} = T(k,t), \tag{7}$$

became the basis of the self - similar description of decay process inside the energy containing scales (which allow the viscous term neglecting). The equation (7) enables the separation of t, χ variables and description of scaling by the following set of equations

$$\begin{aligned} \chi^3\,\frac{\mathrm{d}\Phi}{\mathrm{d}\chi} &= \frac{3\sqrt{15}}{4}\,C_k^{\frac{1}{2}}\,I[\chi;\Phi], \\ \frac{2C_k}{3}\int_0^\infty \mathrm{d}x\, x^2\Phi(x) &= 1 \end{aligned} \tag{8}$$

for unknown function $\Phi(\cdot)$ and parameter C_k. The set of equations (8) is supplemented by the asymptotic conditions

$$\begin{aligned} \chi^{\frac{11}{3}}\Phi(\chi) &= 1 + O\left(\chi^{-\frac{2}{3}}\right), \quad \text{for } \chi \to \infty, \\ \Phi(\chi) &= O(1), \quad \text{for } \chi \to 0. \end{aligned} \tag{9}$$

The non-local and non-linear character of above set of equations makes the exact calculation of $\Phi(\cdot)$ difficult, but allows obtaining its approximate form

$$\Phi(\chi) = (\chi^4 + 0.863\chi^2 + 6.076)^{-\frac{11}{12}} \left[1 + 0.643\,(\chi^2 + 8.054)^{-\frac{1}{3}}\right]^{-1} \tag{10}$$

with quite realistic value $C_k = 1.577$.

In this report we complete picture about the turbulence decay process transforming $E(k,t)$ into the coordinate representation. In the following the two point, equal-time, radial, pair correlation function of the velocity field $\vec{v}(\vec{x},t)$, defined by the average

$$G(r,t) = \langle\, \vec{v}(\vec{x} + \vec{e}r, t) . \vec{v}(\vec{x}, t)\, \rangle, \tag{11}$$

where $\vec{e}$ is a unit vector, r is a distance between $\vec{x}$ and $\vec{x} + \vec{e}r$ points, is considered as the central statistical object in question.

From the definition

$$G(r,t) = \int \frac{d^3\vec{k}}{2\pi k^2} E(k,t) \exp\left[i\vec{k}.\vec{e}r\right]$$

and the scaling form of $E(k,t)$ (1) we obtain

$$G(r,t) = u^2(t) K(\xi), \quad \xi = \frac{r}{l(t)} \tag{12}$$

with the scaling function given by integral transformation

$$K(\xi) = \frac{2C_k}{3} \int_0^\infty d\chi\, \chi^2 \frac{\sin(\chi\xi)}{\chi\xi} \Phi(\chi). \tag{13}$$

Using the expressions (10) and (13) $K(\xi)$ dependence was calculated numerically for discrete mesh of ξ values. General parametrization of $K(\xi)$ is proposed in the form

$$K(\xi) = \exp\left[-\sum_{k=1}^{n} a_k \xi^{\frac{2k}{3}}\right], \tag{14}$$

where parameters a_k can be determinate numerically. We found a good fit for n=2 (see Fig.1)

$$K(\xi) = \begin{cases} \exp\left(-1.13\xi^{\frac{2}{3}} - 0.75\xi^{\frac{4}{3}}\right), \text{for } \xi \leq 3 \;; \\ 1 - 1.19\xi^{\frac{2}{3}}, \qquad \text{for } \xi \leq 0.1. \end{cases} \tag{15}$$

Two parametric fit $\exp\left(-a_1\xi^{\frac{2}{3}} - a_2\xi^{\frac{4}{3}}\right)$ for $\xi \leq 3$ and one parametric $1 - a_1\xi^{\frac{2}{3}}$ strongly local fit for $\xi \leq 0.1$ have been chosen to satisfy the well founded asymptotic Kolmogorov condition

$$K(\xi) = 1 + O(\xi^{\frac{2}{3}}), \quad \text{for} \quad \xi \to 0 \tag{16}$$

and the normalization $K(0) = 1$. The exponential form (15) also implies the expected property $K(\infty) = 0$.

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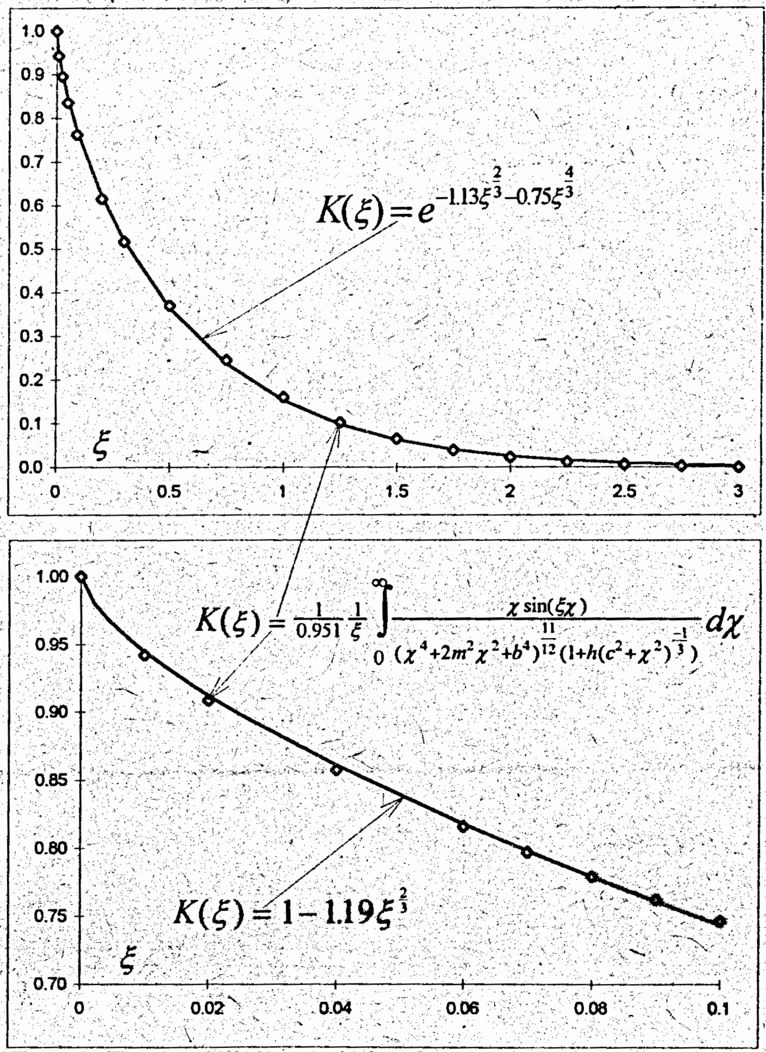

Figure 1: The plot of $K(\xi)$ vs. ξ obtained from (13) and (10) at selected mesh points - diamonds. The parametrizations (15) are presented by solid curves.

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