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SUPERCONDUCTING PAIRING OF SPIN POLARONS IN THE $t-J$ MODEL

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1 Introduction

Recently experimental evidences in favor of a d-wave superconducting pairing in high-T_c cuprates [1] have been supported by theoretical studies of models with strong electron correlations [2]. Many unconventional normal state properties of cuprates can be explained only by proper treatment of strong electron correlations on copper sites which could be also important for superconducting pairing. The simplest model allowing for the electron correlations is a two-dimensional Hubbard model with on-site repulsion U and hopping energy t [3]. Recent studies [4] - [7] of the Eliashberg equations for the Hubbard model in the weak coupling limit, $U \le 4t$, proved a d-wave pairing mediated by spin fluctuation exchange. In the vicinity of antiferromagnetic instability near half filling a superconducting temperature T_c of order $0.02t$ has been obtained.

In the strong coupling limit, $U \gg t$, a $t-J$ model is more appropriate [3, 8]. Exclusion of doubly occupied states in electronic hopping and their strong coupling with spin fluctuations with exchange energy $J \simeq 4t^2/U$ does not allow to apply mean field type approximations or perturbation theory. Exact numerical studies [2, 9, 10] for small clusters within the $t-J$ model show a d-wave superconducting instability. However, to elucidate the nature of this pairing an analytical treatment of the $t-J$ model is needed. For these purpose one can employ a spin polaron model [11, 12] reduced from the $t-J$ model in the limit of low temperature and small hole concentrations. A number of studies of this model [11]- [18] predicts that a doped hole dressed by strong antiferromagnetic spin fluctuations can propagate coherently as a quasi-particle with weight $Z_k \simeq J/t$. In addition to a narrow quasi-particle band of order J there is a broad incoherent band of order $6-7t$ at higher energies. It is quite natural to suggest that the same spin fluctuations could mediate a superconducting pairing of the spin polarons. Recently this problem was treated in the framework of the standard BCS formalism [19, 20]. A simple model of quasiparticles with numerically evaluated spectrum and effective pairing interaction in the atomic limit [19] and mediated by antiferromagnetic magnon exchange [20] has been used. However, since the pairing spin-fluctuation energy is of the same order as a quasi-particle bandwidth J the weak coupling BCS equation is inadequate to treat the problem. A full self-consistent solution of the Eliashberg equations and spin fluctuation susceptibility is needed to resolve this problem.

In this paper for the first time a consistent solution of the strong coupling spin polaron model at finite temperatures and hole concentrations for normal and superconducting states is presented. A numerical solution of a self-consistent system for hole and magnon Green functions for a two sublattice spin polaron model unambiguously demonstrate a singlet d-wave superconducting pairing. The maximum superconducting temperature T_c of order $0.012t$ is obtained around hole concentrations $\delta = 0.25$.

Combining the results for the Hubbard model [4]-[7] obtained in the weak coupling limit with the present one for the strong coupling spin polaron model we can argue that the spin-exchange pairing could be true mechanism for high-temperature superconductivity proposed earlier by several groups on the basis of some phenomenological models (see e.g., [21] - [23]).

2 Polaron model

We start from the $t-t'-J$ model with the Hamiltonian

$$H_{t-J} = -\sum_{ij\sigma} t_{ij} \; \tilde{c}^{+}_{i\sigma}\tilde{c}_{j\sigma} \; + J\sum_{(ij)}(\mathbf{S}_i\mathbf{S}_j - \frac{1}{4}n_i n_j) = H_t + H_J. \qquad (1)$$

where the first term describes electron hopping with energy $t_{ij} = t$ for the nearest neighbors and $t_{ij} = t'$ for the next nearest neighbors sites on a two dimensional square lattice. The electron operators $\tilde{c}_{i\sigma}^+ = c_{i\sigma}^+(1 - n_{i-\sigma})$ act in the space without double occupancy and $n_i = n_{i\uparrow} + n_{i\downarrow}$ is the number operator for electrons. The second term describes spin-1/2 Heisenberg antiferromagnet (AF) with exchange energy J. In the model two main features of a doped hole motion in copper-oxides are properly taken into account: constraint on no double occupancy for holes on lattice sites due to strong electron correlations and interaction of holes with AF spin fluctuations that brings about strong renormalization of the QP spectrum.

For a small concentration of holes when the long range AF order is preserved or at least strong AF correlations for nearest-neighbors still governs the hole motion, the $t - J$ model can be reduced to a more simple spin polaron model as it has been proposed in [11], [12]. To consider superconducting pairing of spin polarons we have to take into account explicitly a two-sublattice structure for the Heisenberg AF. By introducing two sublattices with spin up $(i \in\uparrow)$ and spin down $(i \in\downarrow)$ we define the hole spinless fermion operators for two sublattices by the equation:

$$\tilde{c}_{i\uparrow} = h_i^+,\ \tilde{c}_{i\downarrow} = h_i^+ S_i^+\ (i \in\uparrow); \quad \tilde{c}_{i\downarrow} = f_i^+,\ \tilde{c}_{i\uparrow} = f_i^+ S_i^-\ (i \in\downarrow). \tag{2}$$

where S_i^+, S_i^- are spin operators on the corresponding sublattices. In the linear spin-wave approximation (LSWA) the exchange part of the Hamiltonian (1) can be written as (see e.g., [15]):

$$H_J = \sum_q \omega_q(\alpha_q^+\alpha_q + \beta_q^+\beta_q) + E_0^J \tag{3}$$

where $\alpha_q^+(\alpha_q)$ and $\beta_q^+(\beta_q)$ are the magnon creation (annihilation) operators coupled with the spin lowering operators on two sublattices in LSWA : $S_i^+ \simeq$

a_i, $(i \in\uparrow)$, $S_i^+ \simeq b_i^+$, $(i \in\downarrow)$ by the Bogoliubov canonical transformation:

$$a_{\mathbf{k}} = v_k\alpha_{\mathbf{k}} + u_k\beta_{-\mathbf{k}}^+, \quad b_{\mathbf{k}} = v_k\beta_{\mathbf{k}} + u_k\alpha_{-\mathbf{k}}^+, \tag{4}$$

$$u_k = \left(\frac{1+\nu_k}{2\nu_k}\right)^{1/2}, \quad v_k = -\text{sign}(\gamma_k)\left(\frac{1-\nu_k}{2\nu_k}\right)^{1/2}. \tag{5}$$

with $\nu_k = \sqrt{1-\gamma_k^2}$, $\gamma_k = \frac{1}{2}(\cos ak_x + \cos ak_y)$. The spin-wave energy is given by $\omega_k = SzJ(1-\delta)^2\nu_k$ with δ being a hole concentration and $z = 4$ being the number of the nearest neighbors. The summation over wave-vectors in (3) and below is restricted to $N/2$ points in the AF Brillouin zone. In derivation of the exchange part of the Hamiltonian (3) the contact interaction between holes was taken into account only in the mean field approximation that results in the renormalization of the magnon energy proportionally to the factor $(1-\delta)^2$.

By employing the two sublattice representation (2) for holes and the LSWA we get the following expression for the hopping part of the Hamiltonian (1):

$$H_t \simeq \sum_{kq}(h_k^+ f_{k-q}[g(k,q)\alpha_q + g(q-k,q)\beta_{-q}^+] + H.c.) +$$

$$+ \sum_k(\epsilon_k - \mu)(h_k^+h_k + f_k^+f_k) \tag{6}$$

where

$$g(k,q) = \frac{zt}{\sqrt{N/2}}(u_q\gamma_{k-q} + v_q\gamma_k), \tag{7}$$

and the next nearest neighbour hopping energy $\epsilon_k = 4t'\cos ak_x \cos ak_y$. We ignored two-magnon scattering processes proportional to t' since $|t'|/t \ll 1$. The chemical potential μ should be calculated self-consistently as a function of hole concentration δ and temperature T from the equation:

$$\delta = \langle h_i^+h_i\rangle + \langle f_i^+f_i\rangle \tag{8}$$

3 Hole Green function

To discuss a singlet superconducting pairing within the spin polaron model (3), (6) we consider the equation of motion method for the matrix Green function

$$\hat{G}(k, t-t') = \ll \Psi_k(t) | \Psi_k^+(t') \gg \qquad (9)$$

in terms of the Nambu operators:

$$\Psi_k = \begin{pmatrix} \tilde{c}_{k\uparrow} \\ \tilde{c}^+_{-k\downarrow} \end{pmatrix} = \begin{pmatrix} h_k^+ \\ f_{-k} \end{pmatrix}, \qquad \Psi_k^+ = \left(\tilde{c}^+_{k\uparrow}\ \tilde{c}_{-k\downarrow}\right) = \left(h_k\ f^+_{-k}\right), \qquad (10)$$

where Zubarev's notation for the anticommutator Green function (9) was used [24].

By differentiating the Green function (9) in respect to two times t and t' we obtain the following Dyson equation as described in [16]:

$$\hat{G}(k,\omega)^{-1} = \omega\hat{\tau}_0 + (\epsilon_k - \mu)\hat{\tau}_3 - \hat{\Sigma}(k,\omega), \qquad (11)$$

where $\hat{\tau}_0$ and $\hat{\tau}_3$ are the standard Pauli matrix. The self-energy operator $\hat{\Sigma}(k,\omega)$ is given by the irreducible part of the many-particle Green function. Its components have the form

$$\Sigma_{hh}(k,\omega) = -\Sigma_{ff}(-k,-\omega) = \sum_q \langle\langle f^+_{k-q} Q^+_{k,q} \mid f_{k-q} Q_{k,q}\rangle\rangle^{(ir)}_\omega, \qquad (12)$$

$$\Sigma_{hf}(k,\omega) = (\Sigma_{fh}(k,\omega))^* = -\sum_q \langle\langle f^+_{k-q} Q^+_{k,q} \mid h^+_{q-k} Q_{q-k,q}\rangle\rangle^{(ir)}_\omega, \qquad (13)$$

where

$$Q_{k,q} = g(k,q)\alpha_q + g(q-k,q)\beta^+_{-q}. \qquad (14)$$

The irreducible part in (12), (13) has no parts connected by the single zero-order Green function, $\hat{G}^0(k,\omega) = (\omega\hat{\tau}_0 + (\epsilon_k - \mu)\hat{\tau}_3)^{-1}$.

To obtain the self-consistent equations for the Green function (11) we employ the self-consistent Born approximation (SCBA) which has been proved

to be quite reasonable in calculation of the one-hole spectrum in the normal state (see, e.g. [11]- [18]). For the many-particle time-dependent correlation functions in (12), (13) the SCBA is equivalent to the mode coupling approximation which is just the noncrossing diagram approximation:

$$\langle f^+_{k-q}(t) Q^+_{k,q}(t) f_{k-q} Q_{k,q}\rangle \simeq \langle f^+_{k-q}(t) f_{k-q}\rangle \langle Q^+_{k,q}(t) Q_{k,q}\rangle, \qquad (15)$$

$$\langle f^+_{k-q}(t) Q^+_{k,q}(t) h_{k-q} Q_{q-k,q}\rangle \simeq \langle f^+_{k-q}(t) h_{q-k}\rangle \langle Q^+_{k,q}(t) Q_{q-k,q}\rangle. \qquad (16)$$

By using the spectral representation for the Green function in (12), (13) and the Fourier representation for the correlation functions (15), (16) we get the following equations for the self-energy:

$$\Sigma_{hh}(k,\omega) = \sum_q \iint_{-\infty}^{+\infty} dz d\Omega N(\omega,z,\Omega)\lambda_{11}(k,k-q \mid \Omega) A_{hh}(q,z), \qquad (17)$$

$$\Sigma_{hf}(k,\omega) = \sum_q \iint_{-\infty}^{+\infty} dz d\Omega N(\omega,z,\Omega)\lambda_{12}(k,k-q \mid \Omega) A_{hf}(q,z), \qquad (18)$$

where

$$N(\omega,z,\Omega) = \frac{1}{2}\frac{\tanh(z/2T) + \coth(\Omega/2T)}{\omega - z - \Omega}. \qquad (19)$$

Here by using the symmetry relation for the anticommutator Green functions for fermions

$$\langle\langle h_k^+ \mid h_k\rangle\rangle_{z+i\delta} = \langle\langle f_k^+ \mid f_k\rangle\rangle_{z+i\delta} = -\langle\langle f_k \mid f_k^+\rangle\rangle_{-z-i\delta}$$

and

$$\langle\langle h_k^+ \mid f^+_{-k}\rangle\rangle_{z+i\delta} = -\langle\langle f^+_{-k} \mid h_k^+\rangle\rangle_{z+i\delta} = \langle\langle f_{-k} \mid h_k\rangle\rangle_{z+i\delta}$$

we introduce the spectral density for holes:

$$A_{hh}(k,z) = -\frac{1}{\pi}\mathrm{Im}\ \langle\langle h_k^+ \mid h_k\rangle\rangle_{z+i\delta} = A_{ff}(-k,-z), \qquad (20)$$

$$A_{hf}(k,z) = -\frac{1}{\pi}\mathrm{Im}\ \langle\langle h_k^+ \mid f^+_{-k}\rangle\rangle_{z+i\delta} = A_{fh}(k,z), \qquad (21)$$

and the spin–hole interaction function

$$\lambda_{11}(k,q\mid\Omega) = g^2(q-k,q)B(-q,\Omega) - g^2(k,q)B(q,-\Omega), \qquad (22)$$

$$\lambda_{12}(k,q\mid\Omega) = g(k,q)g(q-k,q)[B(-q,\Omega) - B(q,-\Omega)]. \qquad (23)$$

The spectral density for the magnon Green function is defined by

$$B(q,\omega) = -\frac{1}{\pi}\mathrm{Im}\ \langle\langle\beta_q \mid \beta_q^+\rangle\rangle_{\Omega+i\delta} = -\frac{1}{\pi}\mathrm{Im}\ \langle\langle\alpha_q^+ \mid \alpha_q\rangle\rangle_{-\Omega-i\delta}. \qquad (24)$$

The solution of the Dyson equation (11) can be written in the Eliashberg notation as

$$\hat{G}(k,\omega) = \frac{\omega Z_k(\omega)\hat{\tau}_0 + (\chi_k(\omega)-\epsilon_k)\hat{\tau}_3 + \phi_k(\omega)\hat{\tau}_1}{(\omega Z_k(\omega))^2 - (\chi_k(\omega)-\epsilon_k)^2 - \phi_k(\omega)^2} \qquad (25)$$

where

$$\omega(1 - Z_k(\omega)) = \frac{1}{2}[\Sigma_{hh}(k,\omega) + \Sigma_{ff}(k,\omega)]$$
$$\chi_k(\omega)) = \frac{1}{2}[\Sigma_{hh}(k,\omega) - \Sigma_{ff}(k,\omega)] \qquad (26)$$
$$\phi_k(\omega) = \Sigma_{hf}(k,\omega) = (\Sigma_{fh}(k,\omega))^*$$

and $\Sigma_{ff}(k,\omega) = -\Sigma_{hh}(k,-\omega)$.

To solve the system of equations (26) we have to calculate the magnon Green function in (24).

4 Magnon Green Function

For two sublattice polaron model (3), (6) we have to consider the matrix magnon Green function

$$\hat{D}(q,t-t') = \langle\langle A_q(t); A_q^+(t')\rangle\rangle, \qquad (27)$$

where the two-component magnon operators are

$$A_q = \begin{pmatrix} \alpha_q \\ \beta_{-q}^+ \end{pmatrix}, \quad A_q^+ = (\alpha_q^+ \beta_{-q}). \qquad (28)$$

By using the equation of motion method as in (11) for the commutator Green function (27) we get the following Dyson equation

$$\hat{D}^{-1}(q,\omega) = \omega\hat{\tau}_3 - \omega_q\hat{\tau}_0 - \hat{\Pi}(q,\omega). \qquad (29)$$

The polarization operator is given by the irreducible part of many–particle Green functions

$$\Pi_{11}(q,\omega) = \Pi_{22}(-q,-\omega) = \sum_{k,k'} g(k,q)g(k',q)\langle\langle f_{k-q}^+ h_k \mid h_{k'}^+ f_{k'-q}\rangle\rangle_\omega^{(ir)}, \qquad (30)$$

$$\Pi_{12}(q,\omega) = \Pi_{21}^*(q,\omega) = \sum_{k,k'} g(k,q)g(k-q',-q)\langle\langle f_{k-q}^+ h_k \mid h_{k'}^+ f_{k'-q}\rangle\rangle_\omega^{(ir)}. \qquad (31)$$

The irreducible parts (ir) in (30), (31) has no parts connected by the single zero-order magnon Green function $\hat{D}^0(q,\omega) = (\omega\hat{\tau}_3 - \omega_q\hat{\tau}_0)^{-1}$. To calculate the polarization operator we use also the SCBA which can be written as the mode coupling approximation for time-dependent correlation functions in (30), (31). As a result we get the following expressions for the components (30), (31):

$$\Pi_{11}(q,\omega) = \sum_k \iint_{-\infty}^{\infty} d\omega_1 d\omega_2 N(\omega,\omega_1,\omega_2)\{g^2(k,q)A_{hh}(k,\omega_1)A_{hh}(k-q,\omega_2) -$$
$$- g(k,q)g(q-k,q)A_{fh}(k,\omega_1)A_{hf}(k-q,\omega_2)\} \qquad (32)$$

$$\Pi_{12}(q,\omega) = \sum_k \iint_{-\infty}^{\infty} d\omega_1 d\omega_2 N(\omega,\omega_1,\omega_2)\{g(k,q)g(q-k,q)A_{hh}(k,\omega_1)A_{hh}(k-q,\omega_2) -$$
$$- g^2(k,q)A_{fh}(k,\omega_1)A_{hf}(k-q,\omega_2)\} \qquad (33)$$

where

$$N(\omega,\omega_1,\omega_2) = \frac{1}{2}\frac{\tanh(\omega_2/2T) - \tanh(\omega_1/2T)}{\omega + \omega_1 - \omega_2}. \qquad (34)$$

Therefore the matrix magnon Green function (29) can be written as

$$\hat{D}(q,\omega) = \begin{pmatrix} \omega + \omega_q + \Pi_{22}(q,\omega) & -\Pi_{12}(q,\omega) \\ -\Pi_{21}(q,\omega) & -\omega + \omega_q + \Pi_{11}(q,\omega) \end{pmatrix} \frac{1}{\det(q,\omega)} \qquad (35)$$

where

$$\det(q,\omega) = [\omega - \omega_q - \Pi_{11}(q,\omega)][\omega + \omega_q + \Pi_{22}(q,\omega)] + |\Pi_{12}(q,\omega)|^2 \qquad (36)$$

It should be pointed out that in eqs. (32), (33) the contributions from the anomalous Green functions, which are nonzero only below the superconducting temperature T_c, are also taken into account.

Therefore we got the closed system of equations for the hole Green function (25) and the magnon Green function (35) which should be solved self-consistently.

5 Numerical Results and Discussion

For numerical solution of the system of equations (26), (32), (33) we employ the imaginary frequency representation for the hole Green function (25) with $\omega = i\omega_n = i\pi T(2n+1)$ and the magnon Green function (35) with $\omega = i\omega_n = i\pi T 2n$, $n = 0, \pm 1, \dots$. By using the representation for the function (19)

$$N(i\omega_n, z, \Omega) = -T \sum_m \frac{1}{i\omega_m - z} \; \frac{1}{i(\omega_n - \omega_m) - \Omega} \qquad (37)$$

after integration in (17), (18) we get

$$\Sigma_{hh}(k, i\omega_n) = -T \sum_q \sum_m G_{hh}(q, i\omega_m)\lambda_{11}(k, k-q \mid i\omega_n - i\omega_m), \qquad (38)$$

$$\Sigma_{hf}(k, i\omega_n) = -T \sum_q \sum_m G_{hf}(q, i\omega_m)\lambda_{12}(k, k-q \mid i\omega_n - i\omega_m). \qquad (39)$$

The interaction functions are given by

$$\lambda_{11}(k, q \mid i\omega_\nu) = g^2(k,q) D_{11}(q, -i\omega_\nu) + g^2(q-k, q) D_{11}(-q, i\omega_\nu), \qquad (40)$$

$$\lambda_{12}(k, q \mid i\omega_\nu) = g(k,q) g(q-k, q)\{D_{11}(q, -i\omega_\nu) + D_{11}(-q, i\omega_\nu)\}. \qquad (41)$$

For the magnon Green function we use the representation for (34) in the form

$$N(i\omega_\nu, \omega_1, \omega_2) = T \sum_m \frac{1}{i\omega_m - \omega_1} \; \frac{1}{i(\omega_m + \omega_\nu) - \omega_2} \qquad (42)$$

with $\omega_\nu = 2\pi T \nu$ and $\omega_m = \pi T(2m+1)$. After integration over $d\omega_1$, $d\omega_2$ in eqs. (32), (33) we get

$$\Pi_{11}(q, i\omega_\nu) = T \sum_k \sum_m \{g^2(k,q) G_{hh}(k, i\omega_m) G_{hh}(k-q, i\omega_\nu + i\omega_m) - $$
$$- g(k,q) g(q-k,q) G_{hf}(k, i\omega_m) G_{hf}(k-q, i\omega_\nu + i\omega_m)\} \qquad (43)$$

$$\Pi_{12}(q, i\omega_\nu) = T \sum_k \sum_m \{g(k,q) g(q-k,q) G_{hh}(k, i\omega_m) G_{hh}(k-q, i\omega_\nu + i\omega_m) - $$
$$- g^2(k,q) G_{hf}(k, i\omega_m) G_{hf}(k-q, i\omega_\nu + i\omega_m)\} \qquad (44)$$

Here we have

$$G_{hh}(k, i\omega_m) = -G_{ff}(-k, -i\omega_m), \quad G_{hf}(k, i\omega_m) = G_{hf}(k, -i\omega_m)$$

A linearized system of the Eliashberg equations (26) in the limit $T \to T_c^-$, which can be used to calculate T_c, has the following form

$$G_{hh}(k, i\omega_n) = \frac{1}{i\omega_n + \epsilon_k - \mu - \Sigma_{hh}(k, i\omega_n)}, \qquad (45)$$

$$\Phi(k, i\omega_n) = T \sum_q \sum_m \lambda_{12}(k, k-q \mid i\omega_n - i\omega_m) G_{hh}(q, i\omega_m) G_{hh}(-q, -i\omega_m) \Phi(q, i\omega_m). \qquad (46)$$

In this limit we can also neglect the contributions from the anomalous Green functions in the polarization operator (43), (44).

System of equations (45), (46) was solved by the fast Fourier transformation [25] for a given concentration of holes

$$\delta = \frac{1}{2} + \frac{2T}{N} \sum_k \sum_{n=0}^{\infty} G(k, i\omega_n) \qquad (47)$$

in the range $0.1 \leq \delta \leq 0.35$ .The calculations were performed for the parameters of the spin polaron model (3), (6): $J = 0.4$ and $t' = -0.1$ though we did not find much difference in results for $t' = 0$ (all energies are measured in units of t).

In the numerical calculations we have used a finite mesh of 64×64 k-points in the full BZ and 200-700 points for Matsubara frequencies with a constant cut $\omega_{max} = 10t$ in the summation over it. Usually $10 - 30$ iterations were needed to obtain a solution for the self energy with an accuracy of order 0.001. To calculate the spectral density for holes (20) and the density of states (DOS)

$$A(\omega) = \frac{1}{N}\sum_{k} A(k,\omega) \tag{48}$$

a Pade approximation was used for analytical continuation from Matsubara points on the imaginary axis. At first a self-consistent calculation of the normal Green function (45) was done and then the gap equation (46) was solved for a given concentration of holes.

5.1 Spin polaron quasi-particle spectrum

Self consistent calculation of the Green function (45) with the self-energy operator (38) was performed at first by neglecting magnon renormalization in the interaction function (40) and then a full self-consistent solution by allowing for the polarization operators (43), (44), in (29) was done. The calculations presented in this section was done at finite temperature $T = 0.012$ that is slightly higher then the maximal superconducting temperature given in the next section.

In Fig.1 we present spectral functions at $\mathrm{k} = (\pi/2, \pi/2)$ for several hole concentrations. For small hole concentrations, $\delta = 0.02, 0.04, 0.06, 0.08, 0.10$ there are no much differences for spectral functions calculated from the hole

Green functions with renormalized and unrenormalized magnon energy in the interaction function, eq.(40). So in Fig.1(a) only the results with renormalized magnon spectra are shown as a function of $\omega - \mu$. At higher hole concentrations a negative contribution to the spectral density (24) at $\omega < 0$ energy develops due to excitation of electron-hole pairs that results in negative values for hole spectral functions in the incoherent part of the spectrum. In Fig. 1 (b, c, d) we compare the spectral functions calculated with renormalized (solid line) and unrenormalized (dashed line) magnon spectra. This negative contribution develops at first for long wavelength magnons for small wave-vector as was pointed out already in [17, 18]. In Fig.2 we show the corresponding hole density of states (DOS) (48) at different concentration of holes. In Fig.2(a, b) we compare the results of calculations with renormalized (solid line) and unrenormalized (dashed line) magnon spectra. A negative density of states appears already at $\delta = 0.08$ (Fig.2(b)) In Fig.2(c) we show the density of states in the vicinity of the quasi-particle peak at large hole concentrations calculated with unrenormalized magnon spectra. Since the main quasi-particle peak at $\mathrm{k} = (\pi/2, \pi/2)$ shown in Fig. 1 does not change much in shape with doping even at large hole concentrations the picture of spin polarons as stable quasi-particle seems to be relevant even at large hole concentrations. This robust behaviour of spin polarons with doping can be explained by a small size of the polarons in comparisons with antiferromagnetic correlation length at quite large exchange energy. Here we present calculations for $J = 0.4$ and our estimation for larger exchange energy $J = 1$ show that spin polarons appear to be stable at large hole concentrations. The real part of the hole self energy at zero frequency is shown in Fig.3 for hole concentrations $\delta = 0.02, 0.04, 0.06, 0.08$ (from top to bottom) and in Fig.4 the k-dependence of it is shown in the full Brillouin zone for $\delta = 0.10$. With doping the bandwidth of the hole quasi-particle spectrum increased substantially but does not change much its shape. Nevertheless the rigid band approximation adopted in [19], [20] seems to be inadequate and to obtain reliable numerical results for this strongly correlated system of holes with zero free kinetic energy a self consistent determination of spin polaron spectrum is required. In Fig. 5 the Fermi surface defined as Re $\Sigma(k, \omega = 0) = 0$ is shown for hole concentrations $\delta = 0.1$ (thick line) and $\delta = 0.2$ (thin line). The full BZ shown in the picture consists of two degenerate antiferromagnetic ones marked by the dashed line. We see that at small hole concentrations only hole pockets are filled and the transition from a small hole Fermi surface to a large one develops quite sharply around $\delta = 0.15$. Temperature dependence of the momentum distribution for holes in the spin polaron model was investigated in some details in [16] where it was shown that the Fermi surface washed out at some temperature of the order $T_d = 1.5J\delta$. So at quite low temperatures $T = 0.01$ considered here the Fermi surface does not change much with temperature. It should be also pointed out that a high density of states in the present calculations (see Fig.2) results from a narrowing of a free electron bandwidth due to strong correlations (spin polaron formation) and has nothing to do with the van Hove singularity.

5.2 Superconducting pairing of spin polarons

In the present paper we consider only the linearized Eliashberg equation (46) for the pairing energy $\phi(k, i\omega_n)$ to study the symmetry of the superconducting order parameter and to evaluate the superconducting temperature T_c. Eq.(46) was solved by fast Fourier transforms for different hole concentrations employing the results for the hole spectral functions (20) presented in the previous section. Looking only for even functions of wave-vector k that are realized in the singlet pairing we obtained only d-type symmetry for the gap function. In Fig.6 we show k-dependence of the pairing energy, $\phi(\mathrm{k}, \omega = 0)$, in the quarter of the full BZ. It has typical d-wave symmetry with two ridges resulted from sharp changes of the interaction function at the Fermi surface (cp. Fig.4). In Fig. 7 frequency dependence of the real (imaginary) part of the gap function $\Delta(\mathrm{k}, \omega) = \phi(\mathrm{k}, \omega)/Z(\mathrm{k}, \omega)$ at $\mathrm{k} = (0, \pi/4)$(a), $\mathrm{k} = (0, 3\pi/8)$(b) is shown by solid (dashed) line. The characteristic for the pairing theory cut off energy is of order $J \simeq 0.4$ which is closed to the quasi-particle bandwidth. Therefore we have really a strong coupling limit for spin polarons where all quasi-particle are paired contrary to the weak coupling in conventional superconductors. It is interesting that the same ω-dependence for the gap function with a cut off energy of order $0.2t$ was obtained in the Hubbard model in the weak coupling limit [4, 5, 7]. By examining the temperature dependence of the highest egenvalue in the eq. (46) at different hole concentrations (see Fig.8) we can find the temperature when it crosses the value 1. At this temperature the normal state becomes unstable due to singlet pairing of quasi-particle — spin polarons on different sublattices. In Fig.9 the dependence of superconducting temperature on hole concentrations is shown. We cannot solve our equation at lower temperatures then $T = 0.004$ and therefore has no results for T_c for $\delta < 0.1$. The maximum of T_c at $\delta \simeq 0.25$ is explained by crossing the maximum of the density of hole states by the Fermi level (see Fig.2). This results are quite different with the monotonic increasing of T_c obtained within the weak coupling limit from the BCS equation in [20] and maximum of T_c observed in [10] near half filling $\delta = 0$ for small clusters.

We also investigate T_c-dependence on the exchange energy J which is shown in Fig. 10. T_c increases with J but saturates at larger values. However, we does not obtain a large drop of T_c near $J = 3$ observed in small clusters calculations near phase separation [9]. But the latter fenomenon is beyond the scope of our theoretical approach.

6 Conclusions

In the present paper the hole and magnon spectra for finite temperature and hole concentrations and superconducting pairing of holes in the model with strong electron correlations have been investigated. Numerical solution of the self-consistent equations show a strong renormalization of the hole spectra due to AF spin fluctuations and formation of a narrow quasi-particle spin polaron band (see Figs. 1-3). The same spin fluctuations mediate superconducting d-wave pairing of two holes on different AF sublattices with maximum $T_c \simeq 0.012t$ for $J = 0.4t$ around the hole concentration $\delta = 0.25$. In our calculations we does not observed a strong T_c dependence on J (Fig. 10). Frequency dependence of the gap function (Fig. 7) demonstrates a standard behaviour for the boson-mediated pairing theory. It should be stressed that in our self consistent calculations we does not make any fitting for a model with only two dimensionless parameters, J/t and μ/t.

For a small concentration of holes when the long range AF order is preserved or at least strong AF correlations for nearest-neighbors still governs the hole motion our results obtained for renormalized and unrenormalized magnon spectra do not differ much. For larger hole concentrations, $\delta \geq 0.1$, a negative contribution to the magnon spectral density at $\omega < 0$ due to electron-hole pair excitations results in some instability of the incoherent part of the hole spectrum. Therefore for large concentration of holes we perform calculations with unrenormalized magnon spectrum. To consider magnon spectra for the large hole concentrations region a more elaborate study of spin fluctuation spectra should be done in the original $t-J$ model [26]. However, we believe that spin

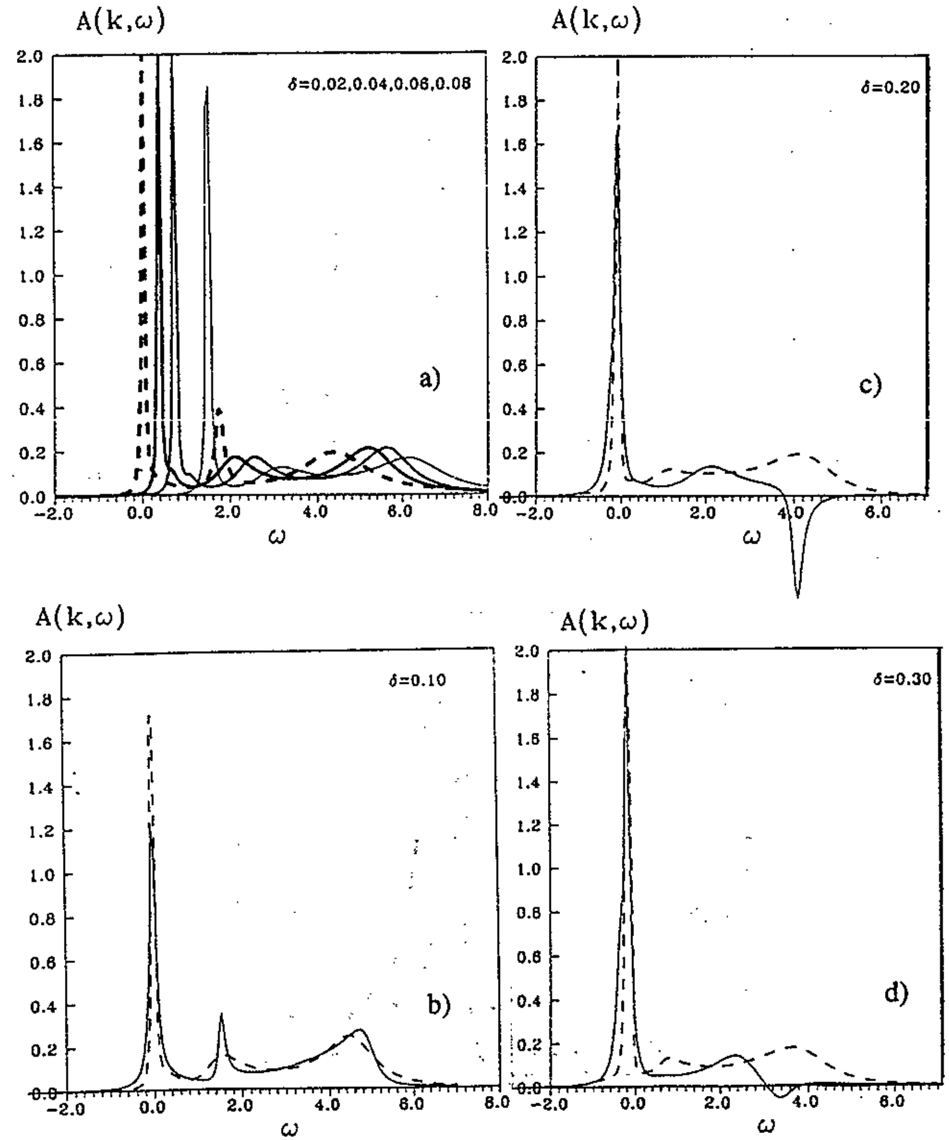

Fig 1. Spectral functions $A(k,\omega)$ at $\mathbf{k} = (\pi/2, \pi/2)$ for hole concentrations: $\delta = 0.02,\ 0.04,\ 0.06,\ 0.08$ (from right to left) as a function of $\omega - \mu$(a); 0.1(b); 0.2(c); 0.3(d). Solid (dashed) lines in (b,c,d) correspond to renormalized (unrenormalized) magnon spectra.

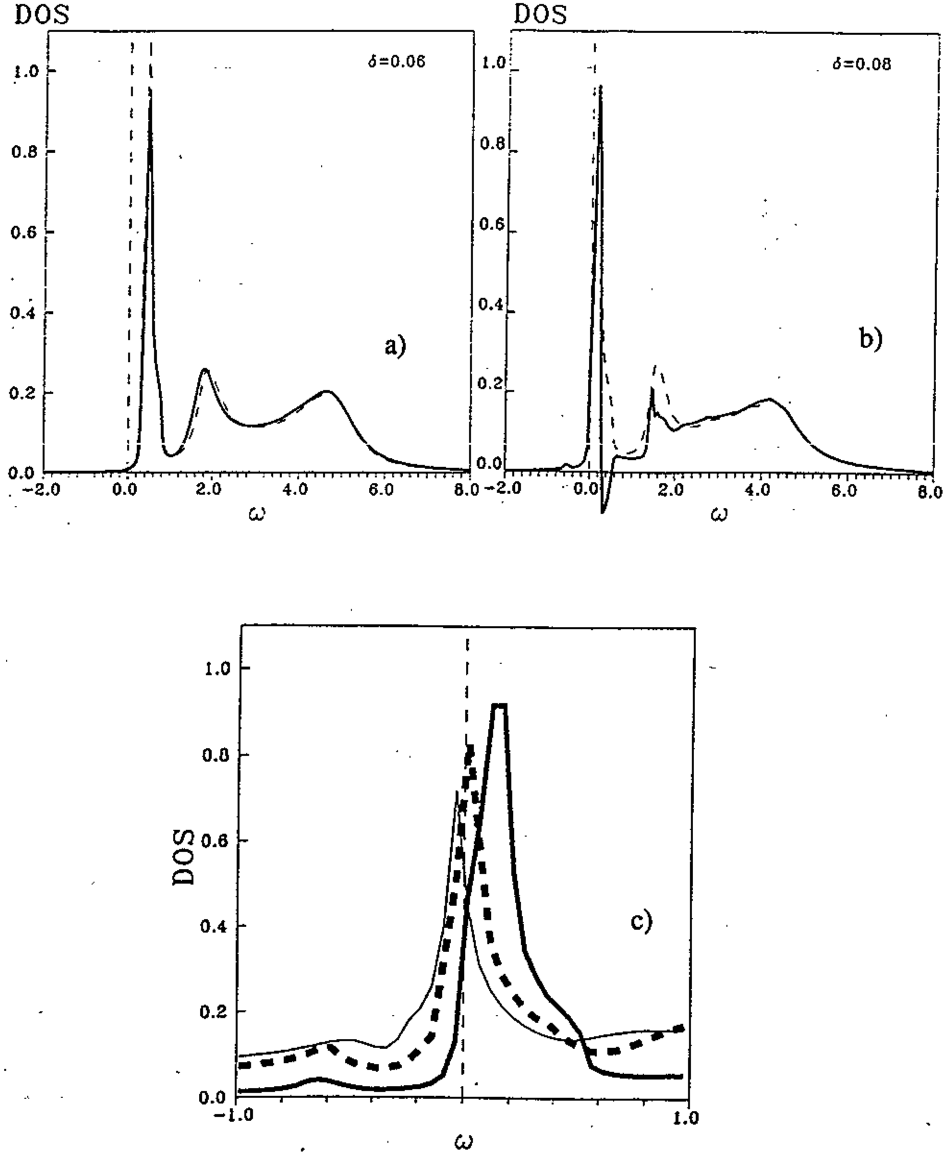

Fig 2. Density of states for hole concentrations $\delta = 0.06$ (a); 0.08(b); 0.1, 0.25, 0.35 (from right to left) (c). Solid (dashed) lines in (a,b) correspond to renormalized (unrenormalized) magnon spectra.

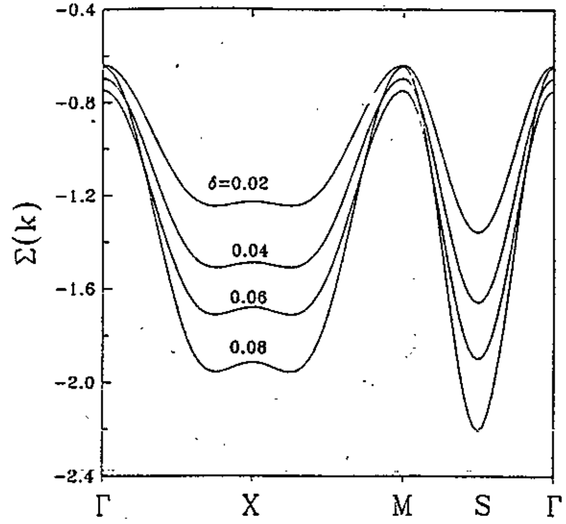

Fig 3. Spin polaron spectra for hole concentrations $\delta = 0.02,\ 0.04,\ 0.06,\ 0.08$ (from top to bottom).

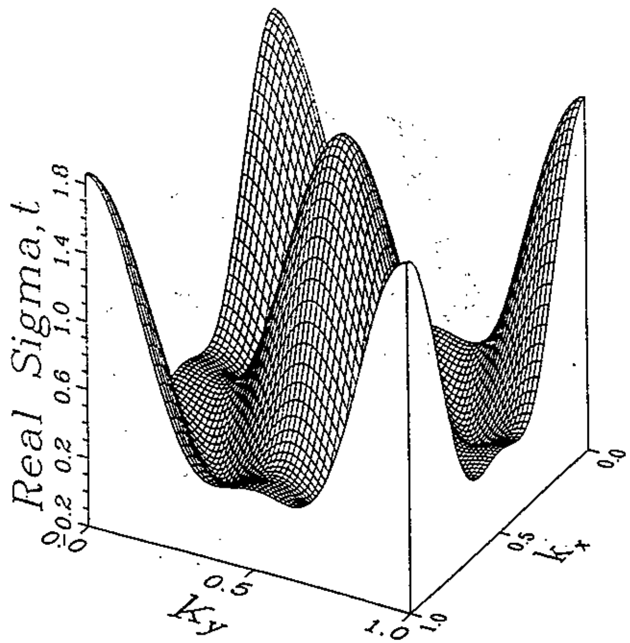

Fig 4. k-dependence of Re $\Sigma(\mathbf{k}, \omega = 0)$ for $\delta = 0.1$.

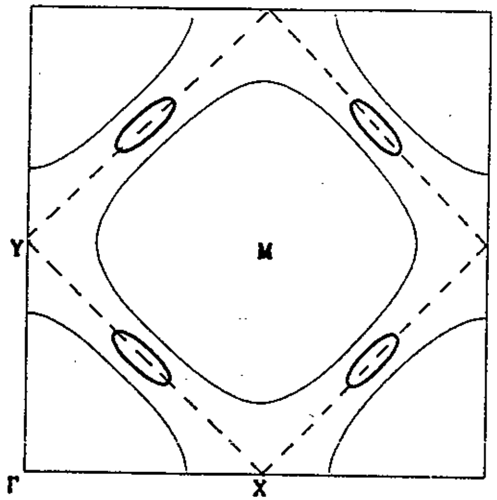

Fig 5. Fermi surface for hole concentrations $\delta = 0.1$ (thick lines) and 0.2 (thin lines). AF Brillouin zone is shown by dashed line.

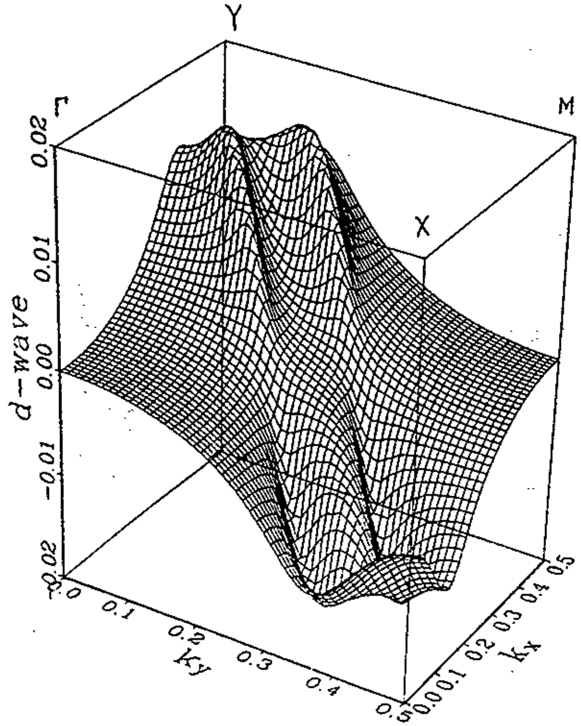

Fig 6. k-dependence of the pairing energy $\phi(\mathrm{k}, \omega = 0)$.

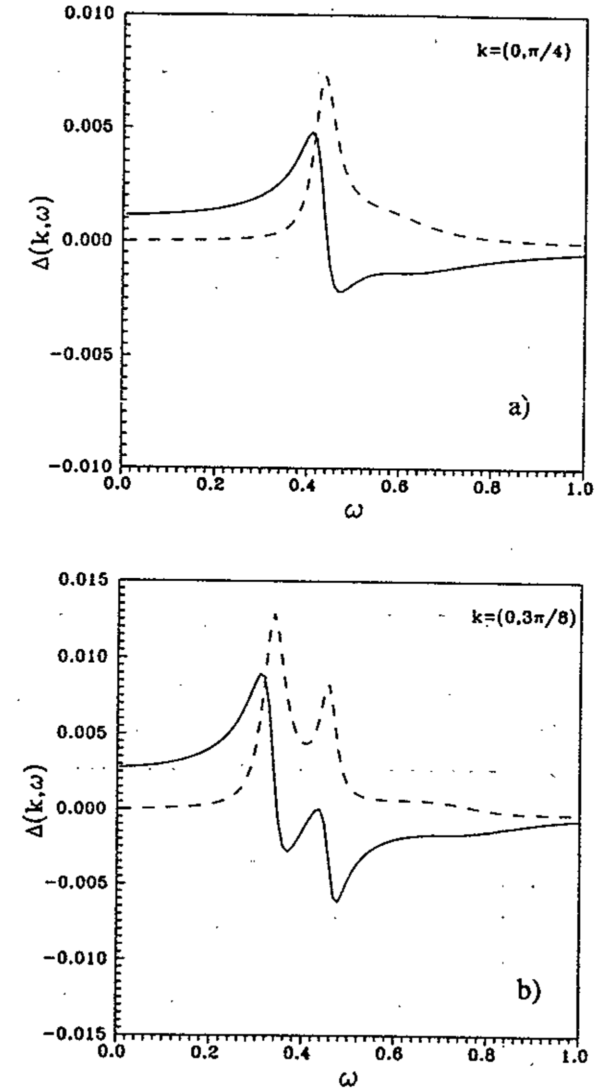

Fig 7. The real (solid) and imaginary (dashed) parts of $\Delta(\mathbf{k},\omega) = \phi(\mathbf{k},\omega)/Z(\mathbf{k},\omega)$ versus ω at $\mathbf{k} = (0, \pi/4)$(a) and $\mathbf{k} = (0, 3\pi/8)$(b) at $T/T_c \simeq 0.8$.

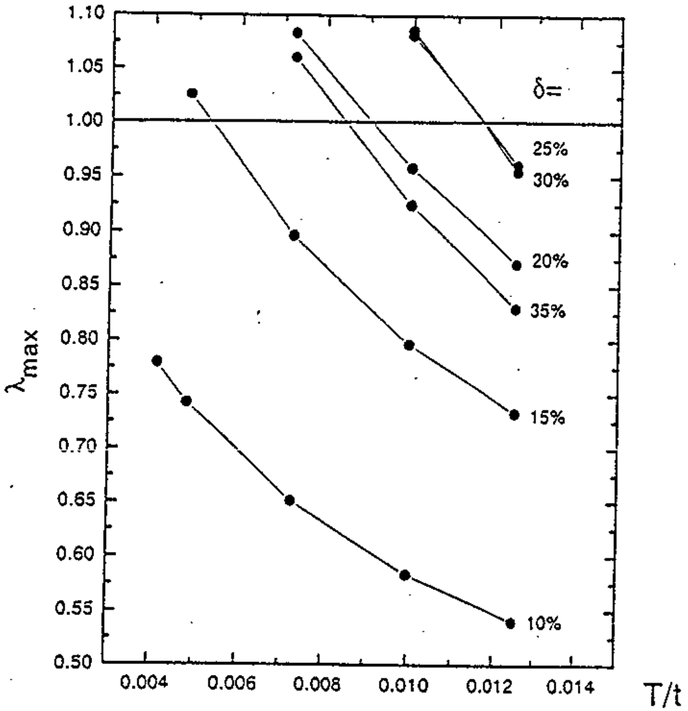

Fig 8. Temperature dependence of egenvalues for hole concentrations $\delta = 0.1;\ 0.15;\ 0.35;\ 0.20;\ 0.3;\ 0.25$ from bottom to top .

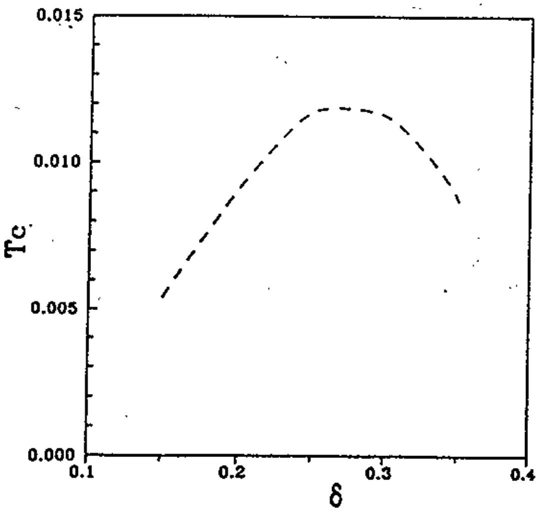

Fig 9. Concentration dependence of superconducting temperature T_c

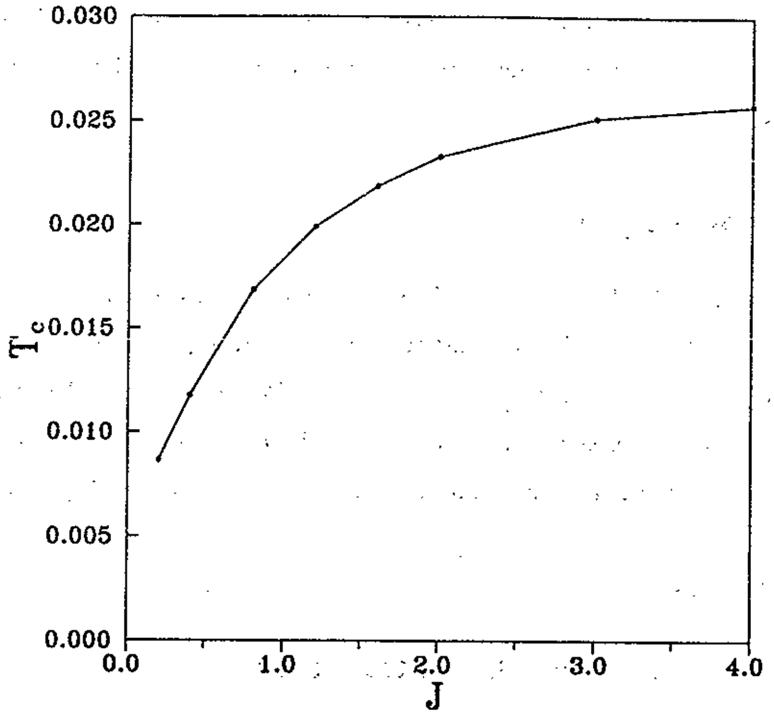

Fig 10. Superconducting temperature T_c versus exchange energy J.

polarons dressed by AF spin fluctuations are the relevant quasi-particles even in this region of large hole concentrations and their pairing mediated by the spin fluctuations, which is also observed in the Hubbard model [4]- [7], could represent the mechanism for high-temperature superconductivity in copper oxides.

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