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THEORY OF THE RANDOMLY FORCED MAGNETOHYDRODYNAMIC TURBULENCE. DOUBLE EXPANSION FROM TWO DIMENSIONAL THEORY

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1 Introduction

In recent years much interest has been focused to the applications of the renormalization group (RNG) techniques which turned out to be so fruitful in critical phenomena theory [1]. At sufficiently large Reynolds numbers the statistical theory can be used to provide an account of the properties of developed turbulence. Since 1977 several authors [2, 3, 4] have applied the RNG method for the investigation of the fully developed turbulence of incompressible continuum described by the Navier-Stokes equation for velocity fluctuation field $\vec{v}$ under the influence of external Gaussian random force $\vec{f^v}$. In this field of interest the RNG is powerfull, general and systematic method achieving remarkable results with a minimum of phenomenological inputs. The principal existence of the stable fixed point (FP) corresponding to celebrated Kolmogorov 1941 scaling, actually observed for a number of experimental and simulational studies, was established using the RNG. Later a large effort has been put into the extension of RNG techniques on a computational problems of stochastic magnetohydrodynamics (MHD) describing the statistics of electrically conducting incompressible flows.

The randomly forced stationary MHD turbulence studied here is described by well known deterministic MHD equations for velocity and magnetic field $\vec{b}$, which are driven by external hydrodynamic $\vec{f^v}$ and magnetic $\vec{f^b}$ Gaussian forces. The RNG calculations based on the $\epsilon-$ expansion [5, 6] commonly predict the existence of two stable FP corresponding to kinetic (Kolmogorov) and magnetic universal critical regimes. For the purpose to perform RNG it is convenient to introduce *one particle irreducible Green functions* Γ These functions, which are defined in usual way in [9] provide a usefull tool for study of RNG problems since they considerably diminish the number of terms in a perturbation series.

As was shown in [6] only the following one particle irreducible Green two point functions: $\Gamma^{v'v}(\vec{k})$, $\Gamma^{vv'}(\vec{k})$, $\Gamma^{b'b}(\vec{k})$, $\Gamma^{bb'}(\vec{k})$ and three point $\Gamma^{v'bb}(\vec{k})$ functions ($|\vec{k}| = k$ is wave number) possess the superficial ultraviolet divergences in 3-d stochastic MHD. In this labeling convention $\vec{v}'$ and $\vec{b}'$ are certain auxiliary vector incompressible fields which are consequence of initial stochastic problem transformation into the quantum field formulation.

In the investigation Ronis has proposed the double expansion with simultaneous deviation $2\epsilon = d - 2$ from the spatial dimension d, and also deviation $\delta = 2 - \lambda$ (following notation [8] δ correspond to usual ϵ) according from the powerlike form of the correlation function $\langle|\vec{f^v}(\vec{k})|^2\rangle \sim k^{2\lambda-d} = k^{2-2\epsilon-2\delta}$ (we use the same notation for the fields and its Fourier components). The investigation [7] devoted to purely Navier-Stokes case is based on the fact that at $d = 2$ a new class of divergent functions $\Gamma^{v'v'}(\vec{k})$ occur, according to which the final prediction for averaged energy dissipation composite operator is nontrivial. Of special relevance to the following investigation is the subsequent paper of Honkonen and Nalimov [8]. They argue that nonlocal term $\vec{v}'(\vec{k})\, k^{2-2\delta-2\epsilon}\, \vec{v}'(\vec{k})$ cannot be renormalized multiplicatively because the counterterms obtained from the diagrams of corresponding theory are always local in space and time. The paper [8] contains not only criticism of the incorrectly applied nonlocal kernel renormalization, but also claim the give constructive natural proposal how to correctly remove ultraviolet divergences by means of *analytical counter term* $\vec{v}'(\vec{k})\, k^2\, \vec{v}'(\vec{k})$.

Inspired by the study [8] we present here an generalized treatment of the stochastic MHD where the analogous nonanalitic term $\vec{b}'(\vec{k})\, k^{2a\lambda-d+4(1-a)}\, \vec{b}'(\vec{k})$ (free parameter a controls the power form of magnetic forcing) requires the introduction of the additional analytical counter term proportional to $\vec{b}'(\vec{k})\, k^2\, \vec{b}'(\vec{k})$ into the action.

The following section we start from the functional formulation of the turbulence problem, which is suitable for the analysis on the RNG basis. In the further section using standard Feynman diagram technique, the Wilson's formulation of the RNG flows [1] under length scales changes in coupling constant space, the formulation of universality, fixed points and scaling have been extended to the stochastic MHD. Finally, the conclusions are presented.

2 Functional model

Our approach is the same as that applied by [6], where the formal equivalence of the stochastic MHD theory and quantum field theory has been recognized. Knowing the MHD equations and statistical properties of the Gaussian random forces in coordinate $\vec{x}, t$ space

$$\begin{aligned}
\langle \vec{f}^{v}(\vec{x},t)\rangle &= 0, \quad \langle \vec{f}^{b}(\vec{x},t)\rangle = 0, \qquad (1)\\
\langle f_j^v(\vec{x}_1,t_1) f_s^v(\vec{x}_2,t_2)\rangle &= g_{v10}\, u_0\, {\nu_0}^3\, D_{js}\left[\vec{x}_1,t_1,\vec{x}_2,t_2,d,\lambda\right]\\
\langle f_j^b(\vec{x}_1,t_1) f_s^b(\vec{x}_2,t_2)\rangle &= g_{b10}\, u_0^2\, {\nu_0}^3\, D_{js}\left[\vec{x}_1,t_1,\vec{x}_2,t_2,d,a\lambda+2\,(1-a)\right],\\
\langle f_j^v(\vec{x}_1,t_1) f_s^b(\vec{x}_2,t_2)\rangle &= 0,
\end{aligned}$$

where the kernel

$$D_{js}\left[\vec{x}_1,t_1,\vec{x}_2,t_2,d,z\right] = \delta(t_1-t_2)\int \frac{d^d\vec{k}}{(2\pi)^d}\, P_{js}(\vec{k})\, \frac{e^{\left[i\vec{k}(\vec{x}_1-\vec{x}_2)\right]}}{k^{d-2z}},$$

with transverse second-rank projector $P_{js}(\vec{k}) = \delta_{js} - k_j k_s/k^2$ (ensuring incompressibility required by $\nabla.\vec{f}^v = 0$ and $\nabla.\vec{f}^b = 0$) all dynamics of the MHD system is governed by the field theoretical action

$$\begin{aligned}
S \;=\; & \frac{1}{2}\int d^d\vec{x}_1\, dt_1\, d^d\vec{x}_2\, dt_2\, \left\{ g_{v10}\, u_0\, {\nu_0}^3\, v_j'(\vec{x}_1,t_1)\; D_{js}\left[\vec{x}_1,t_1,\vec{x}_2,t_2,d,\lambda\right] v_s'(\vec{x}_2,t_2) + \right.\\
& \left. g_{b10}\, u_0^2\, {\nu_0}^3\, b_j'(\vec{x}_1,t_1)\, D_{js}\left[\vec{x}_1,t_1,\vec{x}_2,t_2,d,a\lambda+2\,(1-a)\right]\, b_s'(\vec{x}_2,t_2) \right\} + \\
& \int d^d\vec{x}\, dt \left\{ -\frac{1}{2}\, u_0\, \nu_0^3\, \left(g_{v20}\, v_j'\, \nabla^2 v_j' + u_0\, g_{b20}\, b_j'\, \nabla^2\, b_j' \right) + \right.\\
& \left(-\frac{\partial v_j}{\partial t} + \nu_0\, \nabla^2 v_j - v_s \frac{\partial v_j}{\partial x_s} + b_s \frac{\partial b_j}{\partial x_s} \right)\, v_j' \, +
\end{aligned}$$

$$+ \quad \left(-\frac{\partial b_j}{\partial t} + u_0\,\nu_0 \nabla^2 b_j + b_s \frac{\partial v_j}{\partial x_s} - v_s \frac{\partial b_j}{\partial x_s}\right)\, b'_j \; \Bigg\} \tag{2}$$

which maps the classical Langevin stochastic formalism into the quantum field problem. It is familiar from before that action containing nonanalitic terms proportional to constants g_{v10}, g_{b10} need introduction of two new constants g_{v20} and g_{b20}. All these dimensional constants controlling the amount of randomly injected energy (2) play role of the coupling constants in the perturbative expansion. Their universal values have been determined after the parameter δ was choosen to give the desired power form of forcing.

The factors $\nu_0^3 u_0$ and $\nu_0^3 u_0^2$ containing "bare" (also kinetic or molecular) viscosity ν_0 and "bare" magnetic inverse Prandtl number u_0 have been extracted for convenience of calculations.

The most important measurable quantities in the study of a fully developed turbulence are considered to be the statistical objects represented by correlation and response functions of the fields. They are equivalent to functional averages - Green functions commonly expressed as a terms of Taylor series in A's about $A = 0$ by means of generating functional

$$G\left(\vec{A}^v, \vec{A}^{v'}, \vec{A}^b, \vec{A}^{b'}\right) = \int \mathcal{D}\vec{v}\mathcal{D}\vec{v}'\mathcal{D}\vec{b}\mathcal{D}\vec{b}'\, e^{\left[S + A_j^v v_j + A_j^{v'} v'_j + A_j^b b_j + A_j^{b'} b'_j\right]}\,, \tag{3}$$

where A_j are source fields, $\mathcal{D}\vec{v}\mathcal{D}\vec{v}'\mathcal{D}\vec{b}\mathcal{D}\vec{b}'$ denotes the measure of functional integration. One can derive unlimited number of identities for Green functions from this expression, so the developed turbulence problem can be formulated such as the calculation of functional integral. Such functional formulation is advantageous since the Green functions of the Fourier-decomposed stochastic MHD can be calculated by means of Feynman diagrammatic technique.

3 Renormalization in one loop order, fixed points

Before discussing the main results, we summarize the salient points of the RNG procedure which we have used. Details of applied RNG procedure and connected perturbative techniques have been described elsewhere [9].

The model (2) is renormalizable by the standard power-counting rules, and for limits $\epsilon \to 0, \delta \to 0$, $\lambda \to 2$, $d \to 2$ possesses the ultraviolet divergences which are present in one-particle irreducible Green functions $\Gamma^{vv'}$, $\Gamma^{v'v}$, $\Gamma^{bb'}, \Gamma^{b'b}, \Gamma^{b'b'}, \Gamma^{v'v'}$ and vertex function $\Gamma^{v'bb}$. Now, we turn out our attention to the removal of these divergences.

In the frame of the quantum field RNG the computing of Feynman diagram includes: association vertexes [nonquadratic terms in (2)] with the diagram edges, association of the free propagators [corresponding to the Fourier transform of quadratic part of (2)] given by

$$
\begin{aligned}
\Delta_{js}^{vv'}(\vec{k},\omega) &= \Delta_{js}^{v'v}(-\vec{k},-\omega) = \frac{P_{js}(\vec{k})}{-i\omega + \nu_0\, k^2}\,, \\
\Delta_{js}^{bb'}(\vec{k},\omega) &= \Delta_{js}^{b'b}(-\vec{k},-\omega) = \frac{P_{js}(\vec{k})}{-i\omega + u_0\,\nu_0\, k^2}\,, \\
\Delta_{js}^{vv}(\vec{k},\omega) &= u_0\,\nu_0^3\, k^2\, \frac{g_{v10}\, k^{-2\delta-2\epsilon} + g_{v20}}{|-i\omega + \nu_0\, k^2|^2}\, P_{js}(\vec{k})\,, \\
\Delta_{js}^{bb}(\vec{k},\omega) &= u_0^2\,\nu_0^3\, k^2\, \frac{g_{b10}\, k^{-2a\delta-2\epsilon} + g_{b20}}{|-i\omega + u_0\,\nu_0\, k^2|^2}\, P_{js}(\vec{k})\,,
\end{aligned}
\tag{4}
$$

with superscripts vv', bb', $v'v$, $b'b$, bb and vv refering to the notation of internal diagram lines. Having in mind that we need to know only divergences and are not interested in the finite parts of the one loop diagrams, we perform the integration over internal momentum space with noninteger dimension $2+2\epsilon$ (dimensional regularization introduced by the shift 2ϵ) and integration over internal frequency (using residuum theorem), and separation the divergent (for $\epsilon \to 0, \delta \to 0$) $1/\epsilon\,, 1/\delta\,$, $1/(\,2\delta+\epsilon)$ contributions according to the minimal subtraction scheme [10].

After the ultraviolet divergences have been removed, the continuation back to initial "physical" values $\epsilon_p \to 1/2, \delta_p \to 2,\ \lambda_p \to 0,\ d \to 3$ is possible. The parameters λ_p and ϵ_p produce desired stochastic energy supply of 3-d system from infrared range of k.

The ultraviolet divergences can be removed by adding suitable counter terms into the basic action S_B which is obtained from (2) replacing

$$
S \to S_B = S\left(e_0 \to e\mu^{[e]}\right)\,,\ e \in (g_{v1}, g_{v2}, g_{b1}, g_{b2}, u, \nu)
\tag{5}
$$

where μ is a scale setting parameter having wave number dimension, the renormalized parameters depending on μ are labeled as e. We see that all parameters $g_{v10}, g_{v20}, g_{b10}, g_{b20}$ having nonzero wave number dimensions $[e]$: $[g_{v10}] = \mu^{2\delta}$, $[g_{v20}] = [g_{b20}] = \mu^{-2\epsilon}$, $[g_{b10}] = \mu^{2a\delta}$ (calculated from naive dimensional analysis) are transformed to undimensional parameters $g_{v1}, g_{v2}, g_{b1}, g_{b2}$. The form of S_B implies the additional counter term part

$$
\begin{aligned}
\delta S \;=\; \int d\vec{x}\, dt\, \big[&\nu\,(1-Z_1)\, v_j'\nabla^2 v_j + u\nu\,(1-Z_2)\, b_j'\nabla^2 b_j + \\
&\frac{1}{2}(\,Z_4-1)u\nu^3 g_{v2}\,\mu^{2\delta}\, v_j'\nabla^2 v_s' + \frac{1}{2}(Z_5-1)\, u^2\nu^3 g_{b2}\,\mu^{-2\epsilon}\, b_j'\nabla^2 b_s' + \\
&(1-Z_3)\, v_j'\left(b_s\,\frac{\partial b_j}{\partial x_s}\right)\big]\,,
\end{aligned}
\tag{6}
$$

determined such as to cancel the surface ultraviolet divergences. Within the ultraviolet renormalization the divergences appearing in form of Laurent series in poles $1/\epsilon$, $1/\delta$ and $1/(2\delta+\epsilon)$ are contained in the constants Z_1, Z_2, Z_4, Z_5 renormalizing the "bare" parameters e_0 and constant Z_3 renormalizing fields $\vec{b}, \vec{b}'$. The remaining fields $\vec{v}', \vec{v}$ are not renormalized due to the Galilean invariance of the model (2).

The counter term part (6) have the same form as the terms of the action (2). ultraviolet divergences may be eliminated by redefining the parameters of the original theory according to (5) which is the essential property needed to apply RNG. Renormalized Green functions are expressed in terms of the renormalized parameters

$$
\begin{aligned}
g_{v1} &= g_{v10}\,\mu^{-2\delta}\,Z_1^2 Z_2, \quad g_{v2} = g_{v20}\,\mu^{2\epsilon}\,Z_1^2 Z_2 Z_4^{-1}, \\
g_{b1} &= g_{b10}\,\mu^{-2a\delta}\,Z_1 Z_2^2 Z_3^{-1}, \quad g_{b2} = g_{b20}\,\mu^{2\epsilon}\,Z_1 Z_2^2 Z_3^{-1} Z_5^{-1}, \\
\nu &= \nu_0\,Z_1^{-1}, \quad u = u_0\,Z_2^{-1} Z_1
\end{aligned} \tag{7}
$$

incoming to renormalized action $S_R = S_B - \delta S$ connected with action (2) by the relations of multiplicative renormalization

$$
S_R(\vec{v}, \vec{b}, \vec{v}', \vec{b}'\, e) = S(\vec{v}, \vec{b}Z_3^{1/2}, \vec{v}', \vec{b}'Z_3^{-1/2}, e_0).
$$

The action S_R and related Green funcions dependent on renormalized parameters $e(\mu)$ lead to the theory without ultraviolet divergences. The RNG is mainly concerned with predicting anomalous dimensions γ_j by use of $\beta-$ functions, both defined via operations

$$
\gamma_j = \mu\frac{\partial \ln Z_j}{\partial \mu}|_0, \quad \beta_g = \mu\frac{\partial g}{\partial \mu}|_0, \text{ with } g \in \{g_{v1}, g_{v2}, g_{b1}, g_{b2}, u\}, \tag{8}
$$

where subscript "0" refers that the partial derivatives are taken at fixed values of e_0.

Using RNG routine mentioned briefly the anomalous dimensions $\gamma_j(g_v, g_b, g_{v2})$ can be extracted from a one loop diagrams.

Definitions (8) and expressions (7) for $d = 2 + 2\epsilon$ imply

$$
\begin{aligned}
\beta_{gv1} &= g_{v1}\,(-2\delta + 2\gamma_1 + \gamma_2), \quad \beta_{gv2} = g_{v2}\,(2\epsilon + 2\gamma_1 + \gamma_2 - \gamma_4), \\
\beta_{gb1} &= g_{b1}\,(-2\,a\delta + \gamma_1 + 2\gamma_2 - \gamma_3), \quad \beta_{gb2} = g_{b2}\,(2\epsilon + \gamma_1 + 2\gamma_2 - \gamma_3 - \gamma_5) \\
\beta_u &= u\,(\gamma_1 - \gamma_2).
\end{aligned} \tag{9}
$$

The partial derivatives with respect to μ in (8) generate ϵ and $\delta-$ dependent terms which cancel some "mixed" poles ϵ, δ, $2\delta + \epsilon$, as a consequence finite result can be written

$$
\begin{aligned}
\gamma_1 &= \frac{1}{32\pi}(u\,g_v + g_b), \quad \gamma_2 = \frac{1}{8\pi}\frac{g_v - g_b}{u + 1}, \\
\gamma_3 &= \frac{1}{16\pi}(g_b - g_v), \quad \gamma_4 = \frac{1}{32\pi}\frac{u\,g_v^2 + g_b^2}{g_{v2}},
\end{aligned} \tag{10}
$$

where substitutions $g_v = g_{v1} + g_{v2}$, $g_b = g_{b1} + g_{b2}$ are used. The calculations show us that divergences of $\Gamma^{b'b'}$ do not occur in one loop approximation (and therefore $\gamma_5 = 0$) directly for $d \to 2$.

The critical behaviour and universality of MHD statistics stems from the existence of the stable FP. The continual RNG transformation (flow) is operation linking the invariant parameters $\overline{g}(s)$ determined by the Gell-Mann Law equation

$$\frac{d\overline{g}(s)}{d\ln s} = \beta_g\left(\overline{g}(s)\right) \quad \text{with} \;\; \overline{g} \in \{\overline{g}_{v1}, \overline{g}_{v2}, \overline{g}_{b1}, \overline{g}_{b2}, \overline{u}\} \tag{11}$$

where scaling variable $s = k/\mu$, $s \in (0,1)$ parametrizes the RNG flow with initial conditions $\overline{g}|_{s=1} \equiv g$ (the critical behaviour correspond to infrared limit $s \to 0$) and the expression of the $\beta(\overline{g}(s))$ function is known in the framework of the ϵ, δ expansion [see Eqs.(10) and also (9)]. A FP $g^*(s \to 0)$ satisfies a system of equations $\beta_g(g^*) = 0$, while a stable FP weakly dependent on initial condition is defined in addition, by positiveness of the real part of matrix $(\partial\beta_g/\partial g)|_{g^*}$, or in other words the FP is stable if all trajectories in the neighbourhood domain approach the FP.

In comparison with traditional ϵ expansion, using double expansion some new facts occur. After the continuation $\epsilon_p \to 1/2, \delta_p \to 2, d \to 3$ we have obtained the nontrivial stable *kinetic* FP of RNG with universal inverse Prandtl number

$$u^* = \frac{\sqrt{17}-1}{2} \simeq 1.562 \;\; (\;\sim 1.393 \text{ in usual } \;\epsilon - \text{expansion})$$

and g^*_{v1} and g^*_{v2} determined as

$$g^*_{v1} = \frac{64\pi}{9\,u^*}\,\frac{\delta_p\,(2\delta_p + 3\epsilon_p)}{\delta_p + \epsilon_p}, \quad g^*_{v2} = \frac{64\pi}{9\,u^*}\,\frac{\delta_p^2}{\epsilon_p + \delta_p} \tag{12}$$

with zero couplings $g^*_{b1} = g^*_{b2} = 0$ and anomalous dimensions

$$\gamma_1^* = \gamma_2^* = \frac{2\delta_p}{3}, \quad \gamma_4^* = 2(\epsilon_p + \delta_p), \;\; \gamma_3{}^* = -\frac{4\delta_p}{3\,u^*} \simeq -1.707\,. \tag{13}$$

The region of the *kinetic* FP *stability* is limited by the parameter $a < 1.42$ (whereas $a < 1.15$ determines FP stability in the usual ϵ expansion). The anomalous dimensions $\gamma^*_{1,2,4}$ determined perturbatively exactly settle on the previously found values [5, 6], but γ^*_3, which has been determined up to order δ is signifficantly different from the value (-0.638) obtained earlier [5, 6, 11].

The important outcome of our recent paper [11] is the statement that in randomly driven MHD the Kolmogorov $k^{-11/3}$ scaling holds not only for the one time pair correlation function $\langle v_j(\vec{k})v_j(-\vec{k})\rangle$ but also for function $\langle b_j(\vec{k})b_j(-\vec{k})\rangle$. Now we recongnize the scaling properties in the case of double expansion.

The renormalized function $\langle b_j(\vec{k})b_j(-\vec{k})\rangle_R$ satisfies the basic equation of RNG

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_{\overline{g}}\beta_g\frac{\partial}{\partial g} - \gamma_1\nu\frac{\partial}{\partial\nu} + \gamma_3\right)\,\langle\, b_j(\vec{k})b_j(-\vec{k})\,\rangle_R = 0$$

which lies beyond perturbation theory and expresses the independence of the unrenormalized theory on the scaling parameter μ. The solution of this equation can be written in the form

$$\langle b_j(\vec{k})b_j(-\vec{k})\rangle_R = \nu^2\, k^{-2\epsilon}\, R\,(\overline{g}_{b1}\,(s))\,\exp\left\{\int_1^s \frac{dx}{x}\,(\,\gamma_3\,[\overline{g}\,(x)] - 2\gamma_1\,[\overline{g}\,(x)])\right\} \qquad (14)$$

where factor $\nu^2 k^{-2\epsilon}$ has been obtained from the zero order propagator by means of integral $\int d\omega \Delta^{bb}_{jj}(\vec{k},\omega)$, [see (4)]. The scaling function $R\,(\overline{g}_{b1})$ which can be computed as power series in $\overline{g}_{b1}(s)$ is up to one loop order proportional to $\overline{g}_{b1}\,(s)$. The importance of further analysis follow from the detail that the trajectory generated in the space of invariant couplings (11) has infrared asymptotics

$$\overline{g}_{b1}(s) \to s^{-\gamma_3^* + 2\delta(1-a)},$$

which may be calculated solving Eqs.(9,10,11) for linearized $\overline{g_{b1}}(s)$ around the fixed $\overline{g}_{b1} = g^*_{b1} = 0$. The knowledge of $s-$ dependence of $\overline{g}_{b1}$ gives

$$R\,(\overline{g}_{b1})\;\sim\;\overline{g}_{b1}\,(s) = s^{2\delta\,(1-a)-\gamma_3^*} = s^{2\delta\,(1-a)}\,\exp\left[-\int_1^s \frac{dx}{x}\,\gamma_3^*\right] \quad \text{for} \quad s \to 0\,. \qquad (15)$$

and consequently after setting of just obtained $R(\overline{g}_{b1})$ into Eq.(14) we have

$$\langle\, b_j(\vec{k})b_j(-\vec{k})\,\rangle_R \sim \nu^2\, k^{-2\epsilon}\, s^{2\delta\,(1-a)}\exp\left\{\int_1^s \frac{dx}{x}\,(\,\gamma_3\,[\overline{g}\,(x)] - \gamma_3^* - 2\gamma_1\,[\overline{g}\,(x)\,])\right\}\,.$$

In the infrared limit the subtraction $\gamma_3(\overline{g})-\gamma_3^*$ vanishes and in the last expression only the dependence on the anomalous dimension γ_1 remains. The correlation function is given by

$$\langle b_j(\vec{k})b_j(-\vec{k})\rangle_R \sim k^{-2\epsilon_p + 2\delta_p(1-a) - 4\delta_p/3} \quad \text{for} \;\; k \to 0. \qquad (16)$$

For the mostly acceptable value of free parameter $a = 1$ [results from FP stability and dimensional arguments in (2)], correlation function $\langle b_j(\vec{k})b_j(-\vec{k})\rangle$ is compatible with the standard Kolmogorov prediction $k^{-11/3}$ as it has been obtained earlier [5] in usual $\epsilon-$ expansion.

From (2) we see, that in contrast to earlier development of RNG on stochastic MHD, there exist different ways how to introduce a forcing. Because the renormalizable MHD action contains the addition of terms and counter terms proportional to field structures $\vec{v}'\vec{v}'$ and $\vec{b}'\vec{b}'$, our simple suggestion is to lump these terms together into the *effective forcing*. In the reverse order formulation the renormalizable action (2) can be constructed directly starting from the reformulated variant containing sum

$$S = \int d^d\vec{k}_1\, d\omega_1\, d^d\vec{k}_2\, d\omega_2\, \left[\vec{v}'\,\langle \vec{f}^v \vec{f}^v\rangle_{\text{eff}}\,\vec{v}' + \vec{b}'\,\langle \vec{f}^b \vec{f}^b\rangle_{\text{eff}}\,\vec{b}'\right] + \text{terms linear in fields }\; \vec{v}',\, \vec{b}'$$

with random force pair correlator

$$
\begin{aligned}
\langle f_j^v(\vec{k}_1,\omega_1)\, f_s^v(\vec{k}_2,\omega_2)\rangle^*_{\rm eff} &= u^*\, \nu^3\, \mu\, P_{js}(\vec{k}) \;\times \\
& \qquad \delta\left(\vec{k}_1+\vec{k}_2\right)\delta\left(\omega_1+\omega_2\right)\left[g^*_{v1}\left(\frac{\mu}{k}\right)^3 + g^*_{v2}\left(\frac{k}{\mu}\right)^2\right], \\
\langle f_j^b(\vec{k}_1,\omega_1) f_s^b(\vec{k}_2,\omega_2)\rangle^*_{\rm eff} &= 0\,, \qquad (17)
\end{aligned}
$$

representing more detailed intrinsic statistical picture of forcing [here $<>^*$ denotes the Fourier transform of correlation function is taken in the kinetic FP (12)]. There are now two principal - low ($k << k_*$) and high-wave-number scale kinetic forcings separated by transition region near the value $k_* = \mu\, [g^*_{v1}/g^*_{v2}]^{1/5} \simeq 1.22\,\mu]$ close to the ultraviolet cut off μ. According to (17) a dominant contribution to high-wave-number forcing is from wave numbers near k_* and these wave numbers are also responsible for the initialization of the *inverse cascade*, where the inverse energy transfer is maintained by the k^2 forcing, since the dissipation of energy in the inertial range is not assumed. In the language of classicall hydrodynamics it corresponds to appearence of a large eddies convected by active small eddies with size close to $2\pi/k_*$. From the RNG standpoint, more complex action (2) seems to be a paradigm how to describe the inverse cascade mechanism. Note, that in conventional closure theories the inverse cascade is accompanied with negative modified viscosity.

The results which has been obtained for *magnetic* FP (where system tends to $u^* = 0$) are fundamentally different from those of [5, 6] and we have reached the surprising conclusion that double expansion leads to *unstable magnetic regime*.

The *unstable* fixed points of the presented model have systematically been determined:

(i.) $g^*_{v1}=0,\ g^*_{v2}=0,\ g^*_{b1}=0,\ g^*_{b2}=0,$ (u^* is free parameter),

(ii.) $g^*_{v1}=0,\ g^*_{v2}=-16\epsilon_p\pi,\ g^*_{b1}=0,\ g^*_{b2}=0,\ u^*=0,$

(iii.) $g^*_{v1}=16\delta_p\pi,\ g^*_{v2}=0,\ g^*_{b1}=0,\ g^*_{b2}=0,\ u^*=0,$

(iv.) $g^*_{v1}=0,\ g^*_{v2}=-64\epsilon_p\pi,\ g^*_{b1}=0,\ g^*_{b2}=-64\epsilon_p\pi,\ u^*=0,$

(v.) $g^*_{v1}=0,\ g^*_{v2}=16\epsilon_p\pi/11,\ g^*_{b1}=0,\ g^*_{b2}=96\epsilon_p\pi/11,\ u^*=0,$

(vi.) $g^*_{v1} = \left[8(1-\sqrt{17})\delta_p(2\delta_p+3\epsilon_p)\pi\right]/(9(\delta_p+\epsilon_p)),$
$g^*_{v2} = (8-\sqrt{817})\delta_p^2\pi/(9(\delta_p+\epsilon_p)),$
$g^*_{b1}=0,\ g^*_{b2}=0,\ u^*=-(1+\sqrt{17})/2,$

(vii.) $g^*_{v1} = \left[(11\delta_p^2+24\delta_p\epsilon_p+4\epsilon_p^2)\pi\right]/(\delta_p+\epsilon_p),$
$g^*_{v2} = \left[(5\delta_p+2\epsilon_p)^2\pi\right]/(\delta_p+\epsilon_p),$
$g^*_{b1}=0,\ g^*_{b2}=8(5\delta_p+2\epsilon_p)\pi,\ u^*=0,$

(viii.) $g^*_{v1} = \left[-8(53\delta_p^3 + 156\delta_p^2\epsilon_p + 153\delta_p\epsilon_p^2 + 54\epsilon_p^3)\pi\right]/(27(\delta_p + \epsilon_p)^2)$,

$g^*_{v2} = \left[(504\delta_p + 432\epsilon_p - (32\delta_p^3)/(\delta_p + \epsilon_p)^2)\pi\right]/27$,

$g^*_{b1} = 0$,

$g^*_{b2} = \left[(336\delta_p + 288\epsilon_p - (32\delta_p^2)/(\delta_p + \epsilon_p))\pi\right]/9$,

$u^* = -7 - 6\epsilon_p/\delta_p$,

(ix.) $g^*_{v1} = [8\delta_p(53\delta_p - 162a\delta_p + 171a^2\delta_p - 54a^3\delta_p - 6\epsilon_p + 18a\epsilon_p)\pi]/(27(-1+a)(\delta_p+\epsilon_p))$,

$g^*_{v2} = \left[(504 + 32/(-1 + a) - 936a + 432a^2)\delta_p^2\pi\right]/(27(\delta_p + \epsilon_p))$,

$g^*_{b1} = [(336 + 32/(-1 + a) - 288a)\delta_p\pi]/9$, $g^*_{b2} = 0$, $u^* = -7 + 6a$, $a \neq 1$,

(x.) $g^*_{v1} = 0$,

$g^*_{v2} = \left[8(-38a\delta_p - 81\epsilon_p \pm 9(20a^2\delta_p^2 + 76a\delta_p\epsilon_p + 81\epsilon_p^2)^{1/2})\pi\right]/11$,

$g^*_{b1} = \left[16(-26a\delta_p - 45\epsilon_p \pm 5(20a^2\delta_p^2 + 76a\delta_p\epsilon_p + 81\epsilon_p^2)^{1/2})\pi\right]/11$,

$g^*_{b2} = 0$, $u^* = 0$,

(xi.) $g^*_{v1} = 36\delta_p\pi - 8a\delta_p\pi - \left[(-5 + 2a)^2\delta_p^2\pi)/(\delta_p + \epsilon_p)\right]$,

$g^*_{v2} = \left[(-5 + 2a)^2\delta_p^2\pi\right]/(\delta_p + \epsilon_p)$,

$g^*_{b1} = 8(5 - 2a)\delta_p\pi$, $g^*_{b2} = 0$, $u^* = 0$.

4 Conclusions

In the presented comparative study we have shown that RNG analysis applied to stochastic MHD is quite sensitive in many points to the type of expansion. The main results of a presented MHD version of the initial theory [8] are: (i) classification of the FP - we have found only one stable FP corresponding to $k^{-11/3}$ scaling, (ii) alternatively formulated description of the inverse cascade of energy, (iii) the Kolmogorov $k^{-11/3}$ scaling for the one time pair correlation function $\langle\vec{b}_j(\vec{k})\vec{b}_j(-\vec{k})\rangle$; the investigated asymptotic properties of velocity and magnetic field correlation functions are consistent with traditional three-dimensional analysis and Kraichnan's *hypothesis* [13] *about equipartition of energy among velocity and magnetic Fourier modes*, (iv) significantly new value of the inverse Prandtl number $(\sqrt{17} - 1)/2$ in the kinetic FP. Unfortunately only the higher order calculations or the appearence of another suitable calculable scheme should help us satisfactory clarify contradictions arising from some pathology of perturbative RNG in the turbulence theory.

The another importaint question is the relevance of the presented theory in physics. The conflicting results achieved using different expansional methods led us to a speculative deduction that double expansion applied here is physically close to description of the quasi two dimensional ($2 < d < 3$) turbulent flows. The realizability of such flows seems to be difficult although not impossible in view of some recent approaches in statistical theory and geometry: fractal and anisotropic systems show effectively behaviour of systems with noninteger dimensionality. This statement is also supported by the new value of the inverse Prandtl number which represents certain decrease of effective conductivity in a quasi two dimensional turbulent flows.

The fact that inverse cascade processess can be expected in nearly 2-d turbulence [12] perhaps supports the idea of the alternative and interesting association of inverse cascade mechanisms with description (17). This aspect, which is direct consequence of inclusion of more couplings and condition that 2-d stochastic MHD should be multiplicatively renormalizable, is absent in traditional formulation of the stochastic hydrodynamics.

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