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BRANCH STRUCTURE OF THE BOSE-CONDENSATE EXCITATIONS SPECTRUM

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The recent experiments [1] on the neutron scattering in superfluid He-4 have shown the branch structure of the narrow peak of its excitation spectrum. The present paper develops the Bogolubov theory of Bose-condensation [2] in order to explain the observed picture.

The Bogolubov Hamiltonian of the classical condensate with amplitudes a, a^* and over-condensate ($k \neq 0$) bosons b_k, b_k^+ is defined as

$$H = \sum_{k\neq 0}\left[\Omega_k b_k^+ b_k + \frac{g_k}{2V}(b_k^+ b_{-k}^+ a^2 + b_k b_{-k} a^{*2} + 2b_k^+ b_k |a|^2) + \frac{g_0}{V} b_k^+ b_k |a|^2\right] \quad (1)$$

$$+g_0\frac{|a|^4}{2V}, \quad \Omega_k = \frac{k^2}{\mu}, \quad \mu = 2m, \qquad [b_k, b_{k'}^+] = \delta_{kk'}, \qquad \left[H, \sum_{k\neq 0} b_k^+ b_k\right] \neq 0.$$

It doesn't conserve the number of quantum particles due to the pair-correlation terms (broken gauge symmetry). Still the sum of quantum particles and particles in a condesate is conserved in a quasiclassical manner [3]

$$\frac{dn}{dt} = \frac{d}{dt}\left(|a|^2 + \sum_{k\neq 0} b_k^+ b_k\right) = \{H, |a|^2\} + i[H, \sum_{k\neq 0} b_k^+ b_k] = 0. \qquad (2)$$

The Poisson bracket is taken over variables a, a^*. Formula (2) leads to the canonical distribution with a consraint, supplied by the integral of motion (2), so the partition function for N atoms in (1) is

$$Q = Sp\left(e^{-\beta H}\delta_{N,n}\right).$$

By using the Fourier decomposition of the δ symbol with the spectral parametr ν, we calculate Sp over the quantum variables [1]

$$Q = V\int d\rho \int_{-\pi}^{\pi} dy \exp\left\{\left[-\beta\frac{g_0}{2}\rho^2 + iy(\rho - R)\right]V\right\}\prod_{k\neq 0}\frac{\exp(\omega_k - \nu)\beta/2}{4\sinh^2(\beta E_k/4)}, \qquad (3)$$

$$\omega_k = \Omega_k + (g_0 + g_k)\frac{|a|^2}{V}, \quad \nu = i\frac{y}{\beta}, \quad R = \frac{N}{V}, \quad \rho = \frac{|a|^2}{V},$$

$$E_k = \left[(\Omega_k + \rho g_0 - \nu)^2 + 2\rho g_k(\Omega_k + \rho g_0 - \nu)\right]^{1/2}.$$

[1] The details of calculation may be found in [3].

The quantities E_k give the spectrum of the over-condensate excitations as a function of the interaction g_k, condensate density ρ and the parameter of constraint ν. The condition of the absence of the energy gap $E_k \to 0$ when $k \to 0$ must be fullfiled for E_k. Two values of the constraint parametr $\nu_1 = \rho g_0$ and $\nu_2 = 3\rho g_0$ satisfy this condition. In this way we get two branches of the spectrum

$$E_k^2 = \begin{cases} E_{k1}^2 = \Omega_k^2 + 2\Omega_k\rho g_k, & \nu_1 = c, c = \rho g_0 \\ E_{k2}^2 = \Omega_k^2 + 2\Omega_k\rho g_k - 4\rho g_0(\Omega_k + \rho g_k - \rho g_0), & \nu_2 = 3c. \end{cases} \qquad (4)$$

These two branches correspond to the matrix structure 2×2 of interaction, that may be expressed explicitly because of the boson pair-correlation terms in (1).

Let us investigate the difference between the two branches (4) with an example of the simplest interaction

$$g_k = \begin{cases} g_0\left(1 - k^2/k_0^2\right) \geq 0, & (0 \leq k \leq k_0) \\ g_0\left[(k - 2k_0)^2/k_0^2 - 1\right] \leq 0 & (k_0 \leq k \leq 2k_0) \end{cases} \qquad (5)$$

The first and second formulae of (5) describe the repulsion and attraction of the interaction potential $g(r)$ of the van-der-Vaals type with the minimum of energy at $r = r_0, r_0 k_0 = 1, g_{k_0} = 0$.

In the region of repulsion forces the energy branches (4) look like

$$E_{k1} = \left(k^2\frac{2c}{\mu} - k^4\frac{d}{\mu^2}\right)^{\frac{1}{2}}, \quad E_{k2} = \left(k^2\frac{2cd}{\mu} - k^4\frac{d}{\mu^2}\right)^{\frac{1}{2}}, \quad d = \frac{2c\mu}{k_0^2} - 1 \geq 0. \qquad (6)$$

The inequality for d in (6) shows the values of parameters c, μ and variable ρ, that correspond the repulsion. It is clear that E_{k1} is the excitation branch of the Bogolubov theory [2]. The E_{k2} branch is of the same type, but is situated under the dispersion curve E_{k1} in the plane (E, k) when

$$c < \frac{k_0^2}{\mu}, \quad d < 1 \qquad (7)$$

or over E_{k1} in the case when

$$c > \frac{k_0^2}{\mu}, \quad d > 1. \qquad (8)$$

The tangent of the phonon part of the E_{k2} angle is the product of the the same angle for $_{k1}$ and d. These branches are presented in Fig.1 where

$k_0 = 1$ and $\mu = 2$ are taken. The maximum values E_1 and E_2 of the E_{k1}, E_{k2} branches and their coordinates k_1, k_2 are equal to

$$E_1^2 = \frac{c^2}{d}, \qquad k_1^2 = \frac{c\mu}{d}, \qquad E_2^2 = c^2 d, \qquad k_2^2 = c\mu.$$

The parameter d is the ratio $q_2 q_1^{-1}$ of coordinates $q_2 = (2c\mu - k_0^2)^{1/2}$ and $q_1 = k_0$ of the intersection points for the E_{k2} and E_{k1} branches with the free boson dispersion curve Ω_k. In the region of attraction ($k > k_0$, $d < 0$) formulae of the type (6) based on the second line of (5) are used. Note that there is no complete analogy between the situations (7) and (8). Really, the point $k = q_1$ of the intersection of the E_{k1} branch with dispersion curve Ω_k is always equal to k_0. Therefore the alternative situations of inequalities (7), (8) are presented by two series of inequalities

$$E_1 > E_2, \quad k_2 < k_1, q_2^< < k_0 = q_1 \quad \textit{and} \quad E_1 < E_2, \quad k_1 < k_2, q_1 = k_0 < q_2^>,$$

so that in the first case ($d < 1$) the point k_1 of the branch E_{k1} is in the region of attraction, while in the second ($d > 1$) the same happens with the point k_2 of the branch E_{k_2}. There is the degeneracy of spectrum when $d = 1$: dispersion curves E_{k1} and E_{k2} coincide and have their maximum value $E_1 = E_2 = c^2$ at k_0 point.

To make the difference between two branches (4) more clear we consider now the influence of quantum fluctuations of bosons in the ground state on the spectrum. For this purpose the term h additional to (1) is introduced

$$H_1 = H + h, \quad h = \frac{g_0}{V}(b_0^+ a^* a^2 + b_0 a a^{*2} + 2b_0^+ b_0 |a|^2), \quad [b_0, b_0^+] = 1. \quad (9)$$

This term describes the interaction between the classical condensate and those atoms with $k = 0$ that remain out of condensate and keep their quantum nature because of the non-ideal character of the gas. The operator ${}_1$ may be derived from the exact two-particle interaction by the substitution $b_0 \to a + b_0$, $b_0^+ \to a^* + b_0^+$ (and eliminating the higher powers of quantum variables as well as the pair correlations between atoms b_0, b_0^+). One may see, that like for (1) the model (9) has an integral of motion n_1

$$\frac{dn_1}{dt} = \frac{d}{dt}\left(|a|^2 + \sum_{k=0} b_k^+ b_k\right) = \{H_1, |a|^2\} + i[H_1, \sum_{k=0} b_k^+ b_k] = 0. \quad (10)$$

The partition function of model (9) with the constraint, generated by the integral of motion (10) is equal to

$$Q_1 = Sp\left(e^{-\beta H_1} \delta_{N,n_1}\right). \qquad (11)$$

The partition function (11) may be calculated similarly to the partition function (3). The additional integral over the trajectories b_0, b_0^* appears and the frequency ω_0 for this oscillator is equal to $(2c - \nu)$. The values of the frequency are $\omega_0 = c$ and $\omega_0 = -c$ for the first and second branches of the formula (4). It means that the second branch E_{k2} is unstable with respect to quantum fluctuations of bosons in the ground state.

The disposition of the branch E_{k1} in the plane (E,k) depends on the inequalities (7,8), which means the critical correspondence between the kinetic energy of an atom with the momentum k_0 and condensate density ρ on the scale of interaction potential g_0. When these quantities are equal $\rho g_0 = k_0^2 \mu^{-1}$ (d=1, the degeneracy of spectrum), the change of the E_{k1} branch disposition in (E,k) happens: the maximum of energy $E_1 = E_1^>$ moves from the point $k_1 < k_0$ (large condensate density, repulsion region of g_k) to the point $k_1 > k_0$ with energy $E_1 = E_1^<$ (small condensate density, attractive region of g_k). So, the reorganisation of the branch E_{k1} of the condensate excitation spectrum (5) takes place when $\rho = k_0^2 (g_0 \mu)^{-1}$.

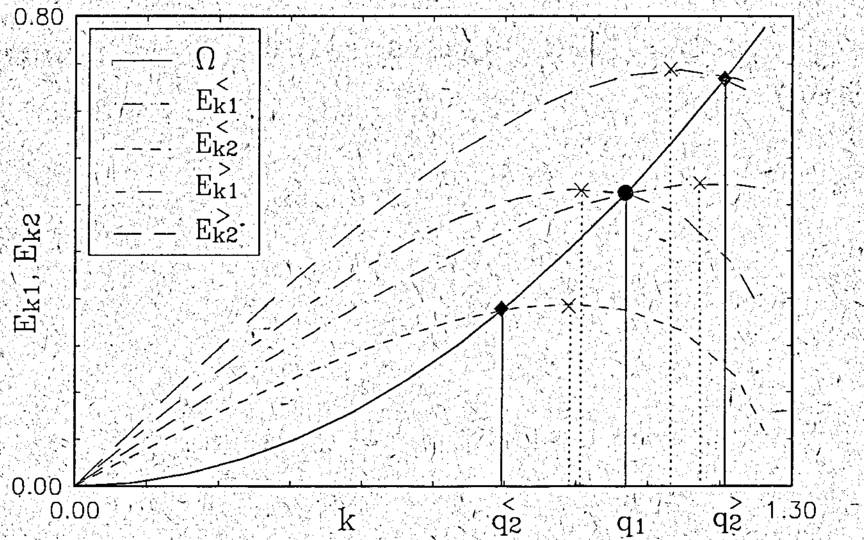

Fig.1. - $E_{k1}^<, E_{k2}^<$ (small condensate density, $d = 0.6$) and $E_{k1}^>, E_{k2}^>$ (large condensate density, $d = 1.4$) branches of spectrum (4); Ω_k - free boson dispersion curve.

It is interesting to look for the accordance between the data of the paper [1] and the theoretical spectrum we have obtained above. The non-stable branch E_{k2} may be associated with the wide branch of the sharp peak in [1], while the E_{k1} branch seems to correspond to the narrow one in [1]. The sign of the inequality (7) or (8) must be found experimentally.

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