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ON THE STATE OF A FERMI-SYSTEM WITH CORRELATION OF PAIRS OF PARTICLES WITH PARALLEL SPINS.

I.Theory of Ground State. II.Thermodynamics

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ON THE STATE OF A FERMI-SYSTEM WITH CORRELATION OF PAIRS OF PARTICLES WITH PARALLEL SPINS.

I.Theory of Ground State. II.Thermodynamics

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In 1972 the superfluidity of ^{3}He has been discovered by D.D.Osheroff, W.J.Gully, R.C.Richardson, and D.M.Lee (Phys. Rev.Lett., 28, 885 (1972)). From experimental and theoretical considerations (see recent review articles by A.J.Leggett, Rev.Modd.Phys., 47, 331 (1975) and J.C.Wheatley, ibid. 47, 415 (1975)) it follows that at the temperatures of the order of 1 mK liquid ^{3}He forms an anisotropic superfluid. For the first time the theory of an anisotropic Fermi superfluids has been described just in the Joint Institute for Nuclear Research (Dubna, 1959) in the preprint which is now reedited.

Later on an anisotropic Fermi superfluid, as a model of superfluid ^{3}He, has been considered by P.W.Anderson and P.Morel, Phys.Rev., 123, 1911 (1961), by P.W.Anderson and W.F.Brinkman, Phys.Rev.Lett., 30, 1108 (1973), R.Balian and N.R.Werthamer, Phys.Rev, 131, 1553 (1963), etc. (see rev. articles).

So, in the case of supefluid ^{3}He, it seems that it would be more proper to speak about the G-A-M state than, as it is used now, about the A-M state.

I. THEORY OF GROUND STATE *

1. Introduction

Consider a dynamical system of Fermi-particles with the Hamiltonian

$$H = \sum_{f,f'} T(f,f') a_f^+ a_{f'} + \frac{1}{2} \sum_{\substack{f_1, f_2 \\ f_1', f_2'}} U(f_1, f_2; f_2', f_1') a_{f_1}^+ a_{f_2}^+ a_{f_2'} a_{f_1'} ,$$

$$T(f,f') = I(f,f') - \lambda \delta(f-f'), \tag{1}$$

where I is the Hamiltonian of particle, U - the interaction energy, λ - the chemical potential, a_f , a_f^+ - the Fermi-amplitudes and f is a set of indices characterizing one-particle states. In our case $f=(\vec{p}, \sigma_z)$, where $\vec{p}$ is a wave vector and σ_z - a spin index.

Similarly as in /1/ (and also in /2,3/) we transform the Hamiltonian (1) by means of the transformation

$$a_f = \sum_\nu (u_{f\nu} \alpha_\nu + v_{f\nu} \alpha_\nu^+). \tag{2}$$

To secure the canonical character of the transformation (2) the functions $\{u, v\}$ must be connected by the orthonormality relations

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$$\sum_{\nu}\{u_{f\nu}u^*_{f'\nu}+v_{f\nu}v^*_{f'\nu}\}=\delta(f-f'),$$

$$\sum_{\nu}\{u_{f\nu}v_{f'\nu}+u_{f'\nu}v_{f\nu}\}=0. \qquad (3)$$

We find the functions $\{u,v\}$ from the additional equations obtained from the compensation principle of dangerous graphs /2/

$$\langle \alpha_{\nu_1}\alpha_{\nu_2}H\rangle_0=0 \qquad (4)$$

(the expectation value corresponds to the vacuum state in the α-representation) which are equivalent to the equations for the functions $F(f,f')$, $\Phi(f,f')$

$$\sum_f\{\zeta(f_1,f)\Phi(f,f_2)+\zeta(f_2,f)\Phi(f_1,f)+S(f_1,f_2)-$$

$$-\sum_f\{F(f,f_1)S(f,f_2)+F(f,f_2)S(f_1,f)\}=0, \qquad (5)$$

where

$$F(f,f')=\sum_{\nu}v^*_{f\nu}v_{f'\nu},\quad \Phi(f,f')=-\Phi(f',f)=\sum_{\nu}u_{f\nu}v_{f'\nu} \qquad (6)$$

and

$$S(f_1,f_2)=\sum_{f'_1,f'_2}U(f_1,f_2;f'_2,f'_1)\,\Phi(f'_1,f'_2), \qquad (7)$$

$$\zeta(f_1,f)=T(f_1,f)+\sum_{f',f''}\{U(f_1,f''_j f',f)-U(f_1,f''_j f,f')\}F(f'',f').$$

In order to represent the functions F,Φ in the form (6) these functions must satisfy the additional subsidiary conditions /2/

$$F(f_1,f_2)=\sum_f\{F(f_1,f)F(f,f_2)+\Phi(f,f_1)\Phi(f,f_2)\}, \qquad (8a)$$

$$\sum_f\{F(f_1,f)\Phi^*(f,f_2)+F(f_2,f)\Phi^*(f,f_1)\}=0. \qquad (8b)$$

From formula (1) of paper /2/ we knew that

$$T(f,f')=(E(p)-\lambda)\delta(f-f')=\epsilon(p)\delta(f-f'),$$

$$U(f_1,f_2;f'_2,f'_1)=\frac{1}{V}J(\vec p_1,\vec p_2;\vec p'_2,\vec p'_1)\delta(\vec p_1+\vec p_2-\vec p'_1-\vec p'_2)\times$$

$$\times\delta(\sigma_1-\sigma'_1)\,\delta(\sigma_2-\sigma'_2),$$

$$J(\vec p_1,\vec p_2;\vec p'_2,\vec p'_1)=J(\vec p_2,\vec p_1;\vec p'_1,\vec p'_2)=J(-\vec p_1,-\vec p_2;\ -\vec p'_2,-\vec p'_1). \qquad (9)$$

The solution of the equation (5) with the subsidiary conditions (8a,b) has been put in the form

$$F(f,f')=F(\vec p)\delta(f-f'),\quad \Phi(f,f')=\Phi(f)\delta(f+f'),$$

$$\Phi(\vec p,+)=-\Phi(\vec p),\quad \Phi(\vec p,-)=\Phi(\vec p). \qquad (10)$$

The functions $F(\vec p)$ and $\Phi(\vec p)$ are assumed to be real and invariant under the transformation of momentum reflection and to satisfy the following equations

$$2\zeta(p)\Phi(\vec p)+\frac{1-2F(\vec p)}{V}\sum_{\vec p'}J(\vec p,-\vec p;-\vec p',\vec p')\Phi(\vec p')=0,$$

$$F(\vec p)=F^2(\vec p)+\Phi^2(\vec p), \qquad (11)$$

where

$$\zeta(p)=\epsilon(p)+\frac{1}{V}\sum_{\vec p'}[2J(\vec p,\vec p';\vec p',\vec p)-J(\vec p,\vec p';\vec p,\vec p')]F(\vec p'). \qquad (12)$$

The solution of these equations leads to the usual formulae of the theory of superconductivity.

2. *The Solution of the Compensation Relation in the Case $\Phi(\vec{p}, \sigma_z)$ is an Odd Function of $\vec{p}$*

From the equations (11) we see that the solution of these equations exists also if $\Phi(\vec{p}, \sigma_z)$ is an odd function of $\vec{p}$. So, instead of (10) we put

$$F(f,f') = F(\vec{p})\delta(f-f'), \quad \Phi(f,f') = \Phi(f)\delta(\vec{p}+\vec{p}')\delta(\sigma-\sigma'),$$

$$F(\vec{p}) = \tilde{F}(-\vec{p}),\ \Phi(\vec{p},\sigma_z) = -\Phi(\vec{p},-\sigma_z)',\ \Phi(\vec{p},\sigma_z) = -\Phi(-\vec{p},\sigma_z).$$

The functions of the form (13)

$$\Phi(\vec{p},\sigma_z) = e_z\sigma_z \sum_{s_z} S_{\sigma_z}(s_z)\phi(\vec{p}) = \sigma_z \cos\theta\phi(\vec{p}),\ \phi(\vec{p}) = \phi(-\vec{p}) \tag{13'}$$

satisfy the conditions, where $(\vec{e} = \vec{p}/p)$, $S_{\sigma_z}(s_z)$ is a spin function ($S_{\sigma_z}(s_z) = 1$, $S_{\sigma_z}(s_z) = 0$ for $\sigma_z \neq s_z$, $\hat{\sigma}_z S = \sigma_z S$, $\sigma_z = \pm 1$). θ is the angle between the wave vector and the quantization axis of electron spins. Substituting (13) in (5) and (8a,b) we get for the functions $F(\vec{p})$, $\Phi(\vec{p},\sigma_z)$ the equations (11) (further we shall denote the spin index by σ). We define

$$C(\vec{p},\sigma) = -1/V \sum_{\vec{p}'} J(\vec{p}, -\vec{p}_j\ -\vec{p}', \vec{p}')\,\Phi(\vec{p}',\sigma). \tag{14}$$

We see that $C(\vec{p},\sigma)$ is an odd function of $\vec{p}$ and σ

$$C(\vec{p},\sigma) = -C(-\vec{p},\sigma),\ C(\vec{p},\sigma) = -C(\vec{p},-\sigma),\ C^2(\vec{p},\sigma) = C^2(\vec{p}). \tag{15}$$

From the equations (11) we obtain

$$\Phi(\vec{p},\sigma) = \frac{C(\vec{p},\sigma)}{2\sqrt{C^2(\vec{p})+\zeta^2(p)}},\quad F(\vec{p}) = \frac{1}{2}\Big\{1 - \frac{\zeta(p)}{\sqrt{C^2(\vec{p})+\zeta^2(p)}}\Big\} \tag{16}$$

hence

$$C(\vec{p},\sigma) = -\frac{1}{2V}\sum_{\vec{p}'} J(\vec{p}, -\vec{p}_j\ -\vec{p}', \vec{p}')\frac{C(\vec{p}',\sigma)}{\sqrt{C^2(\vec{p})+\zeta^2(p)}}. \tag{17}$$

We take

$$J(\vec{p}, -\vec{p}_j\ -\vec{p}', \vec{p}') = J(|\vec{p}-\vec{p}'|) = \sum_n J_n(p,p')P_n(\cos\gamma), \tag{18}$$

where γ is the angle between the vectors $\vec{p}$ and $\vec{p}'$

$$\cos\gamma = \cos\theta\cos\theta' + \cos\phi\sin\theta\sin\theta' \tag{19}$$

θ, θ' are the angles between the vectors $\vec{p}$ and $\vec{p}'$ and the polar axis, and ϕ is the angle round the polar axis. According to the addition theorem

$$P_n(\cos\gamma) = P_n(\cos\theta)\,P_n(\cos\theta') + 2\sum_{m/1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta)\times P_n^m(\cos\theta')\cos m\phi. \tag{20}$$

Substituting (20) into (17) and writing the result in the integral form we obtain

$$C(\vec{p},\sigma) = -\frac{1}{2}\frac{1}{(2\pi)^2}\sum_n P_n(\cos\theta)\int_0^\infty\int_0^\pi J_n(p,p')P_n(\cos\theta)\times \frac{C(p',\cos\theta',\sigma)p'^2dp'}{\sqrt{C^2(p',\cos\theta')+\zeta^2(p')}}\sin\theta' d\theta'. \tag{21}$$

We take the solution of (21) in the form

$$C(\vec{p},\sigma) = \sigma\cos\theta\Psi(p). \tag{22}$$

Moreover we consider the case when the interaction is effective only for the momentum p near p_F and take into account only $J_1(p,p') \simeq J_1(p_F)$ (the contribution from $J_0(p,p')$ vanishes). For the radial part of $C(\vec{p},\sigma)$ we obtain the equation

$$\Psi(p) = -\frac{1}{2}\frac{p_F^2}{(2\pi)^2} J_1 \int_{-1}^{1} x^2 dx \int_{p_F-\Delta}^{p_F+\Delta} \frac{\Psi(p')dp'}{\sqrt{\Psi^2(p')x^2 + \zeta^2(p')}} . \tag{23}$$

Looking for a solution which changes slowly with p near p_F and taking $\zeta(p) \cong E'(p-p_F)$ we get

$$\Psi = 2\omega e^{1/3} e^{3/\tilde{\rho}_1} = 2\omega e^{1/3} e^{-3/\rho_1} \tag{24}$$

where

$$-\tilde{\rho}_1 = -\frac{dn}{dE} J_1 = \rho_1 > 0, \quad \frac{dn}{dE} = \frac{p_F^2}{2\pi^2 E'}, \quad \omega = E'\Delta$$

(we have to replace the interaction of electrons with sound quanta by an effective electron-electron interaction, which is different from zero in a narrow layer near the Fermi surface $E_F \pm \omega$). In order that $\Psi \to 0$ for $\tilde{\rho}_1 \to 0$ the effective interaction must be attractive ($J_1(p_F) < 0$). Having obtained the solution of (23) we get the function $C(\vec{p},\sigma)$ and so the functions $F(\vec{p})$ and $\Phi(\vec{p},\sigma)$. Moreover, as can be seen, we obtain another solution of (11) by putting $\Phi(\vec{p},\sigma) = 0$ and $F(\vec{p}) = 1$ for $p < p_F$ and $F(\vec{p}) = 0$ for $p > p_F$.

We examine now the special form the transformation (2) takes when the functions F and Φ have the form (13). Taking into account (6) we see that

$$u_{f\nu} = u(\vec{p})\,\delta(f-\nu), \quad v_{f\nu} = v(-\vec{p},\sigma)\,\delta(\vec{p}+\vec{\ell})\,\delta(\sigma - s),$$

$$f = (\vec{p},\sigma), \; \nu = (\vec{\ell}, s),$$

$$v(\vec{p},\sigma) = -v(-\vec{p},\sigma), \; v(\vec{p},\sigma) = -v(\vec{p},-\sigma), \; v(\vec{p},\sigma) = \sigma\cos\theta\, w(\vec{p}),$$

$$w(\vec{p}) = w(-\vec{p}) \tag{25}$$

and

$$F(\vec{p}) = v^2(\vec{p},\sigma) = \cos^2\theta\,\omega^2(\vec{p}), \quad \Phi(\vec{p},\sigma) = u(\vec{p})\,v(\vec{p},\sigma). \tag{26}$$

Therefore the transformation (2) has now the form

$$a_{\vec{p}\sigma} = u(\vec{p})\,\alpha_{\vec{p}\sigma} - v(\vec{p},\sigma)\,\alpha^+_{-\vec{p}\sigma}, \; a_{-\vec{p}\sigma} = u(\vec{p})\,\alpha_{-\vec{p}\sigma} + v(\vec{p},\sigma)\,\alpha^+_{\vec{p},\sigma} . \tag{27}$$

If by means of (27) we transform the Hamiltonian

$$H = \sum_{\vec{p},\sigma} \epsilon(p)\, a^+_{\vec{p}\sigma} a_{\vec{p}\sigma} + \frac{1}{2V} \sum_{\substack{\vec{p}_1+\vec{p}_2=\vec{p}_1'+\vec{p}_2' \\ \sigma_1,\sigma_2}} J(\vec{p}_1,\vec{p}_2;\vec{p}_2',\vec{p}_1') \times$$

$$\times a^+_{\vec{p}_1\sigma_1} a^+_{\vec{p}_2\sigma_2} a_{\vec{p}_2'\sigma_2} a_{\vec{p}_1'\sigma_1} \tag{28}$$

(see (1) and (9)), we can see that we must write the compensation equation for the dangerous diagrams for the pairs of fermions with parallel spins and antiparallel momenta.

For the functions $u(\vec{p})$, $v(\vec{p},\sigma)$ we get the first solution for the normal state

$$u(\vec{p}) = \theta_G(\vec{p}) = \begin{cases} 1, & E(\vec{p}) > E_F \\ 0, & E(\vec{p}) < E_F \end{cases},$$

$$v(\vec{p},\sigma) = \theta_F(\vec{p},\sigma) = \begin{cases} 0, & E(\vec{p}) > E_F \\ \pm 1, & E(\vec{p}) < E_F \end{cases} \tag{29}$$

(where $\theta_F(\vec{p},\sigma) = -\theta_F(-\vec{p},\sigma), \dots$), and, taking into account (26) and (11), the second solution

$$u^2(\vec{p}) = \frac{1}{2}\left\{1 + \frac{\zeta(p)}{\sqrt{\Psi^2\cos^2\theta + \zeta^2(p)}}\right\}, \; v^2(\vec{p},\sigma) = \frac{1}{2}\left\{1 - \frac{\zeta(p)}{\sqrt{\Psi^2\cos^2\theta + \zeta^2(p)}}\right\}. \tag{30}$$

This solution is of the type obtained by Bogolubov /1/ but it depends on the direction of the vector $\vec{p}$ and for $\theta = \pi/2$ it goes over into the solution for the normal state.

3. *The Energy Difference for Two States. Energy Gap*

Now we find the mean energy in the vacuum state in α-representation and evaluate the difference for solutions (29) and (30). The general formula for mean value is

$$\bar{E} = \langle H \rangle_0 = \frac{1}{2}\sum_{f,f'} [T(f,f') + \zeta(f,f')] F(f,f') + \tag{31}$$

$$+ \frac{1}{2} \sum_{\substack{f_1,f_2 \\ f_1',f_2'}} U(f_1,f_2;f_2',f_1')\Phi^*(f_1,f_2)\Phi(f_1',f_2').$$

Taking into account (9) and (13) we obtain

$$\langle H \rangle_0 = \sum_{\vec{p}} [\epsilon(p) + \zeta(p)] F(\vec{p}) + \frac{1}{V} \sum_{\vec{p},\vec{p}'} J(\vec{p},-\vec{p};-\vec{p}',\vec{p}')\Phi(\vec{p},\sigma)\,\Phi(\vec{p}',\sigma) \cong$$

$$\cong 2\sum_{\vec{p}} \zeta(p) F(\vec{p}) - \sum_{\vec{p}} \frac{C^2(\vec{p})}{2\sqrt{C^2(\vec{p}) + \zeta^2(\vec{p})}}. \tag{32}$$

We find the energy difference for the two solutions with the help of the identity

$$v^2(\vec{p},\sigma) - \theta_F^2(\vec{p}) = \frac{\theta_G(\vec{p})}{2}\{1 - \frac{\zeta(p)}{\sqrt{\Psi^2\cos^2\theta + \zeta^2(p)}}\} -$$

$$- \frac{\theta_F^2(\vec{p})}{2}\{1 + \frac{\zeta(p)}{\sqrt{\Psi^2\cos^2\theta + \zeta^2(p)}}\}. \tag{33}$$

Finally we get

$$\frac{\Delta\bar{E}}{V} = \frac{2}{V}\sum_{\vec{p}} \zeta(p)[v^2(\vec{p},\sigma) - \theta_F^2(\vec{p})] - \frac{1}{2V}\sum_{\vec{p}} \frac{C^2(\vec{p})}{\sqrt{C^2(\vec{p}) + \zeta^2(\vec{p})}} =$$

$$= -2/3\,\frac{du}{dE}\, e^{2/3}\omega^2 (e^{-2/\rho_1})^3. \tag{34}$$

From this it is clear that the solution (30) leads to the energy state which lies lower than the normal state. This is also the anomalous state connected with the pairs of fermions with parallel spin-moments.

Let us compare the ground state for the case of pairs of fermions with antiparallel spins with the ground state for the case of pairs with parallel spins. We see that the binding energy for the pairs of electrons with parallel spin-moments is proportional to $[\exp(-2/\rho_1)]^3$ while the binding energy for the pairs of electrons with antiparallel spin-moments is proportional to $\exp(-2/\rho_0)$. When the case $J_1(p,p') > 3J_0(p,p')$ is not realized the ground state of the system for pairs of fermions with antiparallel spin-moments has a lower energy than the state for pairs with parallel spin-moments.

We shall now find the formula for the elementary excitation in the superconducting state. The general formula is given by [3]

$$\langle \alpha_\nu | H | \alpha_\nu^+ \rangle_0 = \sum_{f,f'} \zeta(f,f')(u_{f\nu}^* u_{f'\nu} - v_{f\nu}^* v_{f'\nu}) +$$

$$+ \frac{1}{2}\sum_{f_1,f_2} U(f_1,f_2;f_2',f_1')[\Phi(f_2,f_1)(u_{f_1'\nu}^* v_{f_2'\nu}^* -$$

$$- v_{f_1'\nu}^* u_{f_2'\nu}^*) + \Phi^*(f_2,f_1)(u_{f_1'\nu} v_{f_2'\nu} - u_{f_2'\nu} v_{f_1'\nu})]. \tag{35}$$

We remark, by the way, that (35) is identical with the formula for the energy of elementary excitations given by Landau [4]

$$\Omega(\nu) = \frac{\delta \bar{E}}{\delta n_\nu}. \tag{36}$$

Here $\bar{E}$ is the diagonal matrix element of the Hamiltonian (1) transformed by means of the transformation (2) [3] if the expectation value of $a^+_\nu a_\nu$ is not zero but n_ν. In this case the mean energy is given by the formula of the form (31) but the functions F and Φ are now of the form [2]

$$\Phi(f_1,f_2) = \sum_\nu \{ u_{f_1\nu} v_{f_2\nu} (1-n_\nu) + v_{f_1\nu} u_{f_2\nu} n_\nu \},$$

$$F(f_1,f_2) = \sum_\nu \{ v^*_{f_1\nu} v_{f_2\nu} (1-n_\nu) + u^*_{f_1\nu} u_{f_2\nu} n_\nu \}. \tag{37}$$

For $T=0$, that is $n_\nu = 0$, the formulae (37) pass over into (6). Taking also into account that in (31) F and Φ are functions of n_ν we get

$$\frac{\delta \bar{E}}{\delta n_\nu} = \frac{1}{2} \sum_{f,f'} \{ \frac{\delta \zeta(f,f')}{\delta n_\nu} F(f,f') + [T(f,f') + \zeta(f,f')] \frac{\delta F(f,f')}{\delta n_\nu} \} +$$

$$+ \frac{1}{2} \sum_{\substack{f_1,f_2 \\ f_1',f_2'}} U(f_1,f_2;f_2',f_1')[\frac{\delta \Phi^*(f_1,f_2)}{\delta n_\nu} \Phi(f_1',f_2') +$$

$$+ \Phi^*(f_1,f_2) \frac{\delta \Phi(f_1',f_2')}{\delta n_\nu}]. \tag{38}$$

Finding $\frac{\delta F}{\delta n_\nu}$, $\frac{\delta \Phi}{\delta n_\nu}$ from (37) we see that

$$\left(\frac{\delta \bar{E}}{\delta n_\nu}\right)_{n_\nu = 0} = \langle a_\nu | H | a^+_\nu \rangle_0 = \Omega(\nu). \tag{39}$$

Introducing (13) into (35) we obtain

$$\Omega(\vec{p}) = \zeta(p)[u^2(\vec{p}) - v^2(\vec{p},\sigma)] - \frac{2}{V} u(\vec{p}) v(\vec{p},\sigma) \times \tag{40}$$

$$\times \sum_{\vec{p}'} J(\vec{p},-\vec{p};-\vec{p}',\vec{p}') v(\vec{p}',\sigma) u(\vec{p}').$$

Using (11) and (26) we get

$$\Omega(\vec{p}) = \sqrt{\Psi^2 \cos^2\theta + \zeta^2(p)}. \tag{41}$$

On the Fermi-surface

$$\Omega(p,\cos\theta) = |\cos\theta| \, 2\omega e^{1/3} e^{-3/\rho} L = \Delta. \tag{42}$$

Thus, the fermion excitations in the anomalous state are separated from the energy of the ground state by the gap which depends on the direction of the vector $\vec{p}$. Since the energy gap can be equal to zero, there will not exist the current-carrying state of system state with respect to weak perturbations. This means that the anomalous state considered by us is not superconducting.

4. *The Influence of the Electron-Electron Coulomb Repulsion*

Let us now examine the combined effect of the electron-phonon interaction and the Coulomb interaction between the electrons [1,5]. Similarly as in the case of the electron-phonon interaction (which is essential only near the Fermi-surface, $E_{F\pm\omega}$) we replace the Coulomb interaction by a model interaction. Because of the screening of the Coulomb interaction, we replace it by a constant repulsion of electrons (essential only near the Fermi-surface $E_F \pm \omega_1$, $\omega_1 > \omega$). Thus we put

$$J_1(\zeta,\zeta') = \begin{cases} J_{1ph}(\zeta,\zeta') + J_{1c}(\zeta,\zeta'), & |\zeta|,|\zeta'| < \omega \\ J_{1c}(\zeta,\zeta'), & |\zeta| \text{ or } |\zeta'| > \omega. \end{cases} \tag{43}$$

We obtain the equations analogous to (23), from which we can determine $\Psi(\zeta)$ in two intervals ($|\zeta| < \omega$ and $\omega < |\zeta| < \omega_1$)

$$\Psi(\zeta) = -\frac{1}{4}\frac{dn}{dE}\int_{-1}^{1} x^2 dx \int_{-\omega_1}^{\omega_1} J_1(\zeta;\zeta')\frac{\Psi(\zeta')d\zeta'}{\sqrt{\Psi^2(\zeta')x^2+\zeta'^2}}. \quad (44)$$

We denote

$$\Psi(\zeta) = \begin{cases} \Psi_0, & |\zeta| < \omega \\ \Psi_1, & \omega < |\zeta| < \omega_1. \end{cases} \quad (45)$$

From (43) we get

$$-\Psi_0 = \frac{1}{3}[\rho_{1c}\ \Psi_1 \ln\frac{\omega_1}{\omega} + \Psi_0(\tilde{\rho}_{1ph} + \rho_{1c})\ln\frac{2\omega}{\Psi_0} + \frac{1}{3}\Psi_0(\tilde{\rho}_{1ph}+\rho_{1c})],$$

$$-\Psi_1 = \frac{1}{3}[\rho_{1c}\ \Psi_1 \ln\frac{\omega_1}{\omega} + \Psi_0\rho_{1c}\ln\frac{2\omega}{\Psi_0} + \frac{1}{3}\Psi_0\rho_{1c}], \quad (46)$$

where

$$-\tilde{\rho}_{1ph} = -J_{1ph}\frac{dn}{dE} = \rho_{1ph} > 0, \qquad \rho_{1c} = J_{1c}\frac{dn}{dE} > 0.$$

Hence

$$\frac{1}{3}(\ln\frac{2\omega}{\Psi_0} + \frac{1}{3})(\rho_{1ph} - \frac{\rho_{1c}}{1+\frac{1}{3}\rho_{1c}\ln\frac{\omega_1}{\omega}}) = 1. \quad (47)$$

Thus, the condition of appearing of an anomalous state has an analogous form as in paper /1/ for the case of pairs of fermions with antiparallel spin-moments

$$\rho_{1ph} > \frac{\rho_{1c}}{1+\frac{1}{3}\rho_{1c}\ln\frac{\omega_1}{\omega}}. \quad (48)$$

5. Total Spin of a System, Paramagnetic Susceptibility

The spin moment for the unit volume is given by the formula

$$\tilde{S}^z = \frac{1}{V}\sum_{\vec{p}}(a^+_{\vec{p}+}a_{\vec{p}+} - a^+_{\vec{p}-}a_{\vec{p}-}). \quad (49)$$

We pass over the operators α and find (49) for the state C_0 (vacuum state in the α-representation)

$$S^z = \frac{1}{V}\sum_{\vec{p}}\{F(\vec{p},+) - F(\vec{p},-)\}. \quad (50)$$

In the absence of the magnetic field $F(\vec{p},+) = F(\vec{p},-)$ and therefore $S^z = 0$.

Consider now a system of Fermi-particles in the presence of external, weak, constant magnetic field $\delta\mathcal{H}$, applied along the z-axis.

In this case the term is added to the Hamiltonian (28), namely

$$-\frac{e}{m}\delta\mathcal{H}\sum_{\vec{p}}(a^+_{\vec{p}+}a_{\vec{p}+} - a^+_{\vec{p}-}a_{\vec{p}-}), \quad (51)$$

where $+e$ is the charge of the electron and m is its mass.

The compensation equations lead to the formulae

$$F_{\delta\mathcal{H}}(\vec{p},+) = \frac{1}{2}[1 - \frac{\zeta(p) - \frac{e}{m}\delta\mathcal{H}}{\sqrt{[\zeta(p) - \frac{e}{m}\delta\mathcal{H}]^2 + C^2(\vec{p})}}],$$

$$F_{\delta\mathcal{H}}(\vec{p},-) = \frac{1}{2}[1 - \frac{\zeta(p) + \frac{e}{m}\delta\mathcal{H}}{\sqrt{[\zeta(p) + \frac{e}{m}\delta\mathcal{H}]^2 + C^2(\vec{p})}}]$$

$$C(\vec{p},+) = C(\vec{p},-) \cong C(\vec{p}) \quad (52)$$

Hence

$$S^z_{\delta\mathcal{H}} = \frac{1}{2}\frac{dn}{dE}\int_{-\omega}^{\omega}\int_{-1}^{1}\frac{\zeta + \frac{e}{m}\delta\mathcal{H}}{\sqrt{(\zeta+\frac{e}{m}\delta\mathcal{H})^2 + \Psi^2 x^2}}\, d\zeta\, dx\,. \tag{53}$$

On performing the integration, we get

$$S^z_{\delta\mathcal{H}} = \frac{dn}{dE}\,\frac{e}{m}\,\delta\mathcal{H}\,[1-\exp(2/3-6/\rho_1)]. \tag{54}$$

For the paramagnetic susceptibility we get

$$\chi = \frac{2e^2}{m^2}\frac{dn}{dE}\,[1-\exp(2/3-6/\rho_1)]. \tag{55}$$

From (54) we see that in the absence of the magnetic field the total spin of a system equals zero. In the presence of the magnetic field the electron-electron interaction cannot be taken into account by means of perturbation theory since we have obtained the non-analytical dependence of (54) and (55) for ρ_1 near $\rho_1 = 0$.

In this paper we have considered formally the solution of the compensation equations as an odd function of $\vec{p}$. It is provided that from the point of view of the transformation (2) this leads to the state of the system with pairs of fermions with parallel spins. In order to explain the physical picture of this state one has to examine the thermodynamics of this anomalous state.

II. THERMODYNAMICS *

In paper /6/ (referred to as part I) the possibility of an "anomalous" (non-superconducting) state of Fermi-system is studied. This state is connected with creation of pairs with parallel spins. The creation occurs if the electron-electron interaction is attractive; we find that

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in the expansion of the interaction term in spherical harmonics only the coefficients with odd indices give a contribution.

If in the expansion of the interaction in spherical harmonics only terms with even indices J_{2n} are retained (for instance J_0 in paper /7/), then the compensation equation for the pairs with antiparallel spins has two solutions: a trivial one and a nontrivial one (giving the superconducting state). However the compensation equation for the pairs with parallel spins has the trivial solutions only.

If in the expansion of the interaction in spherical harmonics only terms with odd indices J_{2n+1} are retained (paper /6/ $- J_1$), then the compensation equation for the pairs of particles with antiparallel spins has the trivial solution only. On the other hand, the compensation equation for pairs with parallel spins has two solutions: a trivial one and a nontrivial one (giving the "anomalous" state).

The solutions of the compensation equation and of the secular equation for the collective oscillations are in the case J_{2n} even functions of $\vec{p}$ /1,2/ (the superconducting state, longitudinal oscillations) in the case J_{2n+1} odd functions of $\vec{p}$ /6,3/ (the "anomalous" state, the oscillations of the type of the transversal oscillations).

In papers /2,3/ it is proved that for the general form of the interaction term the collective oscillations split into two branches: for the pairs of particles with antiparallel spins and for the pairs of particles with parallel spins. The terms of the type J_{2n} give contribution to the oscillations of the pairs with antiparallel spins only (the solution of the secular equation, the even function of $\vec{p}$, therefore, the longitudinal oscillations), the terms of the type J_{2n+1} give a contribution to the oscillations of the pairs with parallel spins (the solution of the secular equations, the odd functions of $\vec{p}$, therefore, the oscillations of the type of the transversal oscillations). In paper /6/ it is shown that if $3J_1 > J_0$ the energy of the "anomalous" state is lower than the energy of superconducting state and certain properties of this state are studied (for $T=0$).

Now we want to examine the properties of this state for temperature $T \neq 0$.

1. The Compensation Equation

The compensation equation has the form [6]

$$2\zeta(p)\Phi(\vec{p},\sigma) - \frac{1-2F(\vec{p})}{V} C(\vec{p},\sigma)=0, \tag{1}$$

where

$$C(\vec{p},\sigma) = -\frac{1}{V}\sum_{\vec{p}'} J(\vec{p},-\vec{p};-\vec{p}',\vec{p}')\Phi(\vec{p}',\sigma) \tag{2}$$

and

$$\Phi(\vec{p},\sigma)=u(\vec{p})v(\vec{p},\sigma)(1-2n_{\vec{p}\sigma}), \tag{3}$$

$$F(\vec{p}) = v^2(\vec{p},\sigma)(1-n_{\vec{p}\sigma})+u^2(\vec{p})n_{\vec{p}\sigma},$$

$$\zeta(p)=\epsilon(p)+\frac{1}{V}\sum_{\vec{p}'}[2J(\vec{p},\vec{p};\vec{p}',\vec{p})-J(\vec{p},\vec{p}';\vec{p},\vec{p}')] \tag{3a}$$

since the expectation value of $a^+_{\vec{p}\sigma}a_{\vec{p}\sigma}$ is not zero but $n_{\vec{p}\sigma}$ [2]. The functions $\{u,v\}$ are the coefficients of the Bogolubov transformation for transition from the Fermi-operators $a_{\vec{p}\sigma}$, $a^+_{\vec{p}\sigma}$ to $\alpha_{\vec{p}\sigma}$, $\alpha^+_{\vec{p}\sigma}$ these $\{u,v\}$ satisfy the condition

$$u^2(\vec{p})+v^2(\vec{p},\sigma)=1. \tag{4}$$

Similarly as in paper [6] we put

$$J(\vec{p},-\vec{p};-\vec{p}',\vec{p}')=\sum_n J_n(p,p')P_n(\cos\gamma) \cong J_1(p,p')\cos\gamma =$$

$$= J_1(p_F)(\cos\theta\cos\theta'+\cos\phi\sin\theta\sin\theta'),$$

$$C(\vec{p},\sigma)=\sigma\cos\theta\Psi(p), \tag{5}$$

where θ, θ' are the angles between the vectors $\vec{p}$ and $\vec{p}'$ and the polar axis, and ϕ is the angle round this axis.

Equation (2) after taking into account (1), (3), (4) can be written in the integral form

$$1 = -\frac{1}{2}\frac{p_F^2}{(2\pi)^2}J_1\int_{p_F-\Delta}^{p_F+\Delta}\int_0^{\pi}\frac{\cos^2\theta'\sin\theta' d\theta' dp'}{\Omega(p',\cos\theta')}[1-2n(p',\cos\theta')], \tag{6}$$

where

$$n_{\vec{p}\sigma}=n_{\vec{p}}=\frac{1}{e^{\Omega(\vec{p})/T}+1}, \quad \Omega(\vec{p})=\sqrt{\Psi^2\cos^2\theta+\zeta^2(p)}. \tag{7}$$

Hence we get finally for Ψ as the function of temperature the equation

$$1=\rho_1\int_0^1 x^2dx\int\frac{\mathrm{th}\frac{\sqrt{\Psi^2(T)x^2+\zeta^2}}{2T}}{\sqrt{\Psi^2(T)x^2+\zeta^2}}d\zeta=$$

$$=\rho_1\int_0^1 x^2dx\int_{\Psi x}^{\omega}\frac{\mathrm{th}\frac{\Omega}{2T}}{\sqrt{\Omega^2-\Psi^2x^2}}d\Omega, \tag{8}$$

where

$$\rho_1=-\frac{dn}{dE}J_1>0, \quad \frac{dn}{dE}=\frac{p_F^2}{2\pi E'}, \tag{9}$$

this means we assume the interaction to be attractive.

2. *The Dependence of* Ψ *on the Temperature and the Transition Temperature to the "Anomalous" State*

After a change of variables $\Omega=\Psi x \operatorname{ch}\phi$ we get similarly as in /5/

$$\ln\frac{\Psi(0)}{\Psi(T)}=6\int_0^1 x^2\,dx\int_0^\infty\frac{d\phi}{e^{\Psi(T)x\operatorname{ch}\phi/T}+1}=6\int_0^1 x^2\,dx\sum_{m/1}^\infty(-1)^{m+1}K_0\left(\frac{\Psi x m}{T}\right). \tag{10}$$

Hence for $T\sim 0$, making use of the asymptotic formulae for Bessel-functions with great argument, we obtain

$$\ln\frac{\Psi(0)}{\Psi(T)}\cong\sqrt{\frac{18\pi T}{\Psi(0)}}\int_0^1 x^{1/2}e^{x\Psi(0)/T}dx=$$

$$=9\sqrt{\pi/2}\,\Phi\left(\sqrt{\frac{\Psi(0)}{T}}\right)\left(\frac{T}{\Psi(0)}\right)^3\cong 10\left(\frac{T}{\Psi(0)}\right)^3, \tag{11}$$

$$\Psi(T)=\Psi(0)\,e^{-10(T/\Psi(0))^3}, \tag{11a}$$

where $\Phi(z)$ is the error function.

Now we want to find the critical temperature T_c (for $T>T_c$ must be $\Psi(T)=0$). Moreover we want to get $\Psi(T)$ for $T\sim T_c$. For this purpose we make use of a suitable representation of Bessel function $K_0(z)$ /8,5/. We obtain

$$\ln\frac{\Psi(0)}{\Psi(T)}=\ln\frac{e^{1/3}\pi}{\gamma}\,\frac{T}{\Psi(T)}+3\int_0^1 x^2dx\left\{2\pi\sum_{\ell/1}^\infty\left[\frac{1}{(2\ell-1)\pi}-\frac{1}{\sqrt{\Psi^2x^2/T^2+(2\ell-1)^2\pi^2}}\right]\right\}, \tag{12}$$

where $\ln\gamma=0{,}577$, $\gamma=1{,}8$.

Putting in (12) $\Psi=0$, we find

$$\Psi(0)=2\omega\,e^{1/3}\left(e^{-1/\rho_1}\right)^3=e^{1/3}\frac{\pi T_c}{\gamma}. \tag{13}$$

Hence

$$\frac{\Psi(0)}{T_c}=2{,}45. \tag{13a}$$

For $T\sim T_c$

$$\ln\frac{T}{T_c}=\frac{1}{2}\frac{\zeta(3)}{\pi^2}\left(\frac{\Psi(T)}{T}\right)^2+\frac{5}{16}\frac{\zeta(5)}{\pi^4}\left(\frac{\Psi(T)}{T}\right)^4, \tag{14}$$

where $\zeta(n)$ is the zeta function.

Therefore for $T\sim T_c$

$$\frac{\Psi(T)}{T_c}=4{,}1\sqrt{\frac{T_c-T}{T_c}}. \tag{15}$$

From (13a) and (15) it results that the function $\Psi(T)/T_c$ decreases from 2.45 to 0 in the temperature interval $(0,T_c)$.

3. *Entropy and Specific Heat*

Now we want to obtain the formulae for the temperature dependence of the specific heat, especially for $T\sim T_c$. Therefore we must first get the formula for the entropy.

We start from the expression

$$S=-2\sum_{\vec p}\left[n_{\vec p}\ln n_{\vec p}+(1-n_{\vec p})\ln(1-n_{\vec p})\right]. \tag{16}$$

Inserting (7) into (16) and passing to the integral form we obtain

$$s=\frac{S}{V}=-4\frac{dn}{dE}\frac{1}{T}\int_0^1 dx\int_{\Psi x}^\infty\sqrt{\Omega^2-\Psi^2x^2}\,\Omega\frac{dn}{d\Omega}\,d\Omega. \tag{17}$$

After changing the variables similarly as for (10) we have

$$s=4\frac{dn}{dE}\frac{\Psi^2}{T}\int_0^1 x^2dx\sum_{m/1}^\infty(-1)^{m+1}K_2\left(\frac{\Psi x m}{T}\right). \tag{18}$$

Hence for $T \sim 0$, making use for the asymptotic formulae for Bessel-functions with great arguments, we get

$$s = 3\sqrt{2\pi}\,\frac{dn}{dE}\,\Phi\left(\sqrt{\frac{\Psi(0)}{T}}\right)\Psi(0)\left(\frac{T}{\Psi(0)}\right)^2 \cong 6{,}65\,\Psi(0)\frac{dn}{dE}\left(\frac{T}{\Psi(0)}\right)^2. \tag{19}$$

Now we want to find $s(T)$ for $T \sim T_c$. Since for $T \to T_c$, $\Psi(T) \to 0$ we write (17) in the form

$$s(\Psi) = s(0) + \frac{s'(0)}{1!}\Psi + \frac{s''(0)}{2!}\Psi^2 + \frac{s'''(0)}{3!}\Psi^3 + \frac{s''''(0)}{4!}\Psi^4 =$$

$$= 2/3\,\pi^2\frac{dn}{dE}\,T\left[1 - \frac{1}{2\pi^2}\left(\frac{\Psi}{T}\right)^2 + \frac{21}{80}\,\frac{\zeta(3)}{\pi^4}\left(\frac{\Psi}{T}\right)^4\right]. \tag{20}$$

Having found $s(T)$, we can obtain the temperature dependence of specific heat $C(T)$ for $T \sim 0$ and $T \sim T_c$. We denote the specific heat in the normal state for $T = T_c$

$$C_n(T_c) = 2/3\,\pi^2\frac{dn}{dE}T_c\,.$$

For temperature heat $T = 0$ we have

$$\frac{C_a(T)}{C_n(T_c)} = \frac{9e^{1/3}}{\gamma\sqrt{2}}\left(\frac{T}{\Psi(0)}\right)^2 = 4{,}95\left(\frac{T}{\Psi(0)}\right)^2, \tag{21}$$

however, for temperatures $T \sim T_c$ ($T < T_c$) we have

$$\frac{C_a(T)}{C_n(T_c)} = 1{,}83 + 2{,}5\,\frac{T - T_c}{T_c}. \tag{22}$$

From (22) we see that for $T = T_c$ the specific heat has a jump, since $C_a(T_c)/C_n(T_c) = 1.83$.

4. *The Total Momentum of Elementary Excitations*

Let us consider the total momentum $\vec{P}$ of elementary excitations in the case of macroscopic motion with constant velocity

$$\vec{P} = \sum_{\vec{k}} \vec{k}\,(n_{\vec{k},1/2} + n_{\vec{k},-1/2}). \tag{23}$$

Now $n = n(\Omega - \vec{k}\vec{u})$. For small velocities

$$\vec{P} = -2\sum \vec{k}\,(\vec{k}\vec{u})\frac{\partial n}{\partial\Omega(\vec{k})}. \tag{24}$$

In the integral form we have finally

$$p = \frac{\vec{P}}{V} = -p_F^2\frac{dn}{dE}\int_0^1 [\,\vec{u}_\perp(1-x^2) + \vec{u}_{||}2x^2\,]\,dx\int_{-\infty}^{+\infty}\frac{\partial n}{\partial\Omega(x,\zeta)}\,d\zeta, \tag{25}$$

where $\vec{u}_\perp$ is the component of vector $\vec{u}$ perpendicular to the quantization axis of electron spin and $\vec{u}_{||}$ the parallel component of $\vec{u}$. We see from (25) that the vectors $\vec{p}$ and $\vec{u}$ are not parallel and we cannot write the dependence $\vec{p} = N_n m\vec{u}$ and define the number N_n of normal electrons in "anomalous" state.

Consider $\vec{p}$ for $T \sim 0$ and for $T \sim T_c$. After changing variables $\zeta = \Psi\, x\, \mathrm{sh}\phi$ we get

$$\vec{p} = p_F^2\frac{dn}{dE}\,\frac{2\Psi}{T}\int_0^1[\vec{u}_\perp(1-x^2) + \vec{u}_{||}\,2x^2]\,x\,dx \tag{26}$$

$$\times\int\frac{\mathrm{ch}\phi\,d\phi}{(e^{\Psi x\,\mathrm{ch}\phi/T}+1)(e^{-\Psi x\,\mathrm{ch}\phi/T}+1)}.$$

Hence for $T \sim 0$

$$\vec{p} = \vec{p}_\perp + \vec{p}_{||} \cong \vec{p}_\perp, \tag{27}$$

where

$$\vec{p}_{\perp}=p_F^2\frac{dn}{dE}\sqrt{\pi}\left(\frac{T}{\Psi(0)}\right)^{1/2}\vec{u}_{\perp}=N\frac{3}{2}\sqrt{\pi}\left(\frac{T}{\Psi(0)}\right)^{1/2}m\,\vec{u}_{\perp}\,,$$

$$\vec{p}_{||}=3/4\,p_F^2\frac{dn}{dE}\sqrt{\pi}\left(\frac{T}{\Psi(0)}\right)^{3/2}\vec{u}_{||}=N\,9/2\sqrt{\pi}\left(\frac{T}{\Psi(0)}\right)^{3/2}m\,\vec{u}_{||}\,, \tag{28}$$

m is the mass of the electron, N is the number of electrons in unit volume,

$$N=2/3\,\frac{p_F^2}{m}\,\frac{dn}{dE}=\frac{8\pi p_F^2}{3(2\pi)^3} \tag{29}$$

since in formula (9) $E'\sim p_F/m$.

From (28) we see that the perpendicular component of $\vec{p}$ dominates for $T\sim 0$, it is by three orders of magnitude smaller than the parallel component.

In order to obtain (26) for $T\sim T_c$ we must use the following identities

$$\frac{\Psi}{T}\int_0^1 x^3dx\int_0^\infty\frac{ch\phi\,d\phi}{(e^{\Psi x\,ch\phi/T}+1)(e^{-\Psi x\,ch\phi/T}+1)}=\frac{1}{6}\,\frac{\frac{1}{T}\frac{\partial\Psi}{\partial T}}{\frac{\partial}{\partial T}\left(\frac{\Psi}{T}\right)} \tag{29'}$$

and

$$A=\int_0^1 x\,dx\int_0^\infty\frac{ch\phi\,d\phi}{(e^{\Psi x\,ch\phi/T}+1)(e^{-\Psi x\,ch\phi/T}+1)}=-\frac{\frac{\partial f(T)}{\partial T}}{2\frac{\partial}{\partial T}\left(\frac{\Psi}{T}\right)}, \tag{30}$$

where $f(T)$

$$f(T)=2\int_0^1 dx\int_0^\infty\frac{d\phi}{e^{\Psi x\,ch\phi}+1}\,. \tag{31}$$

For $T\sim T_c$

$$f(T)=\ln\left(\frac{e\pi}{\gamma}\,\frac{T}{\Psi}\right)-\frac{7}{24}\,\frac{\zeta(3)}{\pi}\left(\frac{\Psi}{T}\right)^2+\frac{93}{640}\,\frac{\zeta(5)}{\pi}\left(\frac{\Psi}{T}\right)^4. \tag{32}$$

Hence A for $T\sim T_c$

$$A(T)=\frac{1}{2}\,\frac{T}{\Psi}+\frac{7}{24}\,\frac{\zeta(3)}{\pi^2}\left(\frac{\Psi}{T}\right)-\frac{93}{320}\,\frac{\zeta(5)}{\pi^4}\left(\frac{\Psi}{T}\right)^3. \tag{33}$$

Finally $\vec{p}$ for $T\sim T_c$ $(T<T_c)$

$$\vec{p}=Nm\left[\vec{u}+1/4\,(11\vec{u}_{\perp}-8\vec{u}_{||})\,\frac{T_c-T}{T_c}\right]. \tag{34}$$

We see that even for $T\sim T_c$ the parallel component of $\vec{p}$ is smaller than the perpendicular one.

5. *Paramagnetic Susceptibility*

From paper /6/ we have for the paramagnetic susceptibility χ

$$\chi(T)=\frac{2e^2}{m^2}\,\frac{dn}{dE}\left[1-\left(\frac{\Psi(T)}{2\omega}\right)^2\right], \tag{35}$$

where $\Psi(T)$ we get from (10).

For the temperature $T\sim 0$ we obtain from (11a)

$$\chi(T)=\frac{2e^2}{m^2}\,\frac{dn}{dE}\left[1-1{,}96\,e^{-6/\rho_1}e^{-20(T/\Psi(0))^3}\right]. \tag{36}$$

For the temperatures $T\sim T_c$ we get from (15)

$$\chi(T)=\frac{2e^2}{m^2}\,\frac{dn}{dE}\left[1-3{,}25\,e^{-6/\rho_1}\sqrt{\frac{T_c-T}{T_c}}\right]. \tag{37}$$

With the elementary excitations considered here (type J_{2n+1}) we could explain the Reif experiment /9/ which gives the dependence of paramagnetic susceptibility, if we put $\rho_1\sim 3{,}5$.

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References

1. N.N.Bogolubov, V.V.Tolmachev, D.V.Shirkov. A New Method in the Theory of Superconductivity, Izdat. Akad. Nauk SSSR (1958); English copy in Forts. d. Phys., 6, (1958), 605, English edition - Consultants Bureau, INC. New York (1959).
2. N.N.Bogolubov. Uspekhi Fiz.Nauk., 67, 549 (1959).
3. Z.Galasiewicz. Prog. Theor.Phys., 23, 197 (1960).
4. L.D.Landau. Zh.Eksper.Theor. Fiz., 30, 1058 (1956).
6. I.M.Khalatnikov, A.A.Abrikosov. Uspekhi Fiz. Nauk, 65, 551 (1958). English copy in Advices in Phys., 8, 45 (1959).
6. Z.Galasiewicz. Acta Phys.Polon., 19, 467 (1960).
7. J.Bardeen, L.Cooper, J.Schrieffer. Phys.Rev., 108, 1175 (1957).
8. I.M.Ryzik, I.S.Gradstein. Tables of Integrals, Sums and Series. Izdat. Techn. Literat., Moscow, 1951.
9. F.Reif. Phys.Rev., 106, 208 (1957).

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