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STATE EVOLUTION IN THE TWO-PHOTON ATOM-FIELD INTERACTION WITH LARGE INITIAL FIELDS

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I. INTRODUCTION

The Jaynes–Cummings model (JCM) [1] describing the interaction of a single two-level atom with a single-mode quantized radiation field is one of the simplest nontrivial system in quantum optics. In spite of its mathematical simplicity, the model gives rise to an interesting and surprisingly rich dynamical behavior (for a review see [2]). Much attention has been focused on the collapses and revivals of the Rabi oscillations which provide evidence for the quantum nature of the radiation field [3]. Successful realization of the JCM by using highly excited Rydberg atoms enclosed in high-Q superconducting cavities has been reported [4] and the collapse and revival phenomenon has been experimentally tested.

Recently, Gea-Banacloche has studied the evolution of the atomic and field states in the JCM [5]. He showed that an arbitrary initial pure atomic state evolves into a unique pure state in the middle of the collapse region, provided the field is initially in a coherent state with large intensity. Moreover, at the half-revival time, the cavity field represents a coherent superposition of the two macroscopically distinct states with opposite phases. The fact that the atom and field in the JCM most closely return to pure states during the collapse region can be traced out from the behavior of the two quantities: the trace of the square of the density operator [5] and the entropy [6]. Using the entropy concepts, Orszag et al. have pointed out that the pure atomic state can be generated even from the initially mixed ones [7].

The modification of the JCM in which the atom makes two-photon transitions has also attracted considerable interest due to recent development of the two-photon micromasers [8]. Alsing and Zubairy [9], Puri and Bullough [10] have shown that the revivals of the Rabi oscillations in this model are both compact and regular, in contrast with the one-photon case. The effects of the field statistics [11] and cavity damping [12] have been explored. In [13], Sherman and Kurizki have proposed a scheme for preparation and subsequent detection of macroscopic quantum superposition states based on the two-photon JCM. Phoenix and Knight have calculated the entropy in this model and pointed out the periodic recovering of the initial atom-field state occurring under a proper choice of the detuning parameter [14].

In the present paper, we investigate the state evolution of the atom and the field in the two-photon JCM using the effective Hamiltonian approach. We consider both cases when the dynamic Stark shifts are not included and when they are, and compare the obtained results. Various initial field states, namely, the coherent, squeezed vacuum and chaotic ones are treated. We show that when the intensity-dependent energy shifts of the two levels are equal; the atom is prepared initially in a pure state and the field in a highly excited coherent state, the atom and the field most closely return to pure states twice before the revival times and right at the revival times. The atomic and field states at these times are found. The effects of the cavity damping are discussed within the dressed-state approximation. For initial squeezed vacuum and chaotic states, numerical calculations are performed, revealing novel features to be absent in the standard JCM. In the appendix, the atomic and field entropies are calculated with due account for the dynamic Stark shifts.

II. EVOLUTION OF THE FIELD AND ATOMIC STATES

The two-photon JCM under consideration is obtained when a cascade of the atomic transitions $|e\rangle \rightarrow |i\rangle \rightarrow |g\rangle$ is resonant with twice the field frequency, $\omega_{eg} = 2\omega$ whereas the intermediate transition frequencies $\omega_{ei} = \omega + \Delta$ and $\omega_{ig} = \omega - \Delta$ are strongly detuned from ω. After adiabatically eliminating the intermediate state, one arrives at the effective interaction picture Hamiltonian [8,10], in the rotating wave approximation,

$$H = \hbar g \left(a^2|e\rangle\langle g| + a^{\dagger 2}|g\rangle\langle e|\right) + \beta_1 a^\dagger a|g\rangle\langle g| + \beta_2 \left(a^\dagger a + 1\right)|e\rangle\langle e|, \qquad (1)$$

where the Stark shift parameters β_1 and β_2 of the two levels and the effective two-photon coupling g are defined in terms of the coupling constants g_1 (for $|g\rangle \rightarrow |i\rangle$), g_2 (for $|i\rangle \rightarrow |e\rangle$), and Δ as follows

$$\beta_1 = \frac{g_1^2}{\Delta}, \qquad \beta_2 = \frac{g_2^2}{\Delta}, \qquad g = \frac{g_1 g_2}{\Delta}. \qquad (2)$$

Notice that in Eq. (1) the spontaneous contribution to the energy-level shifts has been included [8, 10]. Another form of the effective two-photon Hamiltonian, where $\beta_2 a^\dagger a|e\rangle\langle e|$ stands for $\beta_2\left(a^\dagger a + 1\right)|e\rangle\langle e|$, is also of frequent use. In the high-field limit we are interested in, the two forms obviously lead to identical results.

As has been pointed out by Toor and Zubairy [15], the effective Hamiltonian (1) is valid for strong fields and for times and detunings such that

$$\frac{\bar{n}}{\pi}\left(\frac{r^2+1}{r}\right)^2 gt \ll \frac{\Delta}{g}, \qquad (3)$$

where $\bar{n}$ is the mean number of photons and $r = g_1/g_2$. The conditions and limitations for the validity of the effective Hamiltonian approach have also been discussed in detail in [16] and [17]. In the case of $r = 1$, it is known that the Stark shifts give rise to an additional overall phase factor which can only play a role in the off-diagonal elements of the density matrix. Keeping this point in mind, we put aside for a while the last two terms in Eq. (1) and work with the effective Hamiltonian

$$H = \hbar g \left(a^2|e\rangle\langle g| + a^{\dagger 2}|g\rangle\langle e|\right). \qquad (4)$$

The case with the complete Hamiltonian (1) will be analyzed in the next section.

Using the Hamiltonian (4) to solve the corresponding equations of motion with the initial atomic condition

$$|\psi_{at}(0)\rangle = \alpha|e\rangle + \beta|g\rangle,$$

and initial field condition

$$|\psi_f(0)\rangle = \sum_{n=0}^{\infty} C_n|n\rangle,$$

one can easily find the state vector of the system at time t

$$|\psi(t)\rangle = \sum_{n=0}^{\infty}\Big\{[\alpha C_n\cos(g\sqrt{n+2}t) - i\beta C_{n+2}\sin(g\sqrt{n+2}t)]|e\rangle + [-i\alpha C_{n-2}\sin(g\sqrt{n}t) + \beta C_n\cos(g\sqrt{n}t)]|g\rangle\Big\}|n\rangle, \qquad (5)$$

where $C_n = 0$ for $n < 0$. The exact solution (5) represents a strongly entangled atom-field state. Following [5], we introduce the semiclassical Hamiltonian corresponding to Eq. (4)

$$H = \hbar g\left(v^2|e\rangle\langle g| + v^{*2}|g\rangle\langle e|\right) \tag{6}$$

obtained by replacing the annihilation operator a by a complex number $v = |v|\exp(i\varphi)$. The eigenstates of (6) are

$$|\psi_{SC}^{\pm}\rangle = \frac{1}{\sqrt{2}}[|e\rangle \pm \exp(-2i\varphi)|g\rangle]. \tag{7}$$

If the atom enters the cavity in either of the states (7) and the field is treated classically, the physical observables do not evolve. In a fully quantized theory, though the quantum nature of the electromagnetic field implies that the system would evolve dynamically, one can expect that the states (7) still exhibit features distinguishable from the others. Indeed, let the field be initially in the coherent state

$$|\psi_f(0)\rangle = \exp\left(-\frac{\bar{n}}{2}\right)\sum_{n=0}^{\infty}\sqrt{\frac{\bar{n}^n}{n!}}\exp(in\varphi)|n\rangle \tag{8}$$

which is the most close quantum counterpart of the stable monochromatic excitation in the semiclassical theory. For $\bar{n} \gg 1$, by employing the solution (5) and the relation $C_{n-2} \simeq C_n\exp(-2i\varphi)$ holding for n in the neighborhood of the mean $\bar{n}$, one finds approximately

$$|\psi_{SC}^{+}\rangle|v\rangle\Big|_{t=0} \longrightarrow \frac{1}{\sqrt{2}}[\exp(-i2gt)|e\rangle + \exp(-2i\varphi)|g\rangle]$$

$$\times\exp\left(-\frac{\bar{n}}{2}\right)\sum_{n=0}^{\infty}\sqrt{\frac{\bar{n}^n}{n!}}\exp(in\varphi)\exp\left[-ig\sqrt{n(n-1)}t\right]|n\rangle, \tag{9a}$$

$$|\psi_{SC}^{-}\rangle|v\rangle\Big|_{t=0} \longrightarrow \frac{1}{\sqrt{2}}[\exp(i2gt)|e\rangle - \exp(-2i\varphi)|g\rangle]$$

$$\times\exp\left(-\frac{\bar{n}}{2}\right)\sum_{n=0}^{\infty}\sqrt{\frac{\bar{n}^n}{n!}}\exp(in\varphi)\exp\left[ig\sqrt{n(n-1)}t\right]|n\rangle. \tag{9b}$$

Equations (9) show that starting from the initial conditions (7), the state of the atom–field system can be roughly decoupled into atomic and field parts, each of them evolves, remaining in a pure state. Even more interesting is that the atomic states appearing in equations (9a) and (9b) exactly coincide at times

$$t_1 = (4k+1)\frac{T_R}{4}, \tag{10a}$$

and

$$t_2 = (4k+3)\frac{T_R}{4}, \tag{10b}$$

where k is an integer and T_R is the period of revivals of the Rabi oscillations in the two–photon JCM: $T_R = (\pi/g)$ [9, 10], and are equal to

$$\frac{1}{\sqrt{2}}[i|e\rangle - \exp(-2i\varphi)|g\rangle], \tag{11a}$$

and

$$\frac{1}{\sqrt{2}}[i|e\rangle + \exp(-2i\varphi)|g\rangle], \tag{11b}$$

respectively.

Since the states (7) are orthonormal and can serve as a basis, it follows that any pure initial atomic state will converge into the states (11) at times (10). In other words, we observe the so-called crossings of the atomic "trajectories" in the Hilbert space of the atomic states [5]. In contrast with the one–photon transition case, where the crossings are reached at precisely half the time of the peak revivals, in the system at hand they take place twice in each time interval between a collapse and a subsequent revival, at one and three quarters of the revival time. If the initial atomic state is a linear superposition of $|\psi_{SC}^{\pm}\rangle$, say, the excited state

$$|e\rangle = \frac{1}{\sqrt{2}}\left(|\psi_{SC}^{+}\rangle + |\psi_{SC}^{-}\rangle\right), \tag{12}$$

then the cavity field at times (10) is a coherent superposition of the macroscopically distinct states

$$|\Phi_{\pm}(t)\rangle = \exp\left(-\frac{\bar{n}}{2}\right)\sum_{n=0}^{\infty}\sqrt{\frac{\bar{n}^n}{n!}}\exp(in\varphi)\exp\left[\mp ig\sqrt{n(n-1)}t\right]|n\rangle \tag{13}$$

(a "Schrödinger cat"). The field states given in equation (13) are different from their one–photon counterparts by the phase factors $g\sqrt{n(n-1)}$ which are nothing else but the frequencies of the two–photon Rabi nutations between $|e\rangle$ and $|g\rangle$. Recall that in the one–photon JCM, the Rabi frequencies are of the form $\lambda\sqrt{n}$ with λ being the one–photon coupling constant. By expanding $g\sqrt{n(n-1)}$ in powers of n^{-1}

$$g\sqrt{n(n-1)} = gn\left(1 - \frac{1}{2n} - \frac{1}{8n^2} + \cdots\right), \tag{14}$$

and retaining only terms of the order n we obtain, instead of $|\Phi_{\pm}(t)\rangle$, two coherent states

$$\exp\left(-\frac{\bar{n}}{2}\right)\sum_{n=0}^{\infty}\sqrt{\frac{\bar{n}^n}{n!}}\exp[in(\varphi \mp gt)]|n\rangle \tag{15}$$

which exactly coincide with each other at every time of revivals of the Rabi oscillations

$$t_3 = kT_R, \qquad (k = 1, 2, \ldots). \tag{16}$$

They are identical to the initial state for even k and undergo a global phase change of π from the initial state for odd k whereas the atomic state is the same as that at $t = 0$ for all k.

As a result, there exist three series of the recreations of the field and atomic state vectors in the two–photon JCM, provided that the cavity field is initially in a coherent state with large enough intensity [Under "series" we mean just the first recreations for which the condition (3) is fulfilled]. This is clearly visible in Fig. 1(a), where we have plotted the quantity $\mathrm{Tr}(\rho_{at}^2)$ as a function of the dimensionless time gt for the atom being initially in the upper state and the field in a coherent state with $\bar{n} = 50$. It is not difficult to check that if both the atomic and field subsystems are prepared at $t = 0$ in pure states $\mathrm{Tr}(\rho_f^2) = \mathrm{Tr}(\rho_{at}^2)$.

The results (9)–(12) can be generalized to the field states having a sufficiently well defined phase, for example, the squeezed states [18] with a dominant contribution from the coherent excitation, in the way similar to that employed by Gea-Banacloche for the one–photon transition case [19]. Unfortunately, this asymptotic operator solution approach does not apply for such field states as the squeezed vacuum state [18]

$$|\psi_f(0)\rangle = \sum_{n=0}^{\infty}\frac{(-1)^n\exp(in\varphi)}{\sqrt{\cosh r}}\frac{\sqrt{(2n)!}}{2^n n!}(\tanh r)^n|2n\rangle \tag{17}$$

(where r is the squeezing parameter and $\bar{n} = \sinh^2 r$), which has a double–peaked phase distribution [20], or the chaotic state

$$\rho_f(0) = \sum_{n=0}^{\infty}\frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}|n\rangle\langle n|, \tag{18}$$

whose phase is randomly distributed. For these, we have evaluated the quantity $\mathrm{Tr}(\rho_{at}^2)$ using a computer. The results are presented in Figs. 1(b) and 1(c) for the initial conditions (17) and (18), respectively; for $\bar{n} = 50$ and the atom being initially inverted. Fig. 1(b) shows that when the cavity initially contains a squeezed vacuum state, besides the recreations of the state vectors occurring at t_1, t_2 and t_3, there appears one more series of the recreations at times $k(T_R/2)$ $(k = 1, 2, \ldots)$. As for the initial chaotic state [Fig. 1(c)], we still observe one series of the recreations at times $t_3 = kT_R$. This is a quite nontrivial result: Despite the fact that in our system the atom, which is a two–state system, is coupled to the electromagnetic field, which is a system with an infinite number of degrees of freedom and initially prepared in a completely mixed state, the initial atomic purity is not absorbed forever by the field but periodically returns to the atom. We have also calculated $\mathrm{Tr}(\rho_f^2)$ (not shown in the figure) which in this case is not equal to $\mathrm{Tr}(\rho_{at}^2)$ and saw that it undergoes oscillations near zero, with amplitudes insignificant as compared with those of $\mathrm{Tr}(\rho_{at}^2)$. The recoverings of the state vectors under the initial squeezed vacuum and chaotic states are entirely due to the two–photon character of the atomic transitions and do not appear in the one–photon JCM.

Let us proceed to consider another important aspect of the problem, namely, the effect of the photon leakage from the cavity on the state evolution in the two–photon JCM. It is natural to expect that the purity of the atomic and field states should be very sensitive to the factors of this kind. With the cavity damping taken into account, if the thermal quanta are ignored, the density matrix describing our system is given by the master equation [12]

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] - \kappa\left(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a\right), \tag{19}$$

where the Hamiltonian H is given in Eq. (4) and 2κ is the rate of leakage of photons from the cavity. In the case of finite but very high Q, the master equation (19) can be solved within the dressed–atom approximation [12]. For the solution of (19), we refer the readers to the work [12] by Puri and Agarwal.

We plot in Fig. 2 the quantity $\mathrm{Tr}(\rho_{at}^2)$ for various values of the damping rate, assuming as before an initially excited atom and an initially coherent field with $\bar{n} = 50$. It can be seen that the degree to which the atom for the first time approaches a pure state is about 84% for $\kappa/g = 10^{-3}$ and 96% for $\kappa/g = 10^{-4}$. In the two-photon maser developed by Brune et al. [8], where the maser action between the $40S_{1/2}$ state and the $39S_{1/2}$ state of ^{85}Rb is mediated by the opposite–parity intermediate state $39P_{1/2}$ with $g \sim 4000$ sec^{-1} and $\omega = 68.415$ GHz, these damping rates $[\kappa/g \sim \omega/(2gQ)]$ would correspond to $Q = 10^{10}$ and 10^{11}, respectively, i.e., of two and three orders higher than that really used [8]. It should also be noted that the other factors, such as the presence of the thermal quanta, may worsen further the anew acquired purity of the atom and the field.

III. EFFECTS OF THE DYNAMIC STARK SHIFTS

In the previous section we have restricted our discussion to the simplified effective Hamiltonian (4), which is legitimate for $g_1 \sim g_2$. To investigate the effects of different ac Stark shifts on the state evolution, we use below the complete form (1) of the effective two–photon Hamiltonian. The Hamiltonian (1) can be easily diagonalized with the results [8, 10]

$$\begin{aligned} &H|\psi_n^{\pm}\rangle = \hbar\lambda_n^{\pm}|\psi_n^{\pm}\rangle, \\ &\lambda_n^{+} = \beta_1(n+2) + \beta_2(n+1); \qquad \lambda_n^{-} = 0, \\ &|\psi_n^{\pm}\rangle = \begin{pmatrix} \sin\theta_n \\ \cos\theta_n \end{pmatrix} |n, e\rangle + \begin{pmatrix} \cos\theta_n \\ -\sin\theta_n \end{pmatrix} |n+2, g\rangle, \\ &\tan\theta_n = \left[\frac{\beta_2(n+1)}{\beta_1(n+2)}\right]^{1/2}. \end{aligned} \tag{20}$$

Hence, we have

$$\exp\left(\frac{-iHt}{\hbar}\right)|n,e\rangle = \left(\exp\{-i[\beta_1(n+2)+\beta_2(n+1)]t\}\sin^2\theta_n + \cos^2\theta_n\right)|n,e\rangle$$
$$+\left(\exp\{-i[\beta_1(n+2)+\beta_2(n+1)]t\} - 1\right)\sin\theta_n\cos\theta_n|n+2,g\rangle, \tag{21a}$$

$$\exp\left(\frac{-iHt}{\hbar}\right)|n+2,g\rangle = \left(\exp\{-i[\beta_1(n+2)+\beta_2(n+1)]t\} - 1\right)\sin\theta_n\cos\theta_n|n,e\rangle$$
$$+\left(\exp\{-i[\beta_1(n+2)+\beta_2(n+1)]t\}\cos^2\theta_n + \sin^2\theta_n\right)|n+2,g\rangle. \tag{21b}$$

By replacing the field annihilation operator a by the c-number $v = |v|\exp(i\varphi)$ we get the semiclassical version of (1). As before, the eigenstates of this semiclassical Hamiltonian play a crucial role in the system state evolution under large initial fields. They read

$$|\psi_{SC}^{\pm}\rangle = \frac{\pm\lambda_{SC}^{\pm} \mp \beta_1|v|^2}{\sqrt{\Omega(\pm\lambda_{SC}^{\pm} \mp \beta_1|v|^2)}}|e\rangle \pm \frac{g|v|^2\exp(-2i\varphi)}{\sqrt{\Omega(\pm\lambda_{SC}^{\pm} \mp \beta_1|v|^2)}}|g\rangle \tag{22}$$

where $\lambda_{SC}^{\pm}$ are the corresponding eigenvalues

$$\begin{aligned} \hbar\lambda_{SC}^{\pm} &= \frac{\hbar}{2}\left[\beta_1|v|^2 + \beta_2(|v|^2+1) \pm \Omega\right], \\ \Omega &= \sqrt{[\beta_1|v|^2 - \beta_2(|v|^2+1)]^2 + 4g^2|v|^4}. \end{aligned} \tag{23}$$

Suppose that the atom enters the cavity in one of the states (22), and the field as before is initially in the coherent state (8) with ($\bar{n} \gg 1$). Then, with the aid of Eqs. (21), after some algebra one approximately gets

$$|\psi_{SC}^{+}\rangle|v\rangle\Big|_{t=0} \longrightarrow \left\{\frac{\lambda_{SC}^{+} - \beta_1|v|^2}{\sqrt{\Omega(\lambda_{SC}^{+} - \beta_1|v|^2)}}\exp[-i2(\beta_1+\beta_2)t]|e\rangle + \frac{g|v|^2\exp(-2i\varphi)}{\sqrt{\Omega(\lambda_{SC}^{+} - \beta_1|v|^2)}}|g\rangle\right\}$$
$$\times\exp\left(-\frac{\bar{n}}{2}\right)\sum_{n=0}^{\infty}\sqrt{\frac{\bar{n}^n}{n!}}\exp(in\varphi)\exp\{-i[\beta_1 n + \beta_2(n-1)]t\}|n\rangle, \tag{24a}$$

$$|\psi_{SC}^{-}\rangle|v\rangle\Big|_{t=0} \longrightarrow |\psi_{SC}^{-}\rangle|v\rangle. \tag{24b}$$

In deriving Eqs. (24) we have used the asymptotic relations $(\lambda_{SC}^{+} - \beta_1|v|^2) \sim \beta_2(|v|^2+1)$ and $(-\lambda_{SC}^{-} + \beta_1|v|^2) \sim \beta_1|v|^2$ which hold true in the classical limit ($|v|^2 \gg 1$). As before, Eqs. (24) are of the product form. But now, only when the system starts with the semiclassical eigenstate $|\psi_{SC}^{+}\rangle$, the atom and the field afterwards evolve dynamically. The atomic states appearing in the right-hand sides of (24a) and (24b) do not become identical in the course of time, except for the case of $r = 1$. In this case, one can easily verify that the atomic state originated from $|\psi_{SC}^{+}\rangle$ evolves into the state $|\psi_{SC}^{-}\rangle$ at times t_1 and t_2, with $|\psi_{SC}^{-}\rangle$ being roughly equal to

$$|\psi_{SC}^{-}\rangle \rightarrow \frac{1}{\sqrt{2}}[|e\rangle - \exp(-2i\varphi)|g\rangle], \qquad (25)$$

i.e., the same as in equation (7). Taking into account the sharp location of the photon number distribution around $\bar{n} \gg 1$, we replace $[\beta_1 n + \beta_2(n-1)]$ by $(\beta_1 + \beta_2)n$, that is, we transform the field state in (24a) into a coherent state with the time-dependent phase $[\varphi - (\beta_1 + \beta_2)t]$. This state coincides, independently of the concrete value of r, with the initial state at times

$$t_3 = 2k\pi \left[\left(r + \frac{1}{r} \right) g \right]^{-1}, \qquad (k = 1, 2, \ldots). \qquad (26)$$

Equation (26) is nothing else but the times of revivals of the Rabi oscillations in the two-photon JCM [10]. It reduces to Eq. (16) when setting $r = 1$.

Thus, for $r = 1$, if the atom is initially prepared in a state composed of $|\psi_{SC}^{\pm}\rangle$ (e.g. $|e\rangle$ or $|g\rangle$), we observe three series of recreations of the state vectors at times (10a), (10b) and (16), similarly to the case when the system is driven by the simplified effective Hamiltonian (4). It is clearly seen from a comparison of Fig. 3(a), where we have used the exact solution (21) to plot $\mathrm{Tr}(\rho_{at}^2)$ for the atom initially in the excited state and the field in the coherent state, with Fig. 1(a). However, the two Hamiltonians (4) and (1) predict different state evolutions: The attractor state (25) is not equal to the states (11). Thus, our results are complementary to the conclusions of Toor and Zubairy stating that the effective Hamiltonian (4) is adequate only for describing the quantities in which diagonal elements of the density matrix are involved but is not valid for the description of such quantities as squeezing.

At t_1 and t_2, when the attractor state (25) is reached, the cavity field is a superposition of the two macroscopically distinct states with the relative phase of $\pi/2$. If one uses the state reduction scheme proposed by Sherman and Kurizki [13], i.e., if one projects the atom–field system onto one of the atomic energy states, for example, the upper state, one will have a field macroscopic quantum superposition state with arbitrary relative phase.

In Figs. 3(b) and 3(c), we have depicted $\mathrm{Tr}(\rho_{at}^2)$ for $r = 0.5$ and $r = 0.3$, respectively. As is visible from the figures, the effects of the dynamic Stark shifts are more pronounced when r is deviated from unity. On the one hand, the minimal values of $\mathrm{Tr}(\rho_{at}^2)$ raise indicating that the atomic (and field) state becomes less mixed. On the other hand, the convergences of the atomic state into the unique state is destroyed. These effects resemble those occurring in the standard JCM when the atom–field detuning takes nonzero values [19, 21], which is understandable since the ac Stark shifts can be treated as the intensity-dependent detunings. In contrast with the one-photon JCM, in the two-photon model the atom–field system always returns to its original state at the revival times, regardless of the chosen value of r.

We have also depicted $\mathrm{Tr}(\rho_{at}^2)$ for the field being initially in the squeezed vacuum state (Fig. 4) and chaotic state (Fig. 5), for various values of r. By comparing Fig. 4(a) with Fig. 1(b), and Fig. 5(a) with Fig. 1(c) we see that when $r = 1$, the two Hamiltonians (1) and (4) lead to an identical behavior of $\mathrm{Tr}(\rho_{at}^2)$, as is expected. For $r \neq 1$, when the cavity field initially contains a squeezed vacuum state, only the recreation-of-state-vector series at the revival and half-revival times survive; while when the cavity initially contains a chaotic state, the recreations of the atomic state vector are no more observed.

In the above discussion, we have used the quantity $\mathrm{Tr}(\rho_{at}^2)$ to determine the purity of the atomic state. Another way of looking at this problem is via the entropy [6]. The atomic and field entropies in the two-photon JCM have been calculated and compared with those in the Raman-coupled model [22] by Phoenix and Knight [14]. However, the authors of [14] have ignored the Stark shifts. We present the solution for the case in which the dynamic Stark shifts are taken care of in the appendix.

IV. CONCLUSIONS

We have considered the state evolution in the two-photon JCM with large fields. We have found that for an initially coherent state field and equal Stark shifts of the levels, at 1/4 and 3/4 of the revival time the atom is converged into a unique pure state, no matter how the initial atomic state is chosen, while the cavity field represents coherent superpositions of the two distinguishable components shifted from each other by $\pi/2$. Right at the revival times, the initial atomic and field states recover. The explicit forms have been obtained for the atomic attractor states and the field "Schrödinger cat" states. To make the model more realistic, we have included the leakage of photons from the cavity. The sensitivity of the state purity to the cavity loss has been graphically illustrated for various damping rates. We have also investigated the field being initially in the squeezed vacuum and chaotic states. For both these initial conditions, in contrast with the one–photon transition situation, one still observes the recreations of the state vectors. The model, in which the atom makes two–photon transitions, is not simply a generalization of the one–photon JCM. It provides us with a much richer dynamics.

APPENDIX

In this appendix we derive expressions for the atomic and field entropies in the two–photon JCM in the presence of Stark shifts. In the atomic basis and interaction picture the time evolution operator can be written as

$$\exp\left(\frac{iHt}{\hbar}\right) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \begin{pmatrix} \beta_2(a^\dagger a+1) & ga^2 \\ ga^{\dagger 2} & \beta_1 a^\dagger a \end{pmatrix}^n . \tag{A1}$$

By using the relation $g = \sqrt{\beta_1\beta_2}$, one finds the following identity, after some straightforward algebra,

$$\begin{pmatrix} \beta_2(a^\dagger a+1) & ga^2 \\ ga^{\dagger 2} & \beta_1 a^\dagger a \end{pmatrix}^{n+1} = \begin{pmatrix} \beta_2(a^\dagger a+1)M^n & ga^2N^n \\ ga^{\dagger 2}M^n & \beta_1 a^\dagger a N^n \end{pmatrix}, \tag{A2}$$

where

$$M = \beta_1(a^\dagger a+2) + \beta_2(a^\dagger a+1), \tag{A3}$$

$$N = \beta_1 a^\dagger a + \beta_2(a^\dagger a-1), \tag{A4}$$

and eventually obtain

$$\exp\left(\frac{iHt}{\hbar}\right) = \begin{pmatrix} \frac{\beta_2(a^\dagger a+1)}{M}\exp(iMt) + \frac{\beta_1(a^\dagger a+2)}{M} & \frac{ga^2}{N}\left[\exp(iNt)-1\right] \\ \frac{ga^{\dagger 2}}{M}\left[\exp(iMt)-1\right] & \frac{\beta_1 a^\dagger a}{N}\exp(iNt) + \frac{\beta_2(a^\dagger a-1)}{N} \end{pmatrix} \tag{A5}$$

Assume that the atom is initially in the excited state and the field in the pure state $\rho_f(0) = |\psi_f(0)\rangle\langle\psi_f(0)|$. Then, the density operator of the atom–field system at time t is given by

$$\rho(t) = \begin{pmatrix} C\rho_f(0)C^\dagger & C\rho_f(0)S^\dagger \\ S\rho_f(0)C^\dagger & S\rho_f(0)S^\dagger \end{pmatrix} = \begin{pmatrix} |c\rangle\langle c| & |c\rangle\langle s| \\ |s\rangle\langle c| & |s\rangle\langle s| \end{pmatrix}, \tag{A6}$$

where the notation

$$C = \frac{\beta_2(a^\dagger a+1)}{M}\exp(-iMt) + \frac{\beta_1(a^\dagger a+2)}{M}, \tag{A7}$$

$$S = \left[\exp(-iNt)-1\right]\frac{ga^{\dagger 2}}{N}, \tag{A8}$$

$$|c\rangle = C|\psi_f(0)\rangle, \qquad |s\rangle = S|\psi_f(0)\rangle \tag{A9}$$

has been introduced. Clearly, the field density matrix $\rho_f(t) = \mathrm{Tr}_a[\rho(t)]$ is equal to

$$\rho_f(t) = |c\rangle\langle c| + |s\rangle\langle s|. \tag{A10}$$

According to Phoenix and Knight [6], if the density matrix is of the form (A10), its eigenvalues are

$$\pi_\pm = \langle c|c\rangle \pm |\langle c|s\rangle| \exp(\pm\theta) \tag{A11}$$

where

$$\theta = \sinh^{-1}\left(\frac{\delta}{2}\right),$$

$$\delta = \frac{\langle c|c\rangle - \langle s|s\rangle}{|\langle c|s\rangle|}. \tag{A12}$$

The field entropy is found from Eq. (A11) to be

$$S_f = -(\pi_+ \ln \pi_+ + \pi_- \ln \pi_-). \tag{A13}$$

The atomic entropy can be determined in a simpler way [6, 7, 23]. By tracing the equation (A6) over the field variables one gets for the reduced atomic density matrix

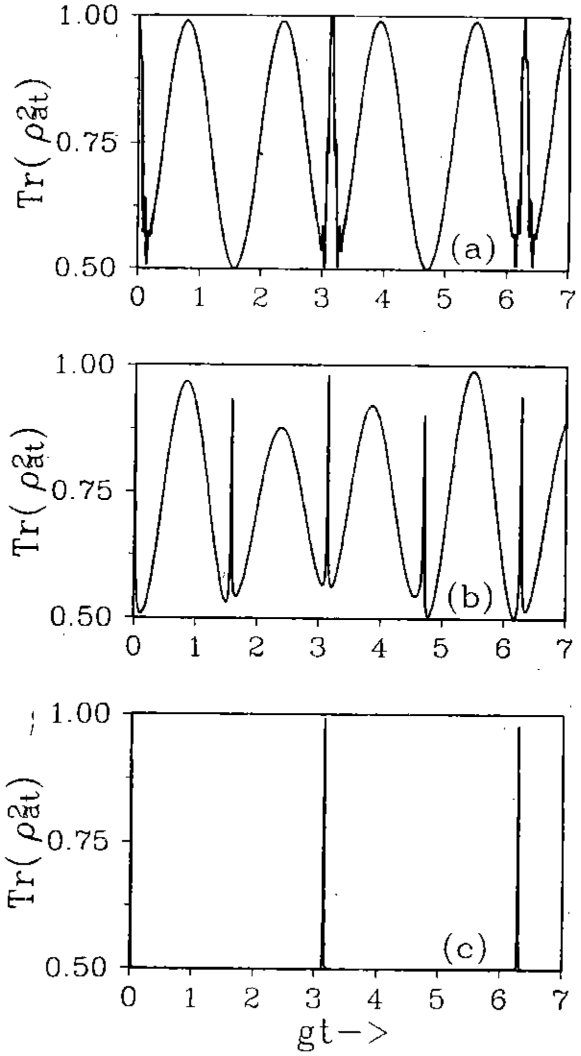

FIG. 1. The time evolution of $\mathrm{Tr}(\rho_{at}^2)$ for the atom initially in the excited state and the field in the (a) coherent state, (b) squeezed vacuum state, and (c) chaotic state. The mean photon number $\bar{n} = 50$.

$$\rho_{at}(t) = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}, \tag{A14}$$

where $\lambda_{ij} = \mathrm{Tr}_f[\rho_{ij}(t)]$, and $\rho_{ij}(t)$ are the matrix elements of $\rho(t)$. The eigenvalues of (A14) read

$$\alpha_\pm = \frac{1}{2}\left[1 \pm \sqrt{(\lambda_{11} - \lambda_{22})^2 + 4|\lambda_{12}|^2}\right], \tag{A15}$$

so that the atomic entropy is given by

$$S_f = -(\alpha_+ \ln \alpha_+ + \alpha_- \ln \alpha_-). \tag{A16}$$

When both the atom and field are initially in the pure states, it is not difficult to prove that Eq. (A15) is identical to Eq. (A11), i.e., the atomic and field entropies are equal. This fact can also be derived from the Araki–Lieb triangle inequality for the entropies of the two interacting quantum systems [24].

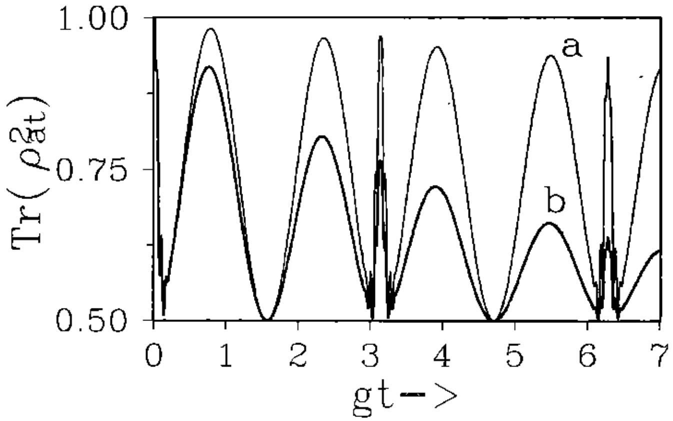

FIG. 2. $\mathrm{Tr}(\rho_{at}^2)$ as a function of time for various values of the cavity relaxation parameter: (a) $\kappa/g = 0.0001$ and (b) $\kappa/g = 0.001$. The atom is initially prepared in the excited state and the field in the coherent state with $\bar{n} = 50$.

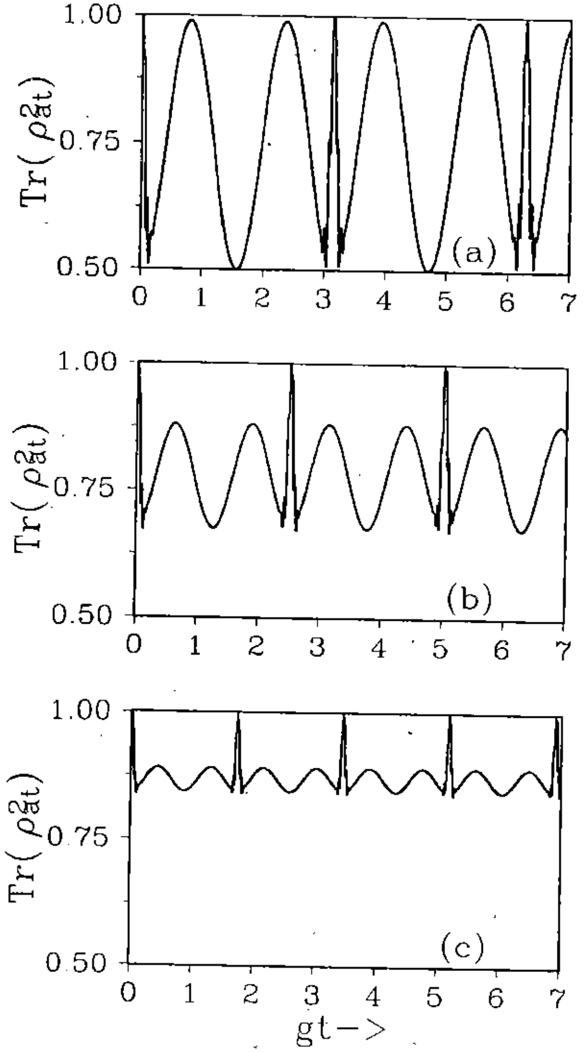

FIG. 3. $\mathrm{Tr}(\rho^2_{at})$ as a function of time in the presence of Stark shifts: (a) $r = 1$, (b) $r = 0.5$, and (c) $r = 0.3$. The atom is initially in its upper state and the field in the coherent state with $\bar{n} = 50$.

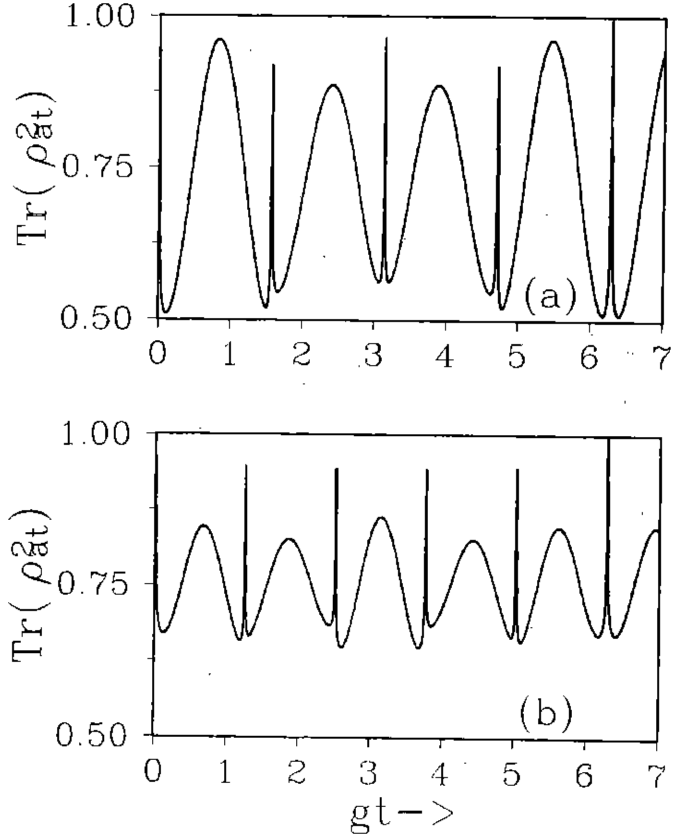

FIG. 4. The same as in Fig. 3 with (a) $r = 1$, (b) $r = 0.5$, but now the field is initially in the squeezed vacuum state.

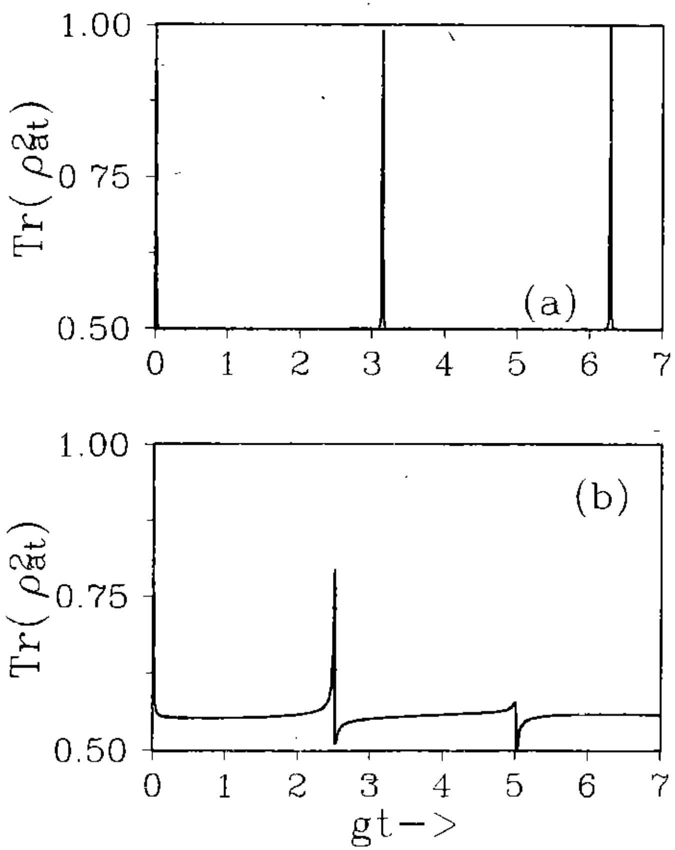

FIG. 5. The same as in Fig. 3 with (a) $r = 1$, (b) $r = 0.5$, but now the field is initially in the chaotic state.

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