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TE-POLARIZED SURFACE WAVES IN NONLINEAR MEDIUM WITH ABSORPTION

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Unusual properties of nonlinear sufrace waves propagating in plane layered structures and their application in nonlinear optical instruments gave rise to a great number of works devoted to their study (e.g., see surveys [1] based on 108 and 74 works respectively). But still the subject of surface wave propagation in nonlinear media with absorption has not been studied sufficiently. In this work, we consider the properties of TE-polarized waves ($\mathbf{E} = (0, E_y, 0)$, $\mathbf{H} = (H_x, 0, H_z)$), guided by the division boundary between two absorbing media. One of the media (in which $z < 0$) is supposed bo be linear by its properties and is characterized by permittivity ε_1 and conductivity σ_1. The other medium ($z > 0$) is nonlinear and complies with the Kerr law:

$$\varepsilon_2 = \varepsilon_{20} + \alpha\, |E_y|^2.$$

It is also supposed that this medium if self-focusing, i.e., $\alpha > 0$. The substance conductivity of this medium is equal to σ_2. Hereafter, the quantities pertaining to a linear medium will be denoted by index «1», and those pertaining to a nonlinear medium, by index «2». TE-polarized waves propagate along the axis x with Z-normal to the surface:

$$E_y(x, z, t) = \frac{1}{2} E_y(z)\, e^{i(\beta k_0 x - \omega t)},$$

where $k_0 = \frac{\omega}{c}$, ω is frequency, c is velocity of light, β is effective refraction index for guided waves [1]. From Maxwell's equations we can derive the following equation for $E_y(z)$, which is true for medium I:

$$\frac{d^2 E_y^{(1)}}{dz^2} - k_0^2 (\gamma_1 + i b_1)\, E_y^{(1)} = 0, \tag{1}$$

where $B_1 = \frac{4\pi\,\sigma_1}{\omega}$, $\gamma_1 = \beta^2 - \varepsilon_1$.

After solving eqaution (1), we have the following expression for $E_y^{(1)}(z)$:

$$E_y^{(1)} = E_0 \exp\left((0.5\gamma_1 + 0.5\,(\gamma_1^2 + b_1^2)^{0.5})^{0.5} k_0 z\right) \times$$
$$\times \exp\left(i\,(-\,0.5\,\gamma_1 + 0.5\,(\gamma_1^2 + b_1^2)^{0.5}\,(z - z_1)\right). \tag{2}$$

Here E_0 and z_1 are constants. As follows from (2), $E_y^{(1)} \to 0$ as $z \to -\infty$. For nonlinear medium II we shall have the following transport equation:

$$\frac{d^2 \widetilde{E}_y^{(2)}}{d\widetilde{z}^2} - (\gamma_2 - ib_2)\, \widetilde{E}_y^{(2)} + |\widetilde{E}_y^{(2)}|^2\, \widetilde{E}_y^{(2)} = 0, \tag{3}$$

where the following values are introduced:

$$\gamma_2 = \beta^2 - \varepsilon_2, \qquad b_2 = \frac{4\pi\,\sigma_2}{\omega}, \qquad \widetilde{z} = zk_0, \qquad \widetilde{E}_y^{(2)} = \sqrt{\alpha}\, E_y^{(2)}.$$

We shall have an approximate finite solution of this equation as $z \to +\infty$. Let us present a complex quantity $\widetilde{E}_y^2$ as

$$\widetilde{E}_y^{(2)} = \left|\widetilde{E}_y^{(2)}\right| e^{i\delta(\widetilde{z})} = S(\widetilde{z})\, e^{i\delta(\widetilde{z})}. \tag{4}$$

After inserting (4) into (3) we have the following system:

$$\begin{aligned} &S'' - \gamma_2 S + S^3 - S\Psi^2 = 0, \\ &S\Psi' + 2S'\Psi + b_2 S = 0, \end{aligned} \tag{5}$$

where $\Psi = \delta'$, the stroke (′) denotes a derivative from $\widetilde{z}$. If we denote $S' = u$, then $S'' = u'_S u$, $\Psi' = \Psi'_S u$. Now we can write down the system (5) as follows:

$$\begin{aligned} &u'_S u - \Psi^2 S - \gamma_2 S + S^3 = 0, \\ &\Psi'_S u S + 2u\Psi + b_2 S = 0. \end{aligned} \tag{6}$$

Now we shall successively introduce the following substitutes:

$$\Psi = \frac{\varphi}{S}, \qquad S^2 = \xi, \qquad V = u^2, \qquad \varphi = \sqrt{\kappa}$$

and write down the system (6) as follows:

$$\begin{aligned} &\xi V'_\xi - \kappa - \gamma_2 \xi + \xi^2 = 0, \\ &\xi\, \kappa'_\xi \sqrt{V} + \kappa\sqrt{V} + b_2 \xi\sqrt{\kappa} = 0. \end{aligned} \tag{7}$$

Deriving the function $\kappa(\xi)$ from the first equation of the system (7) and substituting it into the second equation of the system (7), we have the following equation for $V(\xi)$:

$$b_2^2\, \xi (V'_\xi - \gamma_2 + \xi) = V(\xi V''_{\xi\xi} + 2V'_\xi - 2\gamma_2 + 3\xi)^2. \tag{8}$$

Equation (8) can be simplified by substituting W for $V - \gamma_2\xi + \frac{\xi^2}{2}$:

$$b_2^2\xi W'_\xi = \left(W + \gamma_2\xi - \frac{\xi^2}{2}\right)(\xi W''_{\xi\xi} + 2W'_\xi)^2. \tag{9}$$

We shall search for the solution of equation (9) in the series

$$W = c_1\xi + c_2\xi^2 + c_3\xi^3 + c_4\xi^4 + c_5\xi^5 + \ldots . \tag{10}$$

We have taken into account the fact that deriving a soliton solution from equation (3) in this manner, where $b_2 = 0$ (nonabsorbing medium), leads to the condition $c_0 = 0$.

After substituting the series (10) into (9) we have the following:

$$((c_1 + \gamma_2)\xi + (c_2 - 0.5)\xi^2 + c_3\xi^3 + c_4\xi^4 + c_5\xi^5 + \ldots) \times$$

$$\times [4c_1^2 + 36c_2^2\xi^2 + 144c_3^2\xi^4 + 400c_4^2\xi^6 + 900c_5^2\xi^8 + \ldots +$$

$$+ 24c_1c_2\xi + 48c_1c_3\xi^2 + 80c_4c_1\xi^3 + 120c_5c_1\xi^4 + 144c_2c_3\xi^3 + 240c_2c_4\xi^4 +$$

$$+ 360c_2c_5\xi^5 + 480c_3c_4\xi^5 + 720c_3c_5\xi^6 + 1200c_4c_5\xi^7 + \ldots] =$$

$$= b_2^2\xi\,(c_1 + 2c_2\xi + 3c_3\xi^2 + 4c_4\xi^3 + 5c_5\xi^4 + \ldots).$$

Equating the coefficients at equal powers of ξ we have the following for c_i:

$$c_1 = 0.5(-\gamma_2 + (\gamma_2^2 + b_2^2)^{0.5}),$$

$$c_2 = c_1^2\,(14c_1^2 + 12c_1 - b_2^2)^{-1},$$

$$c_3 = \frac{12c_1c_2 - 60c_2^2c_1 - 36c_2^2\gamma_2}{52c_1^2 + 48\gamma_2c_1 - 3b_2^2},$$

$$c_4 = \frac{9c_2^2 - 18c_2^3 - 108c_2c_3 - 72c_2c_3\gamma_2 + 12c_1c_3}{42c_1^2 + 40c_1\gamma_2 - 2b_2^2}, \ldots .$$

Let us analyze the expression for the coefficients c_i. At $b_2 = 0$, all the coefficients $c_i = 0$, i.e., $W \equiv 0$. Then $V = \gamma_2\xi - 0.5\xi^2$, which holds true for a nonabsorbing medium. If $b_2 \neq 0$, then $c_i \not\equiv 0$ and $W \not\equiv 0$. It follows from the expres-

The condition for eigenvalues of the amplitude E_0 looks like

$$\alpha E_0^2 = \frac{\varepsilon_1 - \varepsilon_2}{0.5 + c_2}. \tag{17}$$

As $c_2 < 0.1$, this condition does not differ much from the corresponding condition for nonabsorbing media $\alpha E_0^2 = 2(\varepsilon_1 - \varepsilon_2)$ even in the extreme case of $b_2 >> \gamma_2$. At $b_2 = 0$ the coefficient $c_2 = 0$, so the condition (17) becomes identical to the condition for eigenvalues E_0 for nonabsorbing media.

Besides (17), from (16) we also have the following additional condition:

$$(\gamma_1^2 + b_1^2)^{0.5} = (\gamma_2^2 + b_2^2)^{0.5} + \frac{3c_2 - 0.5}{c_2 + 0.5}(\gamma_2 - \gamma_1). \tag{18}$$

At $b_1 = b_2 = 0$ this condition changes into the identity $0 \equiv 0$ and does not affect the solution of the problem. Considering the other extreme case of $b_i >> \gamma_i$ from the condition (18) we have $b_1 = b_2$. The condition where the value of b_1 is close to that of b_2 and there is a considerable difference between ε_1 and ε_2 seems to be quite possible [3]. In other cases, the identity (18) is the condition for the quantity β at which the TE-polarized surface wave is realized in the system under consideration.

So, when $b_i << \gamma_i$, from (18) we have the following expression:

$$\beta^2 = \frac{b_2^2 \varepsilon_1 - b_1^2 \varepsilon_2}{b_2^2 - b_1^2}. \tag{19}$$

The energy flux of guided surface waves is determined by the formula [1]:

$$P = \frac{\beta}{2c\mu_0} \int_{-\infty}^{+\infty} |E_y|^2 dz, \tag{20}$$

where μ_0 — the medium magnetic permeability. After inserting expersssions (2), (13) into (20) and taking into account (17) and the expression $\tanh(\nu\tilde{z} - \nu\tilde{z}_0) = \eta$ (derived from the boundary conditions), we have the following formula for the energy flux:

$$P = P_0 \beta \left[\frac{\varepsilon_1 - \varepsilon_2}{0.5 + c_2}(2\gamma_1 + 2(\gamma_1^2 + b_1^2)^{0.5})^{-0.5} + \frac{(c_1 + \gamma_2)^{0.5}}{0.5 - c_2} + \right.$$
$$\left. + \frac{(\gamma_1 + (\gamma_1^2 + b_1^2)^{0.5})^{0.5}}{\sqrt{2}(0.5 - c_2)} \right], \tag{21}$$

where $P_0 = \left(\frac{\varepsilon_0}{\mu_0}\right)^{0.5} (2\alpha\, k_0)^{-1}$ and ε_0 is the electron constant. In particular, when $\gamma_i >> b_i$ and the quantities b_1 and b_2 are of the same order, from expressions (19), (21) we have the following:

$$P = 4P_0 \beta \left(\frac{\varepsilon_1 - \varepsilon_2}{b_2 - b_1}\right)^{0.5} . \tag{22}$$

Let us consider the frequency dependence $P(\omega)$. For metals and, under certain conditions, for semiconductors we can write down the following [2,4]:

$$\varepsilon = 1 - \frac{\omega_P^2}{\omega^2 + \omega_0^2}, \qquad b = \frac{\omega_P^2 \omega_0}{(\omega^2 + \omega_0^2)\,\omega}, \tag{23}$$

where ω_P is the plasma of frequency, $\omega_0 = \frac{1}{\tau}$, τ — time of fading. Taking into account (23), from expressions (19), (22) we have the following functional dependence of energy on frequency:

$$P^2 = 16P_0^2 \left[1 + \omega_{P1}^2 \omega_{P2}^2 \frac{\omega_{P2}^2 \omega_{02}^2 (\omega^2 + \omega_{01}^2) - \omega_{P1}^2 \omega_{01}^2 (\omega^2 + \omega_{02}^2)}{\omega_{P1}^4 \omega_{01}^2 (\omega^2 + \omega_{02}^2)^2 - \omega_{P2}^2 \omega_{02}^2 (\omega^2 + \omega_{01}^2)^2}\right] \times$$

$$\times \frac{\omega_{P2}^2 (\omega^2 + \omega_{01}^2)\,\omega - \omega_{P1}^2 (\omega^2 + \omega_{02}^2)\,\omega}{\omega_{P2}^2 \omega_{02} (\omega^2 + \omega_{01}^2) - \omega_{P1}^2 \omega_{01} (\omega^2 + \omega_{02}^2)} . \tag{24}$$

If $\omega_{01} \sim \omega_{02} \sim \omega_0$, then formula (24) is simplified as follows:

$$P^2 = \frac{16P_0^2}{\omega_0} \left[\omega + \frac{V\omega}{\omega^2 + \omega_0^2}\right], \tag{25}$$

where

$$V = \frac{\omega_{P1}^2 \omega_{P2}^2}{\omega_{P1}^2 + \omega_{P2}^2}.$$

From formula (25) it follows that if the inequality $V > 8\omega_0^2$ is satisfied, then the function $P^2(\omega)$ has the local maximum at frequency

$$\omega_{\max} = \left(- \omega_0^2 + \frac{V}{2} - (0.25V^2 - 2V\omega_0^2)^{0.5}\right)^{0.5}$$

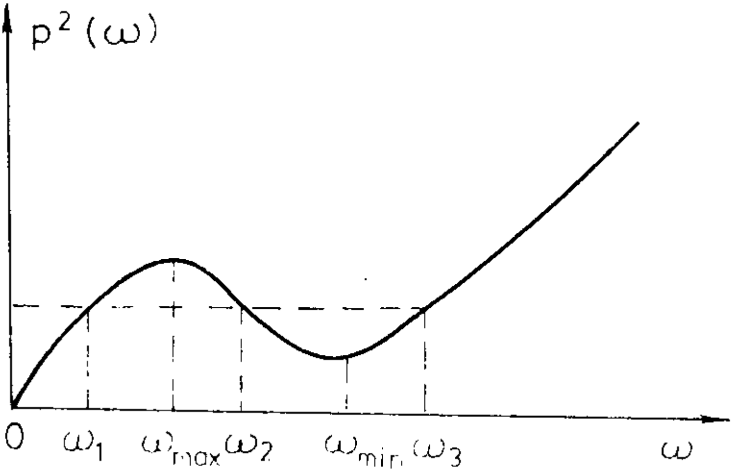

Fig. The character of the dependence $P^2(\omega)$

and the local minimum at frequency

$$\omega_{\min} = \left(-\omega_0^2 + \frac{V}{2} + (0.25V^2 - 2V\omega_0^2)^{0.5} \right)^{0.5}.$$

Thus, there exist 3 frequencies corresponding to one and the same energy flux value for $P(\omega) \leq P_{\max}$. This shows the possibility of choosing an optimum frequency regime from 3 possible regimes at the given value of P.

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