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1. The equations for vertex functions of the Larkin-Migdal [1] approach to superfluid Fermi liquid were solved, in the acoustic limit (i.e., for $|\omega|, kv \ll \Delta$, where $\omega, \vec{k}$ denote the frequency and the wave vector of an external field, respectively, for Δ and v being the energy gap and the quasiparticle velocity on the Fermi sphere, respectively) for systems with isotropic S-pairing in paper [1] . For the system with the Balian-Werthamer [2] pairing * such equations for such a limit were solved by us for scalar and vector vertices [4] and for spin vertices [5] . In papers [1] and [4,5] the effective interaction in the particle-hole channel, coinciding with the effective quasiparticle interaction for normal system,

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* It is known to very few persons only that the main results of this paper were obtained independently by Vdovin [3] .

was of quite general form. On the other hand, the effective interaction in the particle-particle (or hole-hole) channel was restricted only to pairing channel, i.e., to ℓ =0 in /1/ and to ℓ =1 in /4,5/ . Note that according to papers /1,4/ this interaction can be represented by two functions $f^{\xi}_{\varepsilon}(\hat{\vec{p}}\hat{\vec{p}}')$, ε $\pm$ 1, such that

$$f^{\xi}_{\varepsilon}(\hat{\vec{p}}\hat{\vec{p}}') = \sum_{\ell=0}^{\infty}(2\ell+1) f_\ell P_\ell(\hat{\vec{p}}\hat{\vec{p}}'), \tag{1}$$

with the summation over even ℓ for ε =1 and over odd ℓ for ε -1. The Legendre amplitudes of such an interaction, f_ℓ , are equal, for dimensionless interaction, to $[\ln(2\xi/r_\ell)]^{-1}$, where ξ denotes the cut-off energy with r_ℓ being some nonnegative constants (cf.paper /1/). In the pairing channel, i.e., for ℓ =0 for S-pairing and ℓ =1 for pairing /2,3/ , $r_\ell = \Delta$, /1,4/ ; we will denote hereafter ℓ for the pairing channel by ℓ_0 . It can be easily verified for equations for vertex functions in papers /1,4,5/ that if $\min_{\ell\neq\ell_0} |\ln(\Delta/r_\ell)|$, is at least, a number of order of unity, then the harmonics with $\ell \neq \ell_0$ could be neglected in the acoustic limit. On the other hand, if $\min_{\ell\neq\ell_0} |\ln(\Delta/r_\ell)| \ll 1$ and the harmonic fulfilling this inequality appears in the equations for the vertex functions, then the solutions obtained in papers /1,4,5/ are not valid unless $|\omega|$, $kv \ll \Delta\left[\min_{\ell\neq\ell_0}|\ln(\Delta/r_\ell)|\right]^{1/2}$. Note that only in the equations for spin vertices for systems with Balian-Werthamer-Vdovin (BWV) pairing there appear together both functions f^{ξ}_{ε} ε $\pm$ 1 (cf.papers /1,4/) Hence, only for BWV pairing

and spin vertices one can meet the Legendre amplitudes with $\Delta\ell$ =1; in the remaining cases the minimal $\Delta\ell =$ 2. The f_ℓ amplitudes have to diminish beginning from some ℓ , as a result of the centrifugal force /1/ .

Hence, it is rather unexpected to fulfil the condition $|\ln(\Delta/r_\ell)| \ll 1$ for $\ell =$ 2,4,5... (BCS pairing) or ℓ = 3,5,7 ... (BWV pairing). On the other hand, the condition $|\ln(\Delta/r_0)| \ll 1$ seems to be quite natural for the BWV pairing, since the tendency to diminish f_ℓ for increasing ℓ acts here against a tendency for greater f_ℓ in the pairing channel. Hence, it is interesting to solve the equations for the spin vertex for systems with the BWV pairing assuming that $|\ln(\Delta/r_\ell)| \ll 1$ only for ℓ =0. Our results can play a role for spin waves in the B-phase of superfluid ^{3}He. Our previous results /5/ can be obtained from the present ones by a limiting transition. Our present notation coincides with the previous one; the definitions of /5/ will not be repeated, as a rule, here. Our paper /5/ will be hereafter denoted as I; its formulae by (n,I), where n denotes the number of the formula in I.

2. The equations for the tensor describing the normal spin vertex, in the acoustic limit, will be now of the form (cf.I and /3/)

$$J_{ab} = \delta_{ab} - \langle B\{\tfrac{1}{2}(1+\tilde{P})J_{ab} - \tilde{P}\hat{p}_b' J_{ac}\hat{p}_c' - [\omega + (\vec{k}\vec{v}')][i\tau_{ac}\hat{p}_d'\varepsilon_{cdb} + \lambda_a\hat{p}_b']\}\rangle_{\hat{p}'} . \tag{2}$$

The equations for τ_{ac} will coincide with those in I. The equation for the variable λ_a /4/ , for f_1^{T} consisting of only zeroth Legendre harmonic, in the acoustic limit, can be rewritten as follows

$$\lambda_a\left[\ln(\Delta/r) - \langle \omega^2 - (\vec{k}\vec{v})^2 \rangle_{\hat{p}}\right] = -\langle [\omega + (\vec{k}\vec{v})] J_{ab} \hat{p}_b \rangle_{\hat{p}} , \quad (3)$$

where the label "0" near r was omitted for simplicity. It is clear for the symmetry reasons that $\lambda_a = \lambda \hat{k}_a$ and, hence, eq. (3) can be rewritten as

$$\lambda\left[\ln(\Delta/r) + \tfrac{1}{3}k^2v^{-2} - \omega^2\right] = -\langle [\omega + (\vec{k}\vec{v})] \hat{k}_a J_{ab} \hat{p}_b \rangle_{\hat{p}} \quad (4)$$

Let us define $J_{ab} = J^{(1)}_{ab} + J^{(2)}_{ab}$, where $J^{(1)}_{ab}$ is expressed by τ_{ab} , as in I. Hence

$$J^{(2)}_{ab} = \lambda \langle B[\omega + (\vec{k}\vec{v})] \hat{k}_a \hat{p}'_b \rangle_{\hat{p}'} - \langle B\left[\tfrac{1}{2}(1+\tilde{P}) J^{(2)}_{ab} - \tilde{P} \hat{p}'_b J^{(2)}_{ac} \hat{p}'_c\right] \rangle_{\hat{p}'} \quad (5)$$

Using formulae (19.I) one can find that the first term of the right-hand side of eq. (5) is given by

$$\lambda\left[(\omega b_1 + kv b_2 W)\hat{k}_a \hat{p}_b + \tfrac{1}{3}kv(b_0 - b_2)\hat{k}_a \hat{k}_b\right], \quad (6)$$

with $W \equiv (\hat{\vec{k}}\hat{\vec{p}})$. Hence one can find that

$$J^{(2)}_{ab} = (C_0' + C_1' W) + E_0' \hat{k}_a \hat{k}_b , \quad (7)$$

if we take into account that the terms (7) form a closed set with respect to multiplication $\hat{p}_b J^{(2)}_{ac} \hat{p}_c$ appearing in (5). Substituting (7) into (5) one finds

$$C_0' = \lambda\omega \frac{b_1}{1+b_1} ; \; E_0' = \tfrac{1}{3}\lambda kv (b_0 - b_2) S^{-1} ; \; C_1' = \lambda kv\, b_2 (1+b_1) S^{-1} , \quad (8)$$

with

$$S = 1 + \tfrac{2}{3} b_0 + \tfrac{1}{3} b_2 . \quad (9)$$

Taking into account the expression of $J^{(1)}_{ab}$ by the variables X,R describing τ_{ab} (cf. I) and formula (7) one can write

$$J_{ab} = (A_0 + A_1 W)\delta_{ab} + (B_0 + B_1 W)\hat{p}_a \hat{p}_b + D \hat{p}_a \hat{k}_b + [C_0 + C_0' + (C_1 + C_1')W + C_2 W^2]\hat{k}_a \hat{p}_b + (E_0 + E_0' + E_1 W)\hat{k}_a \hat{k}_b , \quad (10)$$

where the variables A_n - E_n are expressed by X,R by formula (17-20, I) and (25-30,I). Substituting (10) into (4) one finds

$$\lambda\left[\ln(\Delta/r) + \tfrac{1}{3}k^2v^2 - \omega^2\right] = -\tfrac{1}{3}kv(A_0 + B_0 + C_1 + C_1' + E_0 + E_0') - \omega(C_0 + C_0') - \tfrac{1}{3}\omega(A_1 + B_1 + E_1 + C_2 + D) . \quad (11)$$

Substituting here C_0', C_1' and E_0' from (8) and $A_n - E_n$ from (17–20,I) and (25–30,I) one obtains

$$\lambda\,(a+k^2v_2^2-\omega^2)-\frac{2}{9}Xkv\omega(b_0-b_2)(1+b_1)S^{-1}=-\frac{1}{3}kv(1+b_1)(1+b_2)S^{-1}, \quad (12)$$

where

$$a \equiv (1+b_1)\ln(\Delta/r), \quad (13)$$

and

$$v_2^2 \equiv \frac{1}{3}v^2(1+b_0)(1+b_1)(1+b_2)(1+\frac{2}{3}b_0+\frac{1}{3}b_2)^{-1}. \quad (14)$$

In order to obtain the equations for X and R, corresponding to the equation for τ_{ac} one should substitute $\mathcal{J}_{ab}$ instead of $\mathcal{J}^{-(1)}_{ab}$ into eqs. (34.I) and (36.1), i.e., substitute E_0+E_0', C_0+C_0' and C_1+C_1' instead of E_0, C_0 and C_1, respectively, with the remaining substitution unchanged. Hence one finds

$$X(\omega^2-\frac{1}{5}k^2v^2)=-\omega(A_0+E_0+E_0')-\frac{1}{5}kv(A_1+D+E_1), \quad (15)$$

(cf. (34.I)) and the equation for R remains unchanged. Thus, the solution for R and the formulae for variables expressing by R only (i.e., A_0, A_1, B_0, B_1, D) remain unchanged too. Substituting formulae (8), (17.I), (20.I), (25.I), (29.I) and (30.I) into (15) one gets

$$\frac{1}{3}\lambda\omega kv(b_0-b_2)+X(\omega^2-v_1^2k^2)=-\omega, \quad (16)$$

where

$$v_1^2=\frac{1}{5}v^2(1+\frac{2}{3}b_0+\frac{1}{3}b_2)(1+b_1)(1+b_3)(1+\frac{2}{5}b_1+\frac{3}{5}b_3)^{-1} \quad (17)$$

denotes the squared velocity of longitudinal spin waves. Solving the system of equations (.2,16) one finds

$$X=\omega(\omega^2-u_1)^{-1}(\omega^2-u_2)^{-1}[a+\frac{1}{3}k^2v^2(1+b_1)(1+b_2)-\omega^2] \quad (18)$$

$$\lambda=\frac{1}{3}kv(1+b_1)(\omega^2-u_1)^{-1}(\omega^2-u_2)^{-1}[\omega^2-\frac{1}{5}k^2v^2(1+b_1)(1+b_2)(1+b_3)(1+\frac{2}{5}b_1+\frac{3}{5}b_3)^{-1}], \quad (19)$$

where

$$u_{1,2}=\frac{1}{2}(a+k^2v_0^2\pm \operatorname{sign} a\sqrt{Q}), \quad (20)$$

with

$$v_0^2\equiv v_1^2+v_2^2+v_3^2, \quad v_3^2\equiv\frac{2}{27}v^2(b_0-b_2)^2(1+b_1)(1+\frac{2}{3}b_0+\frac{1}{3}b_2)^{-1}, \quad (21)$$

and

$$Q=(a+k^2v_0^2)^2-4ak^2v_1^2>0. \quad (22)$$

It can be proved that $u_{1,2}$ are increasing functions of k^2, $u_2 \geqslant 0$ and, for $k^2 << |a|$, $u_2 \approx k^2v_1^2$. Moreover, the sign of u_1 coincides with the sign of a. Hence, we have here always the "acoustic" mode passing, for suitably small k, into the mode determined by us in I. If $a>0$ (i.e., if the effective interaction in the pairing channel is greater than the one in the channel ℓ =0), then also the "optical" mode appears. Since for $k^2 << |a|$ $u_1 \approx a + k^2(v_2^2+v_3^2)$ thus it corresponds to excitations with a gap $2\Delta(1+b_1)^{1/2}[\ln(\Delta/r)]^{1/2} << 2\Delta$. If $a<0$, then the "optical" mode posses into the diffusive one. Note that in all our previous formulae ω and kv are measured in units 2Δ. Hence one should substitute $\omega/2\Delta$ and $kv/2\Delta$ instead of ω and kv, respectively, in order to obtain the formulae without the above restriction. If $|\omega/2\Delta|$, $(kv/2\Delta) << |a|^{\frac{1}{2}}$ then X passes into his previous value, λ tends to zero, and we obtain our previous solution.

3. Let us pass to the tensor of the paramagnetic susceptibility. According to papers [4] and I, in the acoustic limit, it can be written as follows

$$\chi_{ab} = \mu_B^2 \nu \langle J_{ba} - \hat{p}_a J_{bc} \hat{p}_c + [\omega + (\vec{k}\vec{v})][R(\delta_{ab} - \hat{p}_a \hat{p}_b) + (X-R)(\hat{k}_a - \hat{p}_a w)\hat{k}_b + \lambda \hat{k}_b \hat{p}_a] \rangle_{\hat{p}}. \tag{23}$$

Expressing J_{ba} by R, X and λ, by means of formulae (8), (10) and (17-20,I), (25-30,I) after simple integration one finds

$$\chi_{ab} = \chi_{static}\Big[\delta_{ab} + (\delta_{ab} - \hat{k}_a \hat{k}_b)\omega R + \hat{k}_a \hat{k}_b \omega X + \frac{1}{2}\hat{k}_a \hat{k}_b \left(1 + \frac{4}{3} b_0 - \frac{1}{3} b_2\right) k v \lambda\Big]. \tag{24}$$

Note that if we put $4\Delta^2(1+b_1)\ln(\Delta/\tau)$ instead of a, then we obtain χ_{ab} for ω and kv not measured in units 2Δ.

It can be easily checked that, as previously,

$$\lim_{k\to0} X = \lim_{k\to0} R = -\frac{1}{\omega} \text{ for } \omega \neq 0 \text{ and } \lim_{k\to0}\lambda = 0$$

Hence, $\lim_{k\to0}\chi_{ab} = 0$ for $\omega \neq 0$ and this fact has very simple interpretation (cf.I). One can add that even the

appearance of the excitations with a gap much smaller than 2Δ does not lead to terms of order of ω^2/a in $\chi_{ab}(\vec{k}=0,\omega)$ which are here not out of our accuracy. It means that $\lim_{k\to0}\chi_{ab} \sim \omega^2/\Delta^2$, i.e., it is determined by the energy of a pair dissociation. On the other hand, $\lim_{\omega\to0} R = \lim_{\omega\to0} X = 0$ but $\lim_{\lambda\to0}\lambda = -kv(1+b_2)/12\Delta^2\ln(\Delta/\tau)(1+\frac{2}{3}b_0+\frac{1}{3}b_2)$.
Hence

$$\lim_{\omega\to0}\chi_{ab} = \chi_{static}\Big\{\delta_{ab} - \hat{k}_a \hat{k}_b (kv)^2\left(1+\frac{4}{3}b_0-\frac{1}{3}b_2\right)\left[24\Delta^2\ln(\Delta/\tau)\left(1+\frac{2}{3}b_0+\frac{1}{3}b_2\right)\right]^{-1}\Big\}. \tag{25}$$

Note that for normal Fermi liquids and for our previous χ_{ab} it was sufficient to make only one limiting transition, $\omega\to0$ in order to obtain $\chi_{static}\delta_{ab}$. Now it is necessary to perform $\lim_{k\to0}\lim_{\omega\to0}$. It means that if $|a| << 4\Delta^2$ then, in order to obtain the static response, it is unsufficient if the field varies weakly over the correlation length; the field has to vary weakly also over the length $\hbar v/|a|^{1/2}$.

The collective excitations with a gap for superfluid (superconducting) Fermi liquids were considered in the literature (cf.,e.g.,papers [6] and [3]). On the other hand, among them, the collective excitations with a gap much smaller than 2Δ were not considered. It should be noted that the appearance of such excitations is a feature of the Larkin-Migdal approach [1] if the ratio of the Legendre amplitudes of the effective interaction in the particle-particle channel for $l = l_0$ and $l = l' \neq l_0$ is close to unity. It is clear that for consideration of such excitations for superconductors it is necessary to introduce the long-range Coulomb part in the effective quasiparticle interaction.

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