сообщения
объединенного
института
ядерных
исследований
дубна

E17-94-320

L.Ts.Adzhemyan[1], M.Hnatich[2,3], D.Horvath[3], M.Stehlik[2,3]

THE MAXIMAL RANDOMNESS PRINCIPLE IN *d*-DIMENSIONAL TERBILENCE

[1]Faculty of Physics of St.Petersburg University, St.Petersburg, Russia
[2]Joint Institute for Nuclear Research, Dubna, Russia
[3]Institute of Experimental Physics of Slovak Academy of Sciences, Kosice, Slovakia

1994

1 Introduction

The maximum information entropy principle provides a very elegant access to many of the concepts of thermodynamics where it is mostly applied to systems in thermal equilibrium. The analysis presented here generalizes this principle implementing it into the classical hydrodynamic domain. This approach demonstrates the possibility of constructing a sensible and consistent statistical theory of turbulence. The object of the research reported here is reformulation of the approach of Wyld [1], which is often used to describe strongly developed turbulence of incompressible fluids.

Attempts to construct appropriate distribution functions for random velocity field have been made in the Wyld model [6, 7] as well as on the basis of independent statistic assumption [9] where the extremal principle approach has been utilized. General features of this approach are discussed in [11] (see also [12]). In the present paper we consider a based on the principle of maximal randomness of the velocity field statistical model , which has been proposed by one of us [10] and restrict ourself to study statistical features of single-time velocity pulsations. Our statistical approach is the closest one to that proposed in [6]. However, while in [6] the maximal randomness principle is destined to select suitable perturbation approach within the framework of the Wyld model, in present work this principle is the basis of a new statistical approach. That is why the one-loop approximation exploited further coincides neither with the analogous approximation in [6] nor with the direct interaction approximation (DIA) in the Wyld model.

We have investigated the mentioned above statistical model by means of self-consistent skeleton equations in one-loop approximation. It has been shown that the equations are free from divergences and they have a solution with Kolmogorov spectrum in inertial interval. However, this solution is non-physical for space dimension $d = 3$, because Kolmogorov constant is negative. Following the ideas [13] we developed the alternative interpretation of obtained results where the modified (fractal) understanding of dimension occurring in diagrams is assumed.

2 The statistical Model

The utilization of statistical hypothesis of maximal randomness seems to be natural to construct the theory of the strongly developed turbulence. Just this assumption was accepted by Wyld, following Kraichnan [14], in the paper [1] at the elaboration of his well-known diagramatic technique. As Wyld has mentioned, it is desirable to apply the principle of maximal randomness to the velocity field $\mathbf{v}(\mathbf{x})$, selecting the proper statistics of random force $\mathbf{f}(\mathbf{x})$ which simulates the stochasticity of forcing in the Wyld model. To realize this approach is technically difficult that is why Wyld postulated it for the non-perturbated (excluding nonlinearity) velocity field $\mathbf{v}_o(\mathbf{x})$ or, equivalently, for the random force $\mathbf{f}(\mathbf{x})$ with Gaussian statistics. The use of the artificial random force $\mathbf{f}$ may by avoided and the principle of maximal randomness in its initial formulation, i.e. applied to the total velocity field $\mathbf{v}(\mathbf{x})$, will be used to construct the distribution function of velocity pulsations.

The Navier-Stokes equation for the velocity field of randomly stirred incompressible fluid [$\partial_j v_j = 0$] is equivalent to an infinite system of equations for correlation functions. The first simplest equation of this system in the considered stationary case has to be of the form

$$\frac{1}{2}\partial_t\langle E_{ij}(\mathbf{k})\rangle \;=\; -\nu k^2\langle E_{ij}(\mathbf{k})\rangle - \langle T_{ij}(\mathbf{k})\rangle \;+\; \overline{D}_{ij}(\mathbf{k}) = 0, \qquad (1)$$

where

$$E_{ij}(\mathbf{k}) \;=\; v_i(\mathbf{k})v_j(-\mathbf{k}),$$

and transfer is

$$T_{ij}(\mathbf{k}) = \frac{1}{2}\left[v_i(\mathbf{k})(v_s\partial_s v_j)(-\mathbf{k}) + v_j(-\mathbf{k})(v_s\partial_s v_i)(\mathbf{k})\right] .$$

Here, we use the same notation for fields and their Fourier components, therefore $\mathbf{v}(\mathbf{k}) = \int d^d\mathbf{x}\exp(-i\mathbf{k}\mathbf{x})\mathbf{v}(\mathbf{x})$ and traces over the repeated vector subscripts are implied. Brackets $\langle ...\rangle$ denote statistical ensemble averaging. In the homogeneous case all correlators of the form $\langle \mathbf{v}(\mathbf{k}_1)\mathbf{v}(\mathbf{k}_2)\rangle$ are proportional to $\delta^d(\mathbf{k}_1+\mathbf{k}_2)$. We will omit the δ-functions further, implying the integration over the relative coordinates. The quantities as $E_{ij}(\mathbf{k})$, energy transfer $T_{ij}(\mathbf{k})$ and spectral density of energy source $\overline{D}_{ij}$ are proportional to transversal projector $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2$.

The $\overline{D}_{ij}$ is localized at the wave numbers $k \sim 1/L$ (L is correlation length) and can be expressed using equation

$$\overline{D}_{ij}(\mathbf{k}) = \overline{\varepsilon}\;\frac{2(2\pi)^d}{d-1}\,\delta^d\;(\mathbf{k})\;P_{ij}(\mathbf{k})\,, \qquad (2)$$

in the inertial interval $kL \gg 1$. Here $\overline{\varepsilon} = 2^{-1}(2\pi)^{-d}\int d^d\mathbf{k}\,\overline{D}_{jj}(k)$ is the energy injection rate.

The maximal randomness or uncertainity demand itself has no definite physical content until the physical conditions of the search of extreme are not fixed. If one wants to get a reduced description of the physical system, it is necessary to fix the mean values of quantities which are crucial for its interaction with thermostat. In such a way the equilibrium Gibbs ensembles correspond to the fixation of the mean energy and mean number of particles. In the description of the homogeneous isotropic turbulence averaged flow represented 'termostat'. The velocity pulsations interact with this flow. Following the ideas of Kolmogorov, let us suppose, that the most substantial characteristic of this interaction is the mean spectral energy flux with the source localized at the small wave numbers $k \sim 1/L$. The distribution function $\rho(\mathbf{v})$ with normalization $\int D\mathbf{v}\rho(\mathbf{v}) = 1$ ($D\mathbf{v}$ denotes functional integration measure) can be constructed from the demand of information entropy

$$-\langle\ln\rho\rangle = -\int D\mathbf{v}\rho(\mathbf{v})\ln\rho(\mathbf{v})$$

maximum with satisfied condition (1). Introducing the Lagrange multipliers $\lambda_{ij}(\mathbf{k})$ and passing to the unconditional extreme problem

$$\frac{\delta}{\delta\rho\,(\mathbf{v}')}\ \left\langle \ln\rho\,(\mathbf{v})+\int\frac{d^d\mathbf{k}}{(2\pi)^d}\lambda_{ij}\,(\mathbf{k})\ \left(\nu k^2\,E_{ij}(\mathbf{k})+T_{ij}(\mathbf{k})\,\right)\right\rangle=0\,,$$

one obtains the "quantum field" model

$$\rho=\frac{1}{Q}\exp S$$

with the normalization factor Q and "action"

$$\begin{aligned} S &= -\int\frac{d^d\mathbf{k}}{(2\pi)^d}\Big[\nu k^2\lambda_{ij}(\mathbf{k})E_{ij}(\mathbf{k})+ \\ &\quad +\lambda_{ij}(\mathbf{k})v_j(\mathbf{k})(v_s\partial_s v_i)(-\mathbf{k})\Big]\,, \end{aligned} \tag{3}$$

where the λ_{ij} multipliers can be found from (1) and symmetries $E_{ij}(\mathbf{k})=E_{ij}(-\mathbf{k})$ and $T_{ij}(\mathbf{k})=T_{ij}(-\mathbf{k})=v_i(\mathbf{k})(v_s\partial_s v_j)(-\mathbf{k})$, which imply $\lambda_{ij}(\mathbf{k})=\lambda_{ji}(-\mathbf{k})$ have been assumed.

For further manipulations it is convenient to separate a $\lambda_{ij}(\mathbf{k})$ part proportional to $P_{ij}(\mathbf{k})$ through a $\mathbf{k}$ independent coefficient:

$$\lambda_{ij}(\mathbf{k})=\frac{\varphi_0}{2\nu}P_{ij}(\mathbf{k})+\tilde\lambda_{ij}(\mathbf{k})\,. \tag{4}$$

The vertex term of action (3) multiplied by the constant $\varphi_0/2\nu$ becomes total derivative in the coordinate representation (the consequence of energy conservation at $\nu=0$) and therefore it can be omitted. In order to obtain the self consistent equations the action (3) will be rewritten into a more convenient form. For this reason an unit operation $\int\prod Du_i\delta(\,u_i(\mathbf{k})-\tilde\lambda_{ij}(\mathbf{k})v_j(\mathbf{k}))$ and auxiliary field u_i is introduced in a standard way. The functional δ-function $\delta(u-\tilde\lambda v)$ can be expressed in Fourier representation by means of an auxiliary field $\mathbf{f}$ (and then $i\mathbf{f}\to\mathbf{f}$) so we obtain

$$\int\prod_j^d Du_j\delta\left(u_j-\tilde\lambda_{jn}v_n\right)=\int\prod_s^d Du_s\int Df_s\,\exp\left[f_ju_j-f_j\tilde\lambda_{jn}v_n\right]\,.$$

That allows to introduce the "distribution function" $\rho(\mathbf{v},\mathbf{u},\mathbf{f})$ of the three transversal fields - $\mathbf{v}$, $\mathbf{u}$, $\mathbf{f}$ by the relation $\rho(\mathbf{v},\mathbf{u},\mathbf{f})=(1/Q)\exp S$, where

$$\begin{aligned} S &= -\int d^d\mathbf{x}\big[-\frac{1}{2}\varphi_0\,(\partial_jv_i)^2-\nu(\partial_ju_i)(\partial_jv_i)+ \\ &+\ (\partial_ju_i)v_iv_j-f_i\hat\lambda_{ij}v_j+f_iu_i\big]. \end{aligned} \tag{5}$$

The action is written in the coordinate representation, $\hat\lambda$ is an operator with kernel corresponding to $\tilde\lambda(\mathbf{k})$ in the wave number representation.

To clarify the physical meaning of the auxiliary field $\mathbf{f}$ we use the Schwinger equation in form

$$\langle v_i \delta S / \delta u_j \rangle = 0. \tag{6}$$

Taking into account the transversality of $\mathbf{v}$, one obtains:

$$\nu k^2 \langle E_{ij}(\mathbf{k})\rangle - \langle v_i(\mathbf{k})(v_s\partial_s v_j)(-\mathbf{k})\rangle + \langle v_i(\mathbf{k}) f_j(-\mathbf{k})\rangle = 0\,.$$

The relation (1) determines the 'cross' propagator

$$G^{vf}_{ij}(\mathbf{k}) = \langle v_i(\mathbf{k}) f_j(-\mathbf{k})\rangle = \overline{D}_{ij}(\mathbf{k}) \tag{7}$$

proportional to power of stochastic energy forcing. The relation (7) formally coincides with the corresponding relation in Wyld theory, but in the presented the field $\mathbf{f}$ interacts with the $\mathbf{v}$ and therefore the statistics of $\mathbf{f}$ is non-Gaussian.

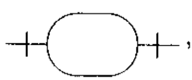
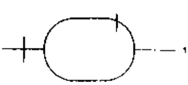
3 Self-consistent equations

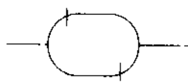
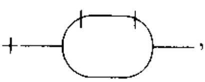
The derived velocity field distribution function has the form of standard quantum field theory of three transversal vector fields with quasilocal action (5). Therefore, the self-consistent equations may be obtained in a standard way, using the Dyson matrix equation for **total (non-perturbated)** propagators (or correlation functions) $\hat{G}$. The special feature of the presented model is that one of these propagators G^{vf} is fixed by relation (7). This relation is a condition allowing the determination of the unknown λ quantity in action (5).

The inverse matrix $\hat{G}^{-1}$ of the fields $\mathbf{v}$, $\mathbf{u}$, $\mathbf{f}$ correlators corresponding to the model (5) are considered in the representation of wave numbers

$$\hat{G}^{-1} = \begin{pmatrix} \varphi_0\, k^2 - \Sigma^{vv} & \nu k^2 - \Sigma^{vu} & \tilde{\lambda} \\ \nu k^2 - \Sigma^{uv} & -\Sigma^{uu} & -1 \\ \tilde{\lambda} & -1 & 0 \end{pmatrix} \quad , \tag{8}$$

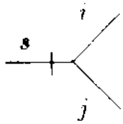
where all matrix elements are proportional to P_{ij} and Σ are the self-energy functions of model (5). The absence of the field $\mathbf{f}$ in the non-quadratic part of (5) also leads to absence of self-energy functions Σ in the matrix elements (8) corresponding to field $\mathbf{f}$. In one-loop approximation for non-vanishing Σ elements the following is valid:

$\Sigma^{uu} = \frac{1}{2}$  , $\quad \Sigma^{uv} =$  ,

$\Sigma^{vv} = \Sigma^{vv}_1 + \Sigma^{vv}_2 =$  +  ,

(9)

where graphic representations are introduced for the symmetrized vertex of interaction and for the total propagators:



$$= \tfrac{1}{2} v_i v_j (\partial_i \delta_{js} + \partial_j \delta_{is}) u_s ,$$

$$=< vv >\equiv G^{vv},$$

$$=< vu >\equiv G^{vu},$$

$$=< uu >\equiv G^{uu}.$$

The inversion of matrix (8) yields

$$\hat{G} = G^{vv} \begin{pmatrix} 1 & \tilde{\lambda} & \nu k^2 - \Sigma^{vu} - \\ & & -\tilde{\lambda}\Sigma^{uu} \\ \tilde{\lambda} & \tilde{\lambda}^2 & \tilde{\lambda}(-\nu k^2 + \Sigma^{uv}) - \\ & & -\varphi_0 k^2 + \Sigma^{vv} \\ \nu k^2 - \Sigma^{uv} - & \tilde{\lambda}(-\nu k^2 + \Sigma^{vu}) - & | - \nu k^2 + \Sigma^{uv}|^2 - \\ -\tilde{\lambda}\Sigma^{uu} & -\varphi_0 k^2 + \Sigma^{vv} & -\Sigma^{uu}(\varphi_0 k^2 - \Sigma^{vv}) \end{pmatrix} , \qquad (10)$$

where

$$(G^{vv})^{-1} = -\mathrm{Det}(\hat{G})^{-1} = \varphi_0 k^2 - \Sigma^{vv} - \tilde{\lambda}^2 \Sigma^{uu} + \tilde{\lambda}\left(2\nu k^2 - \Sigma^{uv} - \Sigma^{vu}\right) . \qquad (11)$$

The propagators G^{vv}, G^{vu} and G^{uu} are coupled to each other with the relations $G^{vu} = \tilde{\lambda} G^{vv}$ and $G^{uu} = \tilde{\lambda}^2 G^{vv}$, which follow from the definition of the field $\mathbf{u} = \hat{\lambda}\mathbf{v}$. Consequently, it is sufficient to obtain a closed set of equations for the propagator G^{vv} and function $\tilde{\lambda}(\mathbf{k})$. The relation (11) becomes the first equation of this set. Substituting G^{vf} from (10) into (7) one obtains the second equation:

$$-\nu k^2 G^{vv} + G^{vv}\left(\Sigma^{uv} + \tilde{\lambda}\Sigma^{uu}\right) + \overline{D} = 0 , \qquad (12)$$

where expressions on both sides are proportional to P_{ij}. The viscosity ν may be neglected in the inertial interval, then from (11) and (12) follows:

$$\begin{aligned} (G^{vv})^{-1} &= \varphi_0 k^2 - \Sigma^{vv} - \tilde{\lambda}^2 \Sigma^{uu} - \tilde{\lambda}\left(\Sigma^{uv} + \Sigma^{vu}\right) , & (13) \\ & G^{vv}\left(\Sigma^{uv} + \tilde{\lambda}\Sigma^{uu}\right) + \overline{D} = 0 . & (14) \end{aligned}$$

In statistical models of developed turbulence the power-like form $\overline{D}_{js}(\mathbf{k}) \sim P_{ij}(\mathbf{k})\, k^{-d}$ corresponding to $d-$ dimensional case is widely used [3, 4, 5]. If we write $\delta^d(\mathbf{k})$ - function in the power-like form

$$\delta^d(\mathbf{k}) = \frac{1}{\mathcal{S}_d} \lim_{\epsilon \to 0^+} \epsilon\, k^{\epsilon - d} , \qquad (15)$$

then using equation (2) we obtain

$$\mathcal{D}_{ij}(\mathbf{k}) = \mathcal{D}_d\,\bar{\varepsilon}\,\epsilon\,k^{\epsilon-d}\,P_{ij}(\mathbf{k}), \quad \text{with} \quad \mathcal{D}_d = \frac{2(2\pi)^d}{(d-1)\mathcal{S}_d}, \quad \mathcal{S}_d = \frac{2\pi^{d/2}}{\Gamma[d/2]}, \tag{16}$$

where $\mathcal{S}_d$ is the surface of $d-$dimensional unit sphere. Note that this energy injection form leads to an expression for turbulent energy flux used in [15]. Solving the self-consistent equations (13) and (14) creating the closed system for G^{vv} and $\tilde{\lambda}$ determination for given (16) and taking the limit $\epsilon \to 0^+$ one reconstructs the necessary information about the amplitudes (see below). The solution will be searched to have the form

$$\begin{aligned} G^{vv}_{ij}(\mathbf{k}) &= c_0\,(\bar{\varepsilon}\,\mathcal{D}_d)^{2/3}\mathcal{K}_d^{-1/3}\,k^{-2\gamma}\,P_{ij}(\mathbf{k}),\\ \tilde{\lambda}_{ij}(\mathbf{k}) &= \lambda_0\,(\bar{\varepsilon}\,\mathcal{D}_d)^{-1}\,k^{2\eta}\,P_{ij}(\mathbf{k}), \end{aligned} \tag{17}$$

with real unknown amplitudes λ_0, c_0 and numerical factor $\mathcal{K}_d = (4\pi)^{-d/2}$ introduced for further convenience. From (1) and (17) using identity

$$E(k) = C_k\,\bar{\varepsilon}^{2/3}\,k^{-5/3} = \frac{\mathcal{S}_d\,k^{d-1}}{2(2\pi)^d}\,G^{vv}_{jj}(k)$$

we find relationship between the amplitude c_0 and Kolmogorov constant C_k:

$$C_k = \frac{(d-1)\,\mathcal{S}_d\,\mathcal{D}_d^{2/3}}{2(2\pi)^d\,\mathcal{K}_d^{1/3}}\,c_0\,. \tag{18}$$

The power form of solution of Eq. (17) exists only if the corresponding integrals are convergent. Of course, these integrals have natural physical infrared ($1/L$) and ultraviolet (Λ) cut-offs. But the basic problem how to found the Kolmogorov spectrum in inertial interval is to prove the existence of finite limits for $1/L$ and Λ tend to zero and infinity, respectively (see [6, 7, 8],e.g.).

Let us discuss the convergence of the integrals in (9), which give contribution into the equations (13), (14), near the Kolmogorov value $2\gamma = 2/3 + d$, $2\eta = d$. The strong infra-red (IR) singularity of the propagator $G^{vv}(2\gamma > 3)$ leads to the IR divergence of diagrams $\Sigma^{uu}, \Sigma^{uv}, \Sigma^{uu}_2$. All these are proportional to $L^{\frac{2}{3}}$ It can be easily shown that the coefficients of the divergent parts of Σ^{uu} and Σ^{vv}_2 are the same, and they differ from that of Σ^{uv} only in sign. As a result the IR divergences cancel each other not only in the balance energy equation (14), as it is expected [6, 7], but also in the equation (13).

In the ultraviolet (UV) range only the diagrams Σ^{vv}_1 and Σ^{vv}_2 proportional to $\Lambda^{11/3}$ (Λ - the UV cut-off parameter) are dangerous. Anyway, external momentum square may be factored out of integral in the sum of the diagrams and therefore the divergent part of Σ^{vv} becomes to be $\sim k^2\Lambda^{5/3}$. According to [10] this is a common property of all contributions to Σ^{vv} because in model (5) each external v-line of any one-particle irreducible diagram is proportional to an external momentum k. Consequently, the divergent part of Σ^{vv} has the same form as the term $\varphi_0 k^2$ in (13), and may be eliminated by a suitable choice of the parameter φ_0.

From the previous text follows that the system of equations (13) and (14) is divergence-free and the calculations can be carried out by the dimensional regularization, resulting in

$$\begin{aligned}
\Sigma^{uu}_{ij}(\mathbf{k}) &= c_0^2\,(\bar{\varepsilon}\,\mathcal{D}_d)^{4/3}\mathcal{K}_d^{-1/3}\,k^{2+d-4\gamma}P_{ij}(\mathbf{k})\,A^{uu}\,,\\
\Sigma^{uv}_{ij}(\mathbf{k}) &= \lambda_0\,c_0^2\,(\bar{\varepsilon}\,\mathcal{D}_d)^{1/3}\mathcal{K}_d^{1/3}\,k^{2+d+2\eta-4\gamma}P_{ij}(\mathbf{k})\,A^{uv}\,,\\
\Sigma^{vv}_{ij}(\mathbf{k}) &= \lambda_0^2\,c_0^2\,(\bar{\varepsilon}\,\mathcal{D}_d)^{2/3}\,\mathcal{K}_d^{-1/3}\,k^{2+d+4\eta-4\gamma}P_{ij}(\mathbf{k})\,A^{vv}\,,
\end{aligned} \qquad (19)$$

where the amplitudes A^{uu}, A^{uv} and A^{vv} are expressed through $\overline{\Gamma}$ functions defined by means of combination of standard Γ - functions:

$$\overline{\Gamma}\,[x_1,x_2,x_3;x_4,x_5,x_6] \equiv \frac{\Gamma(x_1)\Gamma(x_2)\Gamma(x_3)}{\Gamma(x_4)\Gamma(x_5)\Gamma(x_6)}\,.$$

Calculating the Feynman diagrams we find

$$\begin{aligned}
A^{uu}[\gamma,\eta,d] &= \Big(((d-1)(d/2-\gamma)\gamma - (2(d/2-\gamma)^2\gamma)/(1+d-2\gamma))\,\times\\
&\times\; \overline{\Gamma}\big[d/2-\gamma, d/2-\gamma, -d/2+2\gamma;\\
&\quad\; 1+\gamma, 1+\gamma, 1+d-2\gamma\big]\Big)/2\\
A^{uv}[\gamma,\eta,d] &= \Big(-((d-1)(\gamma-\eta))\,/2 + ((d/2-\gamma)\gamma)/(1+d-2\gamma+\eta)\Big)\,\times\\
&\times\; \overline{\Gamma}\big[1+d/2-\gamma+\eta, d/2-\gamma, -d/2+2\gamma-\eta;\\
&\quad\; 1+\gamma-\eta, 1+\gamma, 1+d-2\gamma+\eta\big]\\
A^{vv}[\gamma,\eta,d] &= \Big(1+d-2\gamma+2\eta\,+\\
&+\; ((d-1)(\eta-\gamma)^2)\,/\,(2+d-4\gamma+4\eta)\Big)\,\times\\
&\times\; \overline{\Gamma}\big[1+d/2-\gamma+\eta, 1+d/2-\gamma+\eta, -d/2+2\gamma-2\eta;\\
&\quad\; 1+\gamma-\eta, 1+\gamma-\eta, 2+d-2\gamma+2\eta\big]\,-\\
&-\; \Big(((-1+2\gamma)(2-\gamma+2\eta)\,+\\
&+\; (d-1)(-2+(\gamma-2\eta)(2+d/2-2\gamma+2\eta))\,+\\
&+\; ((d/2-\gamma)(2-\gamma+2\eta+(-1+d)(-d+3\gamma-\gamma^2-\\
&-\; 2\eta+2\gamma\eta)))/(1+d-2\gamma+2\eta))\,\times\\
&\times\; \overline{\Gamma}\big[1+d/2-\gamma+2\eta, d/2-\gamma, -1-d/2+2\gamma-2\eta;\\
&\quad\; 1+\gamma, 1+\gamma-2\eta, 1+d-2\gamma+2\eta\big]\Big)/2\,.
\end{aligned} \qquad (20)$$

Equating the powers in the equations (13), (14) the k-dependent terms have been excluded using (20) and (21). As result we find

Table 1:

C_k	d	λ_0
1.5	2.442	−1.856
2	2.328	−0.942
2.5	2.221	−0.636

$$2\eta = d - \epsilon, \quad 2\gamma = \frac{2+3d-2\epsilon}{3}. \tag{21}$$

Since $\epsilon \to 0^+$ we find Kolmogorov exponent $2\gamma = \frac{2+3d}{3}$. Rewriting (13),(14) with help of (21) we obtain two equations for amplitudes

$$\lambda_0^2 c_0^3 \left(A^{uu} + 2A^{uv} + A^{vv}\right) + 1 = 0, \quad \lambda_0 c_0^3 \left(A^{uu} + A^{uv}\right) + \epsilon = 0. \tag{22}$$

The solution of this system has the form

$$\lambda_0 = -\frac{1}{z}\left(\frac{z_\epsilon}{\epsilon}\right)_{\epsilon\to 0^+}, \quad c_0 = z^{1/3}\left(\frac{\epsilon}{z_\epsilon}\right)^{2/3}_{\epsilon\to 0^+}, \tag{23}$$

where

$$\begin{aligned} z(d) &= -A^{uu}\left[(2+3d)/6, d/2, d\right] - 2A^{uv}\left[(2+3d)/6, d/2, d\right] - \\ &\quad -A^{vv}\left[(2+3d)/6, d/2, d\right], \\ z_\epsilon(d,\epsilon) &= A^{uu}\left[(2+3d-2\epsilon)/6, (d-\epsilon)/2, d\right] + \\ &\quad A^{uv}\left[(2+3d-2\epsilon)/6, (d-\epsilon)/2, d\right]. \end{aligned} \tag{24}$$

Using the first order l'Hospital rule for calculation of the limits

$$\left.\frac{z_\epsilon}{\epsilon}\right|_{\epsilon\to 0^+} = \left.\frac{\partial z_\epsilon}{\partial \epsilon}\right|_{\epsilon\to 0^+}$$

can be calculated. This relation follows from the fact that for $\epsilon \to 0$ the mean energy transfer $\langle T_{ij}\rangle \sim \epsilon$ vanishes [16] in the inertial interval of wavenumbers. The above defined $z(d)$ function presents the following properties

$$z(d) = \begin{cases} \text{is negative for } d > d_* = 2.55695 \\ \text{is positive for } 2 < d < d_* \end{cases}, \tag{25}$$

and

$$\left.\frac{z_\epsilon}{\epsilon}\right|_{\epsilon\to 0} \text{ is positive for } d > 2.$$

Then the solution (17) of the equations (13) and (14) for $d = 3$ (with Kolmogorov indices) leads to non-physical (negative) c_0 and C_k. From many examples we know that curious behavior mentioned above, can be nothing but an artifact of the method, which should not have any physical significance. Nevertheless, the aspect of C_k positiveness [see eq.(25) and table 1] should have the central role in preliminary reformulation of theory.

The partial answers on question of destabilization we try to found in the literature. The crossover dimension $d = 8/3 \sim 2.67$ bellow which intermittency disappears and Kolmogorov theory becomes exact had previously been suggested by Nelkin [17]. At this point of discussion a question had been raised: Could the theory highly simplified by homogeneity and isotropy hypothesis describe the intermittent system? Even Kolmogorov [18] tried to explain the intermittency effect connected with inhomogeneous redistribution of energy dissipation in the cascade process on the phenomenological level. Intermittency models generally correspond to fractal description of turbulence, which opens an area for alternative geometrical formulation. The historically significant theoretical explanation of intermittency which is called in the literature the β model has been proposed [19].

In the explanation presented here the redistribution of turbulence energy among modes is characterized by certain fractal dimension d_F which is a global measure of system spatial activity.

In the present work we show that there could be some close relationship between our model stabilization and fractal aspects of turbulence. Here we will try to bring together fractal and maximum randomness concept. The relationship between turbulence and fractality studied by Mandelbrot [20] has become increasingly important in the past decade when the turbulence investigators have become concerned with how this formalism employ on the phenomenological level of turbulence describtion. The monofractal and multifractal approach based on more complex scaling form has become a paradigm to describe a large variety of complex systems. But the traditional developments in this field of interest link mainly phenomenology and experiment for the present. The direct link between the Navier Stokes equation (which is defined for integer dimensions only) and fractal nature of turbulence is still rather unclear, although there is no doubt that simulations made on massive computers show the roots of fractality which can be found in strongly nonlinear Navier Stokes dynamics. In this we introduce an argument different, to the our knowledge, from other phenomenology, although in another context and with many still open aspects. We have consider turbulent system is characterized by the occurrence of very complex spatial structures which can be described by a new order parameter - d_F. The analytic continuing of Feynman diagrams in Euclidean dimension d allows in principle calculate them not only for the whole $\mathbf{k} - space$ with $d^d\mathbf{k}$ measure, but also over the "smaller objects" with measure $k_d^{d-d_F}\, d^{d_F}\mathbf{k}$ (where k_d is dissipative microscale) of dimension $d_F < d$ [20]. For each d the formal correspondence rule φ (is unknown function) can be constructed

$$\varphi : \qquad d \rightarrow d_F = \varphi(d) \ , \tag{26}$$

where φ is particularly determined by the constraint

$$C_k(d_F) = C_k(\varphi(d)) > 0 \ . \tag{27}$$

It is clear that reformulation (26) and (27) is not satisfactory, especially in connection with the following arbitrariness: in the study limited to inertial range of spectra there are no knowledge or evident reasons how to fix the free parameter - d_F to its physical value. Therefore, at this stage of theory only reverse determination from known experimental values of C_k is possible.

For comparison with experiment it is natural to restrict the following considerations on the $d = 3$ case. Our theoretical expectations on the fractality as a source of turbulence stability are correct in the context of predictions [13] and [19] suggested the regions of largest dissipation in homogeneous turbulence are confined to a fractal set with fractal dimension $2 < d_F = \varphi(3) < d = 3$. For experimentally acceptable values of C_k [see table 1] we have found the fractal dimensions $d_F \simeq 2.5$ close to [13] and [19].

4 Conclusion

The statistical model of strongly developed homogeneous isotropic turbulence of incompressible fluid in d-dimensional case has been investigated. This model has been deduced as a consequence of the maximal randomness principle of velocity pulsations utilizing energy spectral flux stationarity condition. The important model property is absence of infrared and ultraviolet divergences in self-consistent equations. This fact is closely related to energy conservation law. The main prediction of the presented model is value of Kolmogorov constant dependent on Euclidean dimension. The negative value of this constant for $d = 3$ (the scaling solution becomes to be unstable) and positive values near $d \simeq 2.5$ led us to speculations about structural connections of fractal geometry and field-theoretical models of turbulence.

We have greatly benefit from discussions with Dr. Altaisky from JINR Dubna. The authors (M.H. and M.S.) are grateful to D.I.Kazakov and to director D.V.Shirkov for hospitality at the Laboratory of Theoretical Physics, Jinr, Dubna.

This work was supported by Fundamental Research Russian Fund, International Scientific Fund (grant R-63000) and by Slovak Grant agency for science (grant 2/550/93).

References

[1] Wyld H.W. *Ann.Phys.* **14** (1961) 143.

[2] De Dominicis C., Martin P.C. *Phys.Rev.A* **29** (1979) 419.

[3] D.Forster, D.R. Nelson, M.J. Stephen, *Phys.Rev.A* **16** (1977) 732.

[4] Yakhot V. *Phys.Rev.* **A23** (1981) 1486.

[5] Adzhemyan L.Ts., Vasil'ev A.N., Pis'mak Yu.M. *Teor. Mat. Fiz.* (Sov.J.) **57** (1983) 268.

[6] Edwards S.F., McComb W.D. *J.Phys.* **A2** (1969) 157.

[7] Yoshisawa A. *J. Phys. Soc. Japan* **40** (1976) 1498.

[8] Belinicher V.N., L'vov B.S. *Zh.Eksp.Teor.Fiz.*(Sov.J.) **93** (1987) 533.

[9] Migdal A.A. *Mod.Phys.Lett.* **A6** (1991) 1023.

[10] Adzhemyan L.Ts., Nalimov M.Yu. Preprint HU-TFT-91-65 Helsinki, *Teor. Mat. Fiz.* (Sov.J.) **91** (1992).

[11] Castaing B. *J. Phys.* (France) **50** (1989) 147.

[12] Castaing B., Gagne Y., Hopfinger E.J. *Physica D* **46** (1990) 177.

[13] Levich E. *Physics Reports* **151** N.3, N.4 (July 1987), pp.149-171.

[14] Kraichnan R.H. *J. Fluid Mech.* **5** (1959) 497.

[15] Fournier J.D., Frisch U. *Phys.Rev. A* **17** (1978) 747.

[16] S.A. Orszag , in *Fluid Dynamics*, edited by R. Balian and J.-L. Peube (Gordon and Breach, New York, 1977), pp. 235-374.

[17] Nelkin M. *Phys.Rev.A* **9** (1974) 388.

[18] Kolmogorov A.N. *J. Fluid Mech.* **13** (1962) 82.

[19] Frisch U., Sulem P.L., Nelkin M. *J. Fluid Mech.* **87** (1978) 719.

[20] Mandelbrot B.B *The fractal Geometry of Nature* (Freeman, New York 1983).

Received by Publishing Department
on August 9, 1994.