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THE SPIN SUSCEPTIBILITY OF THE PSEUDOISOTROPIC (B) PHASE OF SUPERFLUID He^3 IN THE ACOUSTIC LIMIT (SPIN WAVES)

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Summary

The tensor of the spin susceptibility is calculated using the theory of the Larkin-Migdal type, at zero temperature, in the acoustic limit and collisionless regime. The theory of such a type for Balian-Werthamer pairing has been developed a few years ago by the present author. Any restrictions on the effective quasiparticle interaction in the particle-hole channel are not imposed. It is checked that the spin susceptibility has always two poles, corresponding to spin waves of different polarizations. Their frequencies have been computed by Vdovin and Combescot, but under restricting assumptions. It is checked that these frequencies are dependent on exchange Landau amplitudes for $0 \leqslant \ell \leqslant 3$ and the form of dependence is found. At zero frequency and nonzero wave vector of the external magnetic field the susceptibility pass into the static value.

The purpose of the present paper is to consider the spin waves for the system with Balian-Werthamer (BW) [1] pairing, without any assumption imposed on the effective quasiparticle interaction in the particle-hole channel, but in the collisionless limit and at zero temperature. This topic becomes now interesting, because of the identification of the B-phase of superfluid ^{3}He [2,3] as the BW state, performed preliminarily by Anderson and Brinkman [4]. This identification has been a matter of controversy, but the proposal of spin-singlet D-pairing [5] is inconsistent with recent measurements of the spin susceptibility [6,7] whereas the spin-triplet F-pairing [8] met serious arguments against it in paper [9]. So, the BW state [1] remains till now the best candidate to describe the B-phase of superfluid ^{3}He.

The collective excitations of the BW state were considered firstly (1963) by Vdovin [10], for a weak interaction. This last restriction was not imposed by us in papers [11,12]. There we have developed, for BW pairing, analog of Larkin-Migdal [13] and Larkin himself [14] theories, which were developed previously for systems with isotropic S-pairing. We have solved [11], the equations for vertex functions describing the scalar and vector vertices in the acoustic limit $(|\omega|, kv \ll \Delta$, where ω, k denote the frequency and the wave vector respectively, v is the quasiparticle velocity on the Fermi sphere and Δ the energy gap). It was shown there that all the above vertex functions have one pole at the zero sound frequency, coinciding with the first sound frequenoy, obtained by means of the thermodynamic formula for Fermi liquids[15]. Our results were generalized recently by Maki[16], for nonzero temperatures. On the other hand, our paper[11] oontains erroneous statement that the spin vertex, in the aooustic limit has no pole, i.e., that the propagation of spin waves is impossible for systems with BW pairing.

The results of Vdovin[10] for spin waves were generalized by Combesoot[17], for a system with the angle independent effective quasiparticle interaotion. Moreover, our equations[11] for vertex functions were rederived recently by Gongadze, Gurgenishvili and Kharadze[18] under the silent assumption that the spin-antisymmetric part of the effective quasipartiole interaction in the particle-particle (or hole-hole) ohannel vanishes. These authors have solved the equation for the spin vertex assuming, as Combescot[17], that the effective quasipartiole interaction, in the partiole-hole ohannel, is angle-independent. Their expressions for the frequenoies of spin waves coinoide with those of Combesoot[17] but, nevertheless, the solutions the vertex function obtained in[18] are incorreot. It will become olear for the reader from our further caloulations, but the incorreotness of the results[18] oan be understood quite simply. The equations solved in[18] form the system of inhomogeneous linear integral equations with the degenerate matrix kernel. Such a system is equivalent to the algebraio system of inhomogeneous linear equations. Nevertheless, aoording to the

results of[18], the solution of such a system, describing the response to the external magnetic field is not unique, which is a physical nonsense.

Aooording to Leggett and Rice[19], Leggett[20] and Corruoini et al.[21] the spin exohange Landau amplitude for $\ell=1$ is very small. This is the most physioal argument in favour of disregarding all exohange Landau amplitudes, exoept for $\ell=0$. Nevertheless, both experimental[6] and theoretical estimations by Ostgaard[22] show that the situation is far from the above one. Moreover, the Landau amplitudes for ^{3}He have to be such that the sum rule is fulfilled. On the other hand, only the general solution of the problem gives us the possibility to verify whether the stability conditions[23] guarantee the existenoe of poles of the response funotion (i.e., suitable elementary exoitations). This is a topic of a particular interest since spin waves in the B-phase of superfluid ^{3}He remain still undetected.

1. The discussion of basic equations

Our equations for spin vertices as well as the expression for the spin susoeptibility obtained by us in[11], by the application of methods developed in[13], will not be rederived here. The deduction of equivalent equations for spin vertices and the expression for the spin susoeptibility, by means of methods developed in[14] can be found in Appendix. Choose the usual phase of the Δ- matrix of BW, i.e., $\hat{\Delta}=(\vec{\sigma}\hat{\vec{p}})(i\sigma^y)\Delta$. Here a circumflex over a letter denotes a spin matrix, with the exception of letters with arrows on, where it denotes a unit veotor parallel to the vector under a oiroumflex. For the above ohoice of $\hat{\Delta}$, one oan express the anomalous vertioes $\hat{\tau}_{1,2}$

(i.e., with two incoming and outgoing particle lines, respectively), conjugate to normal spin vertex, as follows:

$\hat{\tau}_1 = (\hat{\tau} + \lambda)(i\sigma^y)$, $\hat{\tau}_2 = (i\sigma^y)(\hat{\tau} - \lambda)$, where $\hat{\tau}$ is a traceless matrix. The term λ is equal to zero if the spin-antisymmetric part of the effective quasiparticle interaction in the particle-particle channel vanishes [11] . On the other hand, if zeroth Legendre amplitude of the spin-antisymmetric dimensionless effective interaction in the particle-particle channel is not equal to $[\ln(2\xi/\Delta)]^{-1}$, where ξ is the cut-off parameter, then it can be proved that disregarding the λ term we obtain an error negligibly small in the acoustic limit. (Cf. papers [11] and [13]). According to important but till now nonformulated principle of theoretical physics any parameter is not equal to anything else if it does not have to. Hence, in the acoustic limit, the variable λ can be put zero with simultaneous omitting the equation for λ . The remaining equations, with λ put equal zero, have the form

$$\hat{J}_a(\hat{p}) = \hat{J}_a^{\omega} + \langle B(\hat{p}\hat{p}')\{L\hat{J}_a(\hat{p}') + O(\vec{\sigma}\hat{p}')\hat{J}_a(-\hat{p}')(\vec{\sigma}\hat{p}') - M[\hat{\tau}_a(\hat{p}'),(\vec{\sigma}\hat{p}')]_-\}\rangle_{\hat{p}'}, \quad (1)$$

$$\hat{\tau}_a(\hat{p}) = \langle f_{-1}^{\xi}(\hat{p}\hat{p}')\{[N + \ln(2\xi/\Delta)]\hat{\tau}_a(\hat{p}') - O(\vec{\sigma}\hat{p}')\hat{\tau}_a(\hat{p}')(\vec{\sigma}\hat{p}') + M[\hat{J}_a(\hat{p}'),(\vec{\sigma}\hat{p}')]_-\}\rangle_{\hat{p}'}. \quad (2)$$

Here $B(\hat{p}\hat{p}')$ denotes spin-exchange part of the dimensionless effective interaction in the particle-hole channel *, $f_{-1}^{\xi}(\hat{p}\hat{p}')$ -

*) It coincides with the effective interaction for a normal system.

the spin antisymmetric part of the dimensionless effective interaction in the particle-particle channel, consisting of only odd Legendre harmonics, $\langle \dots \rangle_{\hat{p}'}$ the average over solid angles connected with the vector $\hat{p}'$ and $\vec{\sigma}$ the pseudovector of Pauli matrices. In the acoustic limit it is sufficient to put $-L = O = \frac{1}{2}$, $2M = -\omega - (\vec{k}\vec{v}')$
$N = -\frac{1}{2} + \omega^2 - (\vec{k}\vec{v}')^2$ where $\vec{v}' = v\hat{p}'$
and ω, kv are measured in the units 2Δ ; the definitions of the above functions were given in [13,14] (cf.also [11]). Our vertex functions are chosen here such that $\hat{J}_a^{\omega}$, being the vertex $\hat{J}_a$ for the system without pairing for $|\omega| \gg kv$ is equal to σ_a . For such vertex functions the tensor of paramagnetic susceptibility is given by [11]

$$\chi_{ab} = -\mu_B^2 \nu \frac{1}{2} Tr\langle \sigma_a \{L\hat{J}_b + O(\vec{\sigma}\hat{p})\hat{J}_b(\vec{\sigma}\hat{p}) - M[\hat{\tau}_b,(\vec{\sigma}\hat{p})]_-\}\rangle_{\hat{p}}, \quad (3)$$

where μ_B denotes the Bohr magneton, ν the density of states on the Fermi sphere and the trace is taken over spin indices. The kernels $B(\hat{p}\hat{p}')$ and $f_{-1}^{\xi}(\hat{p}\hat{p}')$ will be determined here by its Legendre amplitudes (i.e., Landau amplitudes). They will be defined as follows

$$B(\hat{p}\hat{p}') = \sum_{l=0}^{\infty}(2l+1)\, b_l\, P_l(\hat{p}\hat{p}') \quad (4)$$

in order to avoid too complicated denominators. The kernel $f_{-1}^{\xi}(\hat{p}\hat{p}')$ will be determined by f_l in the manner (4) but with $f_{2k} = 0$. Note that the analog of the gap equation is given by $f_1 = [\ln(2\xi/\Delta)]^{-1}$ [11,13]. Note also that new b_l are equal to $b_l/(2l+1)$ in our old notation[11] and $Z_l/4(2l+1)$ in Leggett's notation [24]. In the further part of this paper we restrict ourselves to $f^{\xi}_{-1}(\hat{p}\hat{p}') = 3(\hat{p}\hat{p}')[\ln(2\xi/\Delta)]^{-1}$, i.e., negleot all remaining Legendre amplitudes of interaction in this channel. Note that all concrete caloulations up to now were performed under an analogous restriction. On the other hand, we will not impose any restriction on $B(\hat{p}\hat{p}')$.

As follows from (1,2) $\hat{J}_a$ and $\hat{\tau}_a$ are a-th components of traceless pseudoveotor and vector respectively. As a result of the above restriction on f^{ξ}_{-1}, $\hat{\tau}_a$ has to be a linear function of the veotor $\hat{p}$, i.e., $\hat{\tau}_a = \tau_{abc}\sigma_b\hat{p}_c$, where the pseudotensor τ_{abc} of third rank is only $\omega, \vec{k}$ - dependent. Note that hereafter the summation convention over repeated vector indioes is assumed. Analogously one can write $\hat{J}_a = J_{ab}\sigma_b$, where J_{ab} is $\hat{p}, \vec{k}$ and ω - dependent tensor. Its general form can be written as follows

$$J_{ab} = A\delta_{ab} + B\hat{p}_a\hat{p}_b + C\hat{k}_a\hat{p}_b + D\hat{p}_a\hat{k}_b + E\hat{k}_a\hat{k}_b , \tag{5}$$

where the functions A-E depend only on $(\hat{p}\hat{k}) \equiv w$, k and ω. The most general $\hat{p}$- independent pseudotensor of third rank can be expressed by

$$\tau_{abc} = iR\varepsilon_{abc} + i(X-R)\varepsilon_{dbc}\hat{k}_d\hat{k}_a + iY\varepsilon_{adc}\hat{k}_d\hat{k}_b + iZ\varepsilon_{abd}\hat{k}_d\hat{k}_c , \tag{6}$$

where ε_{abc} denotes the Levi-Civita pseudotensor. As a result of the following identity

$$\varepsilon_{abc} = \hat{k}_d(\hat{k}_a\varepsilon_{dbc} + \hat{k}_b\varepsilon_{adc} + \hat{k}_c\varepsilon_{abd}) \tag{7}$$

the pseudotensor τ_{abc} depends, in fact, only on three oombinations of variables R,X,Y,Z and hence our choice Y = 0 can be made without any loss of generality. In order to prove (7) let us remark that both sides of (7) are of the same tensor charaoter,and in the reference frame with $\hat{k}$ parallel to the Z -axis, (7) has the form

$$\varepsilon_{abc} = \delta_{3a}\varepsilon_{3bc} + \delta_{3b}\varepsilon_{a3c} + \delta_{3c}\varepsilon_{ab3} . \tag{8}$$

This relation oan be verified by inspection and, if we remark that (8) is equivalent to (7), then we complete our proof. Substituting $\hat{J}_a$ and $\hat{\tau}_a$ in the above form into (2) one finds

$$\langle \hat{p}_f\{U\hat{p}_c[R\varepsilon_{abc} + (X-R)\varepsilon_{dbc}\hat{k}_d\hat{k}_a] + UZ\varepsilon_{abd}\hat{k}_d w - 2OZ\hat{p}_b w\varepsilon_{acd}\hat{p}_c\hat{k}_d\}\rangle_{\hat{p}} = -2\langle \hat{p}_f M[A\varepsilon_{adb}\hat{p}_d + D\varepsilon_{cdb}\hat{p}_a\hat{p}_d\hat{k}_c + E\varepsilon_{cdb}\hat{k}_a\hat{k}_c\hat{p}_d]\rangle_{\hat{p}} , \tag{9}$$

where $U = N + O$ and $w = (\hat{k}\hat{p})$. Note that the equation (9) is not restricted to the acoustic limit and that the variables R,X,Z are $\hat{p}$- independent. The analogous expression for the spin susceptibility has the form

$$\chi_{ab} = \mu_B^2\nu\langle (O-L)J_{ba} - 2O\hat{p}_a\hat{p}_c J_{bc} - 2M[R(\delta_{ab} - \hat{p}_a\hat{p}_b) + (X-R)\hat{k}_b(\hat{k}_a - \hat{p}_a w) + Zw(w\delta_{ab} - \hat{k}_a\hat{p}_b)\rangle_{\hat{p}} . \tag{10}$$

Taking into account the symmetry properties and (5) one can rewrite (10) in the form

$$\chi_{ab} = \chi_{\perp}(\delta_{ab} - \hat{k}_a \hat{k}_b) + \chi_{\parallel} \hat{k}_a \hat{k}_b \,, \tag{11}$$

where

$$\chi_{\perp} = \mu_B^2 \nu \langle A(-L+OW^2) - \tfrac{1}{2} B(O+L)(1-W^2) - DW(O+L-OW^2) - M[R(1+W^2)+2ZW^2] \rangle_{\hat{p}} \,, \tag{12}$$

$$\chi_{\parallel} = \mu_B^2 \nu \langle (A+\tilde{D}W+E)(O-L-2OW^2) - (BW^2+CW)(O+L) - 2MX(1-W^2) \rangle_{\hat{p}} \,.$$

2. The transformation and solution of equations in the acoustic limit

In the acoustic limit $O \gg |U|$ and, from the equation (9), $|X|, |R| \gg Z$. Hence the term proportional to UZ in (9) and the terms proportional to Z in (10) and (12) should be neglected, as well as in $\hat{\tau}_a$ substituted into (1). This equation, written in terms of J_{ab} and R, X, passes in the acoustic limit into

$$J_{ab} = \delta_{ab} - \langle B[\omega + (kv')][R(\delta_{ab} - \hat{p}'_a \hat{p}'_b) + (X-R)\hat{k}_a(\hat{k}_b - \hat{p}'_b W')] \rangle_{\hat{p}'} - \tag{13}$$

$$\langle B[\tfrac{1}{2}(1+\hat{\tilde{P}})J_{ab} - \hat{\tilde{P}} \hat{p}'_b J_{ac} \hat{p}'_c] \rangle_{\hat{p}'} \,,$$

with B depending on $(\hat{\vec{p}}\hat{\vec{p}}')$ and J_{ab} depending on $\hat{\vec{p}}'$ or $\hat{\vec{p}}$, respectively, under or out of symbol $\langle \dots \rangle_{\hat{p}'}$; the operator $\hat{\tilde{P}}$ changes $\hat{\vec{p}}'$ to $-\hat{\vec{p}}'$. Applying the following relations

$$\langle B \rangle_{\hat{p}'} = b_0 \,, \quad \langle B \hat{p}'_a \rangle_{\hat{p}'} = b_1 \hat{p}_a \,, \quad \langle B \hat{p}'_a \hat{p}'_b \rangle_{\hat{p}'} = \tfrac{1}{3}(b_0 - b_2)\delta_{ab} + b_2 \hat{p}_a \hat{p}_b \,,$$

$$\langle B \hat{p}'_a \hat{p}'_b \hat{p}'_c \rangle_{\hat{p}'} = \tfrac{1}{5}(b_1 - b_3)(\hat{p}_a \delta_{bc} + \hat{p}_b \delta_{ac} + \hat{p}_c \delta_{ab}) + b_3 \hat{p}_a \hat{p}_b \hat{p}_c \,, \tag{14}$$

(Cf.(4)), one can rewrite the second term on the right-hand side of (13) as

$$-\delta_{ab} R \left[\frac{\omega}{3}(2b_0 + b_2) + \frac{kv}{5}(4b_1 + b_3)W\right] + \hat{p}_a \hat{p}_b R(\omega b_2 + kv\, b_3 W) +$$

$$\hat{k}_a \hat{p}_b \left[X \frac{kv}{5}(b_1 - b_3) + (X-R)(\omega b_2 W + kv\, b_3 W^2)\right] + \tag{15}$$

$$\hat{p}_a \hat{k}_b R \frac{kv}{5}(b_1 - b_3) - \hat{k}_a \hat{k}_b (X-R)\left[\frac{\omega}{3}(2b_0 + b_2) + \frac{kv}{5}(3b_1 + 2b_3)\right].$$

Comparing formulae (13) and (15) with (5) one can remark that in the solution J_{ab} of eq. (13) the following terms, at least, should appear

i) the A-terms with $\ell =$ 0,1,

ii) the B-terms with $\ell =$ 0,1,

iii) the C-terms with $\ell =$ 0,1,2,

iv) the D-terms with $\ell =$ 0,

v) the E-terms with $\ell =$ 0,1,

where ℓ denotes the orders of Legendre polynomials $P_\ell(W)$ appearing in the functions A-E. Taking into account now the terms of J_{ab} transform one into another in the procedure $\hat{p}_b J_{ac} \hat{p}_c$ appearing in (13) one can also remark that the terms i)-v) of (5) are sufficient to solve eq. (13). Let us define

$$F(w)=\sum_{n=0}^{m(F)} F_n w^n , \tag{16}$$

where F is one of functions A–E. Substituting (5) into (13) one remarks that formulae (14) are sufficient to calculate all appearing integrals. Hence, comparing all linearly independent terms, we find immediately

$$A_0=[1-R\omega(S-1)]S^{-1}, \tag{17}$$

$$B_0=b_2(1+\omega R)S^{-1}, \tag{18}$$

$$C_1=\omega(X-R)\,b_2\,S^{-1}, \tag{19}$$

$$E_0=-\omega(X-R)(S-1)S^{-1}, \tag{20}$$

where

$$S=1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_2 . \tag{21}$$

Moreover,

$$C_2+b_3G=(X-R)kv\,b_3\ ,\ C_2-G\left[1+\tfrac{1}{5}(b_1-b_3)\right]=(X-R)\frac{kv}{5}(3b_1+2b_3), \tag{22}$$

with $G\equiv E_1+C_2$, and

$$A_1+\tfrac{1}{5}(b_1-b_3)F=-\tfrac{1}{5}Rkv(4b_1+b_3)\ ,\ B_1+b_3F=Rkv\,b_3, \tag{23}$$
$$A_1+B_1-F\left[1+\tfrac{1}{5}(b_1-b_3)\right]=-\tfrac{1}{5}Rkv(b_1-b_3),$$

with $F\equiv A_1+B_1+D$. We have also

$$C_0=\tfrac{1}{5}(b_1-b_3)\left(Xkv-F-G\right). \tag{24}$$

Solving the systems (22) and (23) and substituting the results into (24) one finds

$$A_1=-\tfrac{1}{5}Rkv(b_1^2+4b_1b_3+4b_1+b_3)P^{-1}, \tag{25}$$

$$B_1=Rkv\,b_3(1+b_1)P^{-1}, \tag{26}$$

$$C_0=\tfrac{1}{5}Xkv(b_1-b_3)P^{-1}, \tag{27}$$

$$C_2=(X-R)kv\,b_3(1+b_1)P^{-1}, \tag{28}$$

$$E_1=-(X-R)kv\left[\tfrac{3}{5}(b_1-b_3)+b_3(1+b_1)\right]P^{-1}, \tag{29}$$

$$D=\tfrac{1}{5}Rkv(1+b_1)(b_1-b_3)P^{-1}, \tag{30}$$

where

$$P\equiv 1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_3 . \tag{31}$$

Let us come back to the equation (9) in the acoustic limit. Taking into account that

$$\langle \hat{p}_a\hat{p}_b\hat{p}_c\hat{p}_d\rangle_{\hat{\vec{p}}}=\tfrac{1}{15}\left(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}\right), \tag{32}$$

one can find

$$(\omega^2-\tfrac{1}{5}k^2v^2)\left[R\varepsilon_{abc}+(X-R)\hat{k}_a\hat{k}_d\varepsilon_{dbc}\right]+\tfrac{1}{5}Z\hat{k}_b\hat{k}_d\varepsilon_{adc}-$$

$$(\tfrac{2}{5}k^2v^2R+\tfrac{1}{5}Z)\hat{k}_c\hat{k}_d\varepsilon_{abd}=-(\omega A_0+\tfrac{1}{5}kvA_1)\varepsilon_{abc}- \qquad (33)$$

$$\left[\omega E_0+\tfrac{1}{5}kv(D+E_1)\right]\hat{k}_a\hat{k}_d\varepsilon_{dbc}+\tfrac{1}{5}kv(D-2A_1)\hat{k}_c\hat{k}_d\varepsilon_{abd}.$$

According to the assumption about the form of anomalous vertex function made in [18] X has to be equal R and $Z=0$. It is quite clear from (33) that this equation cannot be fulfilled under the above assumption. If we eliminate from (33) the term proportional to $\hat{k}_b\hat{k}_d\varepsilon_{adc}$ using (7), then we obtain three equations for variables R,X,Z. The equivalent procedure consists in choosing $\hat{k}$ along the third axis and substituting cyclic permutations of 1,2,3 instead of a,b,c. Hence

$$(\omega^2-\tfrac{1}{5}k^2v^2)X=-\omega(A_0+E_0)-\tfrac{1}{5}kv(A_1+D+E_1), \qquad (34)$$

and

$$(\omega^2-\tfrac{3}{5}k^2v^2)R-\tfrac{1}{5}Z=-\omega A_0-\tfrac{1}{5}kv(3A_1-D), \qquad (35)$$

$$(\omega^2-\tfrac{1}{5}k^2v^2)R+\tfrac{1}{5}Z=-\omega A_0-\tfrac{1}{5}kvA_1 .$$

From (35) one finds

$$(\omega^2-\tfrac{2}{5}k^2v^2)R=-\omega A_0-\tfrac{1}{10}kv(4A_1-D) \qquad (36)$$

Substituting, into eqs. (34) and (36), A_0, E_0 and A_1, D, E_1 expressed by the formulae (17,20) and (25,29,30) one obtains

$$R=-\omega\left[\omega^2-\tfrac{2}{5}k^2v^2(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_2)(1+b_1)(1+\tfrac{1}{4}b_1+\tfrac{3}{4}b_3)(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_3)^{-1}\right]^{-1}, \qquad (37)$$

$$X=-\omega\left[\omega^2-\tfrac{1}{5}k^2v^2(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_2)(1+b_1)(1+b_3)(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_3)^{-1}\right]^{-1}. \qquad (38)$$

Now the formulae (17-20) and (25-30) together with (5,6) and (37,38) serve as the coincide form for the solution.

3. The discussion of solutions and conclusions

Let us obtain first the expression for the spin susceptibility. Substituting our solutions into eq. (10) we find

$$\chi_{ab}=\chi_{static}\left[\delta_{ab}+(\delta_{ab}-\hat{k}_a\hat{k}_b)\omega R+\hat{k}_a\hat{k}_b\omega X\right], \qquad (39)$$

where the value χ_{static} is equal to $2\mu_B^2\nu/3(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_2)$; this value was obtained in our paper [11] for M (cf. (1,2) put equal zero. Here we have the following property:

$$\lim_{\omega\to 0}\chi_{ab}=\chi_{static}\delta_{ab}, \qquad (40)$$

if $k \neq 0$. This property has a simple physical meaning, since the static limit corresponds to the static but slightly inhomogeneous field, as a result of,e.g.,the finiteness of a sample. It should be noted that the tensor (39), is the homogeneous function of variables ω, kv of zeroth degree and, hence, the formula (39) is valid in an arbitrary system of units. We have $\lim_{k\to 0} R = \lim_{k\to 0} X = -\omega^{-1}$ for $\omega \neq 0$ and hence $\lim_{k\to 0} \chi_{ab} = 0$ for $\omega \neq 0$. This result is quite understandable in the acoustic limit; in the homogeneous time-periodic field this quantity has to be of order of $(\omega/2\Delta)^2$ for $\omega \ll 2\Delta$ and this quantity lies beyond the accuracy of the acoustic limit. This property has an analog in the theory of normal Fermi liquids. It is well-known fact that $\lim_{k\to 0} S(\omega,k)=0$ for $\omega \neq 0$ where S denotes an arbitrary response function for conserved quantities. Moreover, the usual accuracy for these theories neglects $(\omega/\mu)^2$ where μ is the chemical potential of the system.

According to (39) the pole of R corresponds to transversal spin waves whereas the pole of X -to longitudinal spin waves. It is clear that transversal waves are twice degenerated whereas the longitudinal waves are undergenerated (cf.[17]). All factors containing amplitudes b_ℓ in R and X can be represented by

$$\frac{m}{n}(1+b_\ell) + \frac{n-m}{n}(1+b_{\ell'}),$$

where $0 \leq m \leq n$ and ℓ, ℓ' denotes whether 0,2 or 1,3. Because the stability conditions for spin Landau amplitudes

defined as here are $1+b_\ell > 0$, [23], all these factors are positive. Hence we obtain that stability conditions guarantee that in (39) always two poles appear. Moreover, the stability conditions guarantee also that the transversal mode is at higher energies than the longitudinal mode with the same k, i.e., that the transversal spin waves are faster than longitudinal ones. Note that the Landau amplitudes b_0 and b_2 appear in (39) only in the same combination as in the static spin susceptibility.

Our calculations demonstrate a property characteristic for theories based on sufficiently general phenomenological approach. It could be called "a principle of maximal freedom of physical systems". Let us demonstrate the action of this principle using the spin susceptibility of ^{3}He. The static susceptibility for the normal system determines b_0, whereas in the B-phase also b_2 [11]. Moreover, the detection of transversal and longitudinal spin waves gives us b_1 and b_3. From this point of view here is not any cross check for the theory; the independent measurements of b_1 [19–21] could be treated as the exception confirming a general rule.

In all our calculations the temperature effects and those of spin-unconserving weak dipole-dipole interaction were disregarded. The first of them could be taken into account by means of methods developed by Leggett[25]; the second ones lead to rather serious difficulties in a similar formulation of the theory (cf.[26]).

The author is greatly indebted to Prof.A.J.Leggett for sending the review article [24] prior to publication.

APPENDIX

We are going to reduce equations for spin vertices and the spin susceptibility so that only integrations over Fermi surface are important, analogously to transformations made in [14]. Note that only the transformation of the equation for the normal vertex will be a subject of our interest. Applying the procedure of Larkin [14] to the equation for normal vertex we find

$$\hat{\mathcal{J}}_\alpha(\hat{\vec p}) = \hat{\mathcal{J}}^k_\alpha + \langle g(\hat{\vec p}\hat{\vec p}')\{ S\hat{\mathcal{J}}_\alpha(\hat{\vec p}') + O(\vec\sigma\hat{\vec p}')\hat{\mathcal{J}}_\alpha(-\hat{\vec p}')(\vec\sigma\hat{\vec p}') - M[\hat{\tau}_\alpha(\hat{\vec p}'),(\vec\sigma\hat{\vec p}')]_- - 2M\lambda_\alpha(\hat{\vec p}')(\vec\sigma\hat{\vec p}')\}\rangle_{\hat{\vec p}'} , \qquad (41)$$

where $\hat{\mathcal{J}}^k_\alpha$ denotes the spin vertex for the normal system taken in the k-limit, i.e., for $\lim\limits_{k\to 0}\lim\limits_{\omega\to 0}$ and $g(\hat{\vec p}\hat{\vec p}')$ is the spin-exchange part of dimensionless scattering amplitude of quasiparticles, $g_\ell = b_\ell/(1+b_\ell)$, $S = L+1$.
The equations for λ_α and $\hat\tau_\alpha$ will be the same as in [11].
According to [27] (cf.also [28]) if the spin vertex is chosen such that $\hat{\mathcal{J}}^\omega_\alpha = \sigma_\alpha$, then $\hat{\mathcal{J}}^k_\alpha = \sigma_\alpha(1-g_0) = \sigma_\alpha/(1+b_0)$.
Let us pass now to the transformation of the expression for the spin susceptibility. According to [11] one can write the spin susceptibility as

$$\chi_{ab} = -\left(\frac{\mu_B}{Z}\right)^2 \int \frac{d^4p}{(2\pi)^4 i}\, Tr\Big\{\hat{\mathcal{J}}^0_\alpha\Big[G\hat{\mathcal{J}}_b G - F\hat D\hat{\mathcal{J}}^-_b\hat D^+ F - G\hat\tau_{1b}\hat D^+ F + F\hat D\hat\tau_{2b} G\Big]\Big\} , \qquad (42)$$

where $\hat{\mathcal{J}}^0_\alpha = Z\sigma_\alpha$ so that $\hat{\mathcal{J}}^\omega_\alpha$ at the Fermi sphere is equal to σ_α ; Z denotes the discontinuity of the occupation number at the Fermi sphere for a normal system , $\hat D = \hat\Delta/\Delta = (\vec\sigma\hat{\vec p})i\sigma^y$, and normal (G) and anomalous (F) Green's functions before the vertex are taken at $\vec p + \vec k/2,\ \varepsilon + \omega/2$ whereas after the vertex at $\vec p - \vec k/2,\ \varepsilon - \omega/2$, $d^4p = d^3p\, d\varepsilon$, $\hat{\mathcal{J}}^-(p) \equiv \hat{\mathcal{J}}^T(-p)$. Performing in (42) the transformation originally proposed by Larkin [14] one finds using the equation (41) and expressing $\hat\tau_{1b}, \hat\tau_{2b}$ by $\hat\tau_b$ and λ_b

$$\chi_{ab} = \chi^k_{ab} - \frac{\mu_B^2\nu}{2} Tr\langle\hat{\mathcal{J}}^k_\alpha\{S\hat{\mathcal{J}}_b + O(\vec\sigma\hat{\vec p})\hat{\mathcal{J}}_b(\vec\sigma\hat{\vec p}) - M[\hat\tau_b,(\vec\sigma\hat{\vec p})]_- - 2M\lambda_b(\vec\sigma\hat{\vec p})\}\rangle_{\hat{\vec p}} . \qquad (43)$$

Here

$$\chi^k_{ab} = -\left(\frac{\mu_B}{Z}\right)^2 \int \frac{d^4p}{(2\pi)^4 i}\, Tr\left[\hat{\mathcal{J}}^0_\alpha (GG)^k \hat{\mathcal{J}}^k_b\right] , \qquad (44)$$

with $(GG)^k \equiv \lim\limits_{k\to 0}\lim\limits_{\omega\to 0}\lim\limits_{\Delta\to 0} GG$, [14] .
Taking into account the results of paper [27] for $\hat{\mathcal{J}}^k_b$ (cf.also [28]) we find that χ^k_{ab} denotes the static susceptibility of a normal system, i.e., $\delta_{ab}\mu_B^2\nu/(1+b_0)$.
Moreover, $\hat{\mathcal{J}}^k_\alpha$ in the second term of the formula (43), i.e., on the Fermi surface, is equal to $\sigma_\alpha/(1+b_0)$. It is clear

that the formula (43) is equivalent to (10) and can be also written in form (11,12). Note that according to eq. (42) $\chi_{\alpha b}$ and ν correspond in Appendix to quantities per unit volume.

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