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Nguyen Dinh Huyen*, Ho Trung Dung

QUASIPROBABILITY DISTRIBUTIONS FOR THE THREE-LEVEL JAYNES — CUMMINGS MODEL

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*Department of Physics, Moscow State University,
Leninskie-gory, Moscow 117234

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1 Introduction

The interaction between a two-level atom and a single mode of the radiation field (Jaynes–Cummings model (JCM) [1]) is a subject of intensive research in the last years [2]–[6]. It has been shown that the dynamics of the system is very sensitive to the statistical properties of the field. If the initial field state is a Fock state, the atomic response is purely classical with periodic Rabi oscillations [1]. However, if the field is a superposition of the Fock states with , for example, coherent photon statistics, the envelopes of these oscillations exhibit collapses and revivals [7]–[9]. Experiments with Rydberg atoms in high–Q microwave cavities [10]–[15] make it now possible to test many predictions in the JCM and experimental observation of the collapse–revival phenomenon has been reported [14].

The collapses and revivals of the Rabi oscillations can also be considered from another point of view. Recently, Eiselt and Risken [16, 17] have proposed a method for calculating the s–parametrized quasiprobability distributions $W(\alpha, s)$ of Cahill and Glauber [18] for the damped JCM. Starting with the cavity field initially in a coherent state and the atom in its upper state, they have found that the initial single–peak function splits into two peaked functions which rotate on a circle in the complex α–plane . When the peaks are well separated, one observes the collapses of the Rabi oscillations, when they overlap the revivals occur. The bifurcation of the quasiprobability distribution has also been mentioned by Phoenix and Knight [19] for the two–photon JCM and the Raman–coupled model [20]. Using the same approach as in [16, 17], Schoendorff and Risken have re-examined this model taking into account the damping of the cavity [21]. Cirac and Sánchez-Soto have analyzed the Q–function for the JCM in the context of the population trapping and Bužek et al. [23] – in the context of the oscillations of the photon number distribution.

In this paper, we consider an expanded version of the JCM, namely, the one–mode three–level system under two–photon resonance (for a review see [3, 5] and refs. therein). This model shows a feature of collapses and revivals with some modifications compared with those in the standard JCM. For example, at far-off-resonance, the collapses and revivals are closely periodic whereas those in the two-level JCM become quasichaotic [3]. It is then interesting to know how the quasiprobability distributions will behave in the three-level system. Below, the formulas for the Q-function and Wigner function are obtained for all three possible types (Ξ-, V-, and Λ-types) of the atomic level configurations. We particularize the Q-function for the Λ-type and establish its connection with the collapses and revivals of the Rabi oscillations in both zero and finite detuning regimes. When the atom is injected into the cavity in the upper state, the dynamics of the quasiprobability distribution Q is shown to be quite similar to that occurring in the two-level JCM. The peculiarities of the three-level system manifest themselves more prominently when the atom is injected into the cavity in one of its lower states. We explain the splitting of the Q-function in terms of the eigenstates of the semiclassical Hamiltonian. We also predict the convergence of the atom into a unique pure state and the generation of the field macroscopic superposition state in the middle of the collapse region under a proper choice of the initial conditions.

2 Dynamics of the Q-function

For a three-level atom with two allowed and one forbidden transitions there exist three possible atomic level configurations: Ξ-, V- and Λ-types. Let us assume that the allowed dipole transitions are those of the atomic levels $1 \leftrightarrow 2$ and of $2 \leftrightarrow 3$, but not of $1 \leftrightarrow 3$. We also assume the two-photon resonance condition where the sum (Ξ-type) or the difference (V- and Λ-types) of the two transition-frequencies is exactly on resonance with the atomic frequency difference between levels 1 and 3 (see Fig. 1). The hamiltonian of the system under consideration is then given by

$$H = H_A + H_F + H_{AF}, \tag{1}$$

where H_A and H_F describe a free atom and a free field, respectively, and H_{AF} describes the atom-field interaction in the rotating wave approximation ($\hbar = 1$)

$$\begin{aligned} H_A &= \sum_{j=1}^{3} \Omega_j b_j^\dagger b_j, \quad H_F = \omega a^\dagger a, \\ H_{AF} &= \xi a b_2^\dagger b_1 + \eta a^\dagger b_3^\dagger b_2 + \text{h.c.} \end{aligned} \tag{2}$$

Here $a^\dagger$ and a are the creation and annihilation operators of a photon in the mode and $b_j^\dagger$ and b_j are those of an electron at level j; ω is the mode frequency and Ω_j is the corresponding level energy; ξ and η are the coupling constants which may be treated as real and positive without any loss of generality.

For a given excitation number N there exist three bare states

$$|j)^{(N)} = |j; N - \mu_j) \qquad (j = 1, 2, 3), \tag{3}$$

where the configuration parameters (μ_1, μ_2, μ_3) are equal to (0,1,2) for Ξ-type, (1,0,1) for V-type and (0,1,0) for Λ-type atoms [3]. These states are used as the basis in the N-subspace which gives the following matrix representation of the time evolution operator in the "quasi-interaction" picture

$$\begin{aligned} U^{(N)}(t) &= \exp\left(\frac{i\Delta t}{2}\right) \\ &\times \begin{pmatrix} \bar{\eta}_N^2 \exp\left(\frac{i\Delta t}{2}\right) + \bar{\xi}_N^2 x_N(t) & \bar{\xi}_N y_N(t) & \bar{\xi}_N \bar{\eta}_N \left[-\exp\left(\frac{i\Delta t}{2}\right) + x_N(t)\right] \\ \bar{\xi}_N y_N(t) & x_N^*(t) & \bar{\eta}_N y_N(t) \\ \bar{\xi}_N \bar{\eta}_N \left[-\exp\left(\frac{i\Delta t}{2}\right) + x_N(t)\right] & \bar{\eta}_N y_N(t) & \bar{\xi}_N^2 \exp\left(\frac{i\Delta t}{2}\right) + \bar{\eta}_N^2 x_N(t) \end{pmatrix}, \end{aligned} \tag{4}$$

where Δ is the detuning parameter, $\xi_N \equiv \xi\sqrt{N}$, $\eta_N \equiv \eta\sqrt{N}$ for V- and Λ-types, $\eta_N \equiv \eta\sqrt{N-1}$ for Ξ-type, and

$$\begin{aligned} g_N &= \sqrt{\xi_N^2 + \eta_N^2}, \quad f_N = \sqrt{g_N^2 + \frac{\Delta^2}{4}}, \\ \bar{\xi}_N &= \frac{\xi_N}{g_N}, \quad \bar{\eta}_N = \frac{\eta_N}{g_N}, \\ x_N(t) &= \cos(f_N t) + i\frac{\Delta}{2 f_N}\sin(f_N t), \\ y_N(t) &= -i\frac{g_N}{f_N}\sin(f_N t). \end{aligned} \tag{5}$$

Note that $U^N(t)$ is nothing but the matrix representation of the time evolution operator in the II-picture $U_{II}^N(t)$ in [3]. This II-picture coincides with the usual interaction picture when the one–photon resonance condition ($\Delta = 0$) is met. If the initial state of the cavity field mode is a coherent state

$$|\alpha_0\rangle = \sum_{n=0}^{\infty} b_{0n}|n\rangle, \tag{6}$$

where

$$b_{0n} = \exp\left(-\frac{|\alpha_0|^2}{2}\right)\frac{|\alpha_0|^n}{\sqrt{n!}}e^{in\varphi}, \tag{7}$$

and the electron is initially at level i, the state of the total atom–field system in the II–picture is given by

$$|\psi(t)\rangle = \sum_n b_{0n}\sum_{j=1}^{3} U_{ji}^{(n+\mu_i)}|j; n+\mu_{ij}\rangle. \tag{8}$$

Here $\mu_{ij} = \mu_i - \mu_j$.

The quasiprobability distribution $Q(\alpha, t)$ is defined as [18]

$$Q(\alpha, t) = \langle\alpha|\rho|\alpha\rangle, \tag{9}$$

which has the form of a shifted Gaussian

$$Q_0(\alpha) = \exp(-|\alpha - \alpha_0|^2) \tag{10}$$

in the case of the initial state $|\alpha_0\rangle$. Since the state of the system is given by Eq. (8), its density operator is $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$. After tracing over the atom, the quasiprobability distribution Q is found to be

$$\begin{aligned} Q(\alpha,t) &= \mathrm{Tr}_A[\langle\alpha|\rho(t)|\alpha\rangle] \\ &= \sum_{n,k}|b_{0n}||b_{0k}|\left\{\cos[(n-k)(\theta-\varphi)]\mathrm{Re}U + \sin[(n-k)(\theta-\varphi)]\mathrm{Im}U\right\}, \end{aligned} \tag{11}$$

where the notation

$$\begin{aligned} U &\equiv U(n,k,i;t) = \sum_{j=1}^{3}|b_{n+\mu_{ij}}||b_{k+\mu_{ij}}|U_{ji}^{(N)}(t)\left[U_{ji}^{(K)}(t)\right]^*, \\ N &= n+\mu_i, \quad K = k+\mu_i, \\ b_n &= \exp\left(-\frac{|\alpha|^2}{2}\right)\frac{|\alpha|^n}{\sqrt{n!}}e^{in\theta}, \end{aligned} \tag{12}$$

has been introduced.

The Wigner distribution can be calculated by using the formula [18]

$$W(\alpha,t) = 2\sum_{n=0}^{\infty}(-1)^n\mathrm{Tr}_A[\langle n|\rho(t)D(2\alpha)|n\rangle], \tag{13}$$

where $D(2\alpha)$ is the displacement operator

$$D(2\alpha) = \exp(2\alpha a^\dagger - 2\alpha^* a). \tag{14}$$

After some algebra we get

$$W(\alpha,t) = 2\sum_{n,k=0}^{\infty} b_{0n}b_{0k}^*\sum_{j=1}^{3}(-1)^{n+\mu_{ij}}U_{ji}^{(n+\mu_i)}(t)\left[U_{ji}^{(k+\mu_i)}(t)\right]^*\langle k+\mu_{ij}|D(2\alpha)|n+\mu_{ij}\rangle \tag{15}$$

with the matrix elements of the exponential operator $D(\alpha)$ being given by [18]

$$\langle k|D(\alpha)|n\rangle = \exp(-|\alpha|^2/2)\times\begin{cases}\sqrt{n!/k!}\,\alpha^{k-n}L_n^{(k-n)}(|\alpha|^2) & \text{for } k\ge n\\ \sqrt{k!/n!}\,(-\alpha^*)^{n-k}L_k^{(n-k)}(|\alpha|^2) & \text{for } n\ge k\end{cases} \tag{16}$$

Here $L_n^{(m)}(x)$ is an associated Laguerre polynomial.

Since in the JCM the Wigner function contains the same phase information as the Q–function [16, 17], in what follows we concentrate our attention on the Q–function. To be definite, we choose the Λ–type – the configuration with the common upper level 2 and two lower levels 1 and 3 (The two–photon resonance implies that the levels 1 and 3 are degenerate). We consider two initial atomic states: when the electron is initially at level 2 or level 1. Level 3, obviously, may be treated equivalently to level 1.

Case 1. The intermediate level 2 is initially occupied (so we put i=2)

From Eqs. (4), (5), and (12) we have

$$\begin{aligned} \mathrm{Re}U &= |b_n||b_k|\Bigg\{\cos(f_Nt)\cos(f_Kt) \\ &+ \frac{1}{f_Nf_K}\left[\frac{|\alpha|^2}{\sqrt{(n+1)(k+1)}}(\xi_N\xi_K+\eta_N\eta_K)+\frac{\Delta^2}{4}\right]\sin(f_Nt)\sin(f_Kt)\Bigg\}, \end{aligned} \tag{17}$$

$$\mathrm{Im}U = |b_n||b_k|\frac{\Delta}{2}\left[\frac{1}{f_K}\sin(f_Kt)\cos(f_Nt) - \frac{1}{f_N}\sin(f_Nt)\cos(f_Kt)\right]. \tag{18}$$

Substituting Eqs. (17) and (18) into Eq. (11), one readily obtains the explicit expression for the quasiprobability distribution $Q(\alpha,t)$. By letting the dipole coupling constant

η between levels 2 and 3 vanish, Eqs. (17) and (18) reduce to those for the two–level JCM. On the other hand, one can see that after letting η=0, Eqs. (17), (18) do not change their time dependence. Thus, for $i = 2$ the time behaviour of the Q–function in the three–level system is quite similar to that in a two–level system consisting of level 2 and the combination of levels 1 and 3, in which the relation between the contribution from level 1 to that from level 3 is ξ^2/η^2. As the time goes on, the initially one–peak quasiprobability function splits into two peaked functions

$$Q(\alpha,t) = Q_+(\alpha,t) + Q_-(\alpha,t) \tag{19}$$

counterrotating in the complex plane. Revivals of the Rabi oscillations are found for those times when the two peaks collide. Explicit forms for $Q_\pm$ at exact one–photon resonance ($\Delta = 0$), in a first approximation, can be obtained by replacing the coupling constant g by $\sqrt{\xi^2+\eta^2}$ in those for the JCM [17]. Notice that the same similarities hold for the time behaviour of the Rabi oscillations in this situation [3].

On resonance, one can easily check that the two components $Q_\pm$ are symmetrical with respect to the real axe, supposing that $\varphi = 0$. When the detuning parameter differs from zero, they have different heights. At far-off-resonance such that

$$\epsilon_\Delta^2 \equiv \frac{4g_{\bar n}^2}{\Delta^2} \ll 1, \tag{20}$$

the quasiprobability distribution Q can be written in the form

$$Q(\alpha,t) = \exp\left[-|\alpha - \alpha_0(t)|^2\right] + O(\epsilon_\Delta^2), \tag{21}$$

where

$$\alpha_0(t) = |\alpha_0| \exp\left(\varphi - \frac{\xi^2+\eta^2}{\Delta}t\right), \tag{22}$$

and $O(\epsilon_\Delta^2)$ is a set of oscillating terms of an order of ϵ_Δ^2 or smaller. This means that when $|\Delta|$ increases, one peak is eventually quenched while the other becomes nearly Gaussian and moves along the circle $|\alpha| = |\alpha_0|$ in the complex plane. Our results are consistent with those of Yoo and Eberly [3] stating that under the condition (20), the reduced field density operator can be roughly written as $\rho_F(t) \simeq |\alpha_0(t)\rangle\langle\alpha_0(t)|$ where $\alpha_0(t)$ is given by Eq. (22).

Case 2. Level 1 is initially occupied

After putting $i = 1$, from Eqs. (4),(5) and (12) we have

$$\begin{aligned} \mathrm{Re}U \;=\; & |b_n||b_k|\left\{\frac{\eta^2}{\xi^2+\eta^2} + \frac{\xi^2}{\xi^2+\eta^2}\left[\cos(f_N t)\cos(f_K t)\right.\right. \\ & \left.\left. + \frac{1}{f_N f_K}\left(\frac{\sqrt{nk}}{|\alpha|^2} g_N g_K + \frac{\Delta^2}{4}\right)\sin(f_N t)\sin(f_K t)\right]\right\}, \end{aligned} \tag{23}$$

$$\mathrm{Im}U = |b_n||b_k|\frac{\xi^2}{\xi^2+\eta^2}\frac{\Delta}{2}\left[\frac{1}{f_N}\sin(f_N t)\cos(f_K t) - \frac{1}{f_K}\sin(f_K t)\cos(f_N t)\right]. \tag{24}$$

The Q–function now can be presented as the sum of time–independent and time–dependent terms

$$Q(\alpha,t) = \frac{\eta^2}{\xi^2+\eta^2}Q_0(\alpha) + \frac{\xi^2}{\xi^2+\eta^2}Q_1(\alpha,t), \tag{25}$$

where $Q_0(\alpha)$ is given by Eq. (10) and $Q_1(\alpha,t)$ is associated with the time–dependent terms in Eqs. (23) and (24). The comparison between Eqs. (23), (24) and Eqs. (17), (18) shows clearly that $Q_1(\alpha,t)$ may also be treated as a Q–function which splits into two counterrotating peaks as the interaction is turned on whereas $Q_0(\alpha)$ represents an unmoved shifted Gaussian bell. This is displayed in Fig. 2 where we have plotted the Q–function in the complex α–plane for exact resonance case and for various moments of the scaled time $T = (t/T_R)$ with T_R being the revival period. When the counterrotating peaks are well–separated, the Rabi oscillations are in the collapse regime, when they collide the Rabi oscillations show revivals. What is more, we observe two series of collidings – one when only two rotating peaks overlap and the other when all three peaks overlap. These correspond to the two series of revivals of the occupation probabilities of levels 1 and 3 [3]. For longer time the two rotating peaks spread over the whole circle and the collisions between them as well as the distinctly isolated revivals do not appear any more. However, in contrast with the standard (lossless) JCM where the Q–function in the long time region spreads over the whole circle $|\alpha| = |\alpha_0|$, here the maximum due to the time–independent term in Eq. (25) always remains [see Fig. 2(f)].

At far-off-resonance, $Q_1(\alpha,t)$ is again simplified to

$$Q_1(\alpha,t) = \exp\left[-|\alpha-\alpha_0(t)|^2\right] + O(\epsilon_\Delta^2), \tag{26}$$

where $\alpha_0(t)$ is now

$$\alpha_0(t) = |\alpha_0| \exp\left(\varphi + \frac{\xi^2+\eta^2}{\Delta} t\right). \tag{27}$$

The time behaviour of $Q_1(\alpha,t)$ in Eq. (26) is closely periodic. It roughly describes a rotating Gaussian which collides with the unmoved one associated with $Q_0(\alpha)$ after each time interval of $2\pi\Delta/(\xi^2+\eta^2)$. This is the time when the revivals of the Rabi oscillations, which are compact and nearly periodic in the large detuning limit [3], occur.

The multipeaked structure of the quasiprobability distribution, its dependence on the initial atomic conditions and detuning parameter are explainable from the dressed–state viewpoint by connecting each contributing component of the Q–function with the corresponding dressed state [19]. This approach, however, does not tell us much about the state of the system itself. As has been pointed out by Gea-Banacloche [24], in order to follow the time evolution of the quasiprobability distributions and of the field and atomic states as well, in the high field limit, it is very instructive to write the initial atomic state in terms of the semiclassical eigenstates. For simplicity, we impose on resonance ($\Delta = 0$). Then, the semiclassical Hamiltonian in interaction picture is obtained by replacing the field annihilation operator in H_{AF} [Eq. (2)] by the complex number $z = |z|e^{i\varphi}$. Its eigenstates read

$$\begin{aligned}
|\psi^1\rangle &= \frac{\eta}{\sqrt{\xi^2+\eta^2}}|1\rangle - \frac{\xi}{\sqrt{\xi^2+\eta^2}}|3\rangle,\\
|\psi^2\rangle &= \frac{1}{\sqrt{2}}\left(\frac{\xi}{\sqrt{\xi^2+\eta^2}}|1\rangle + e^{i\varphi}|2\rangle + \frac{\eta}{\sqrt{\xi^2+\eta^2}}|3\rangle\right),\\
|\psi^3\rangle &= \frac{1}{\sqrt{2}}\left(\frac{\xi}{\sqrt{\xi^2+\eta^2}}|1\rangle - e^{i\varphi}|2\rangle + \frac{\eta}{\sqrt{\xi^2+\eta^2}}|3\rangle\right).
\end{aligned} \tag{28}$$

If the cavity initially contains a highly excited coherent field $|\alpha_0\rangle$ with $\bar{n} = |\alpha_0|^2 \gg 1$, these states evolve as

$$|\psi^1(t)\rangle = |\psi^1\rangle|\alpha_0\rangle, \tag{29}$$

$$\begin{aligned}
|\psi^2(t)\rangle &\simeq \frac{1}{\sqrt{2}}\left(\frac{\xi}{\sqrt{\xi^2+\eta^2}}|1\rangle + e^{i\varphi}\exp\left[-i\sqrt{\xi^2+\eta^2}\frac{t}{2\sqrt{\bar{n}}}\right]|2\rangle + \frac{\eta}{\sqrt{\xi^2+\eta^2}}|3\rangle\right)\\
&\quad\times\sum_{n=0}^{\infty} b_{0n}\exp[-i\sqrt{(\xi^2+\eta^2)n}t]|n\rangle,
\end{aligned} \tag{30}$$

$$\begin{aligned}
|\psi^3(t)\rangle &\simeq \frac{1}{\sqrt{2}}\left(\frac{\xi}{\sqrt{\xi^2+\eta^2}}|1\rangle - e^{i\varphi}\exp\left[i\sqrt{\xi^2+\eta^2}\frac{t}{2\sqrt{\bar{n}}}\right]|2\rangle + \frac{\eta}{\sqrt{\xi^2+\eta^2}}|3\rangle\right)\\
&\quad\times\sum_{n=0}^{\infty} b_{0n}\exp[i\sqrt{(\xi^2+\eta^2)n}t]|n\rangle.
\end{aligned} \tag{31}$$

Equation (29) is exact since $|\psi^1\rangle$ is a coherent trapping state [3] while the equations (30) and (31) are asymptotic in the sense that the differences between their right–hand sides and the exact solutions (8) are vectors whose norms vanish in the limit $\bar{n}\to\infty$. The product form of Eqs. (29)–(31) indicates that if the atom is initially prepared in one of the states $|\psi^i\rangle$ ($i = 1,2,3$), the atom and the field subsequently remain disentangled, despite the fact that they in fact influence each other dynamically. Since the states (28) form a basis set for the atom, the evolution of any other initial state can be expressed as a linear combination of (29), (30), and (31). For instance, when the atom is prepared initially in the upper state $|2\rangle$

$$|2\rangle = \frac{1}{\sqrt{2}}\left(|\psi^2\rangle - |\psi^3\rangle\right) \tag{32}$$

(apart from a global phase factor), the atom–field state vector at time t can be written as

$$|2\rangle_t = \frac{1}{\sqrt{2}}\left[|\psi^2(t)\rangle - |\psi^3(t)\rangle\right]. \tag{33}$$

From Eqs. (30), (31), and (33) it follows that the quasiprobability distribution Q is composed of two components

$$Q_\pm = |\langle\alpha|\Phi_\pm(t)\rangle|^2, \tag{34}$$

where

$$|\Phi_\pm(t)\rangle = \sum_{n=0}^{\infty} b_{0n}\exp[\mp i\sqrt{(\xi^2+\eta^2)n}t]|n\rangle, \tag{35}$$

that is, we recover the double–peaked structure of the Q–function mentioned above. The field states $|\Phi_\pm(t)\rangle$ have the same form as the ones in the JCM do [24] with

$\sqrt{\xi^2+\eta^2}$ standing for the JCM coupling constant g. Their statistical properties have been studied thoroughly in [24]. From Eqs. (30) and (31), it also follows that at half of the revival time

$$t_0 = \frac{\pi\sqrt{\bar{n}}}{\sqrt{\xi^2+\eta^2}} \left(= \frac{T_R}{2}\right), \tag{36}$$

the atomic states become identical and are equal to

$$\frac{1}{\sqrt{2}}\left(\frac{\xi}{\sqrt{\xi^2+\eta^2}}|1\rangle - ie^{i\varphi}|2\rangle + \frac{\eta}{\sqrt{\xi^2+\eta^2}}|3\rangle\right). \tag{37}$$

Consequently, if the initial atomic state is a linear superposition of $|\psi^2\rangle$ and $|\psi^3\rangle$, the interaction forces the atom into the unique attractor state (37) at $t_0 = T_R/2$. This means also that at that time, the state of the cavity field is a coherent superposition of the states $|\Phi_\pm(t_0)\rangle$, which, as has been shown in [24], are macroscopically distinct states with opposite phases.

In contrast with the state $|2\rangle$, in the initial atomic state $|1\rangle$

$$|1\rangle = \frac{\eta}{\sqrt{\xi^2+\eta^2}}|\psi^1\rangle + \frac{1}{\sqrt{2}}\frac{\xi}{\sqrt{\xi^2+\eta^2}}\left(|\psi^2\rangle + |\psi^3\rangle\right) \tag{38}$$

all three semiclassical eigenstates (28) contribute. As a consequence, the corresponding Q–function at time t will be composed of the three components Q_0 and $Q_\pm$ with their weights equal to $\eta^2/(\xi^2+\eta^2)$ and $(1/2)\xi^2/(\xi^2+\eta^2)$, respectively, which is in agreement with Eq. (25). Clearly, the appearance of the time–independent component Q_0 is due to the presence of the third level 3. This peak disappears if the dipole coupling constant η between levels 2 and 3 vanishes.

In conclusion, we have studied the quasiprobability distributions in the one–mode three–level problem. The dynamics of the Q–function has been considered in detail for the Λ–type atom under the high field limit and various initial atomic conditions. It has been shown that when the electron is initially at level 2, the dynamics of the Q–function in the three–level system resembles very much that in the JCM. A noticeably different behaviour takes place when the electron is initially at level 1 or level 3. In both the cases, the time behaviour of $Q(\alpha,t)$ has been connected with the collapses and revivals of the Rabi oscillations. The detuning–dependence of the Q–function has been discussed. The

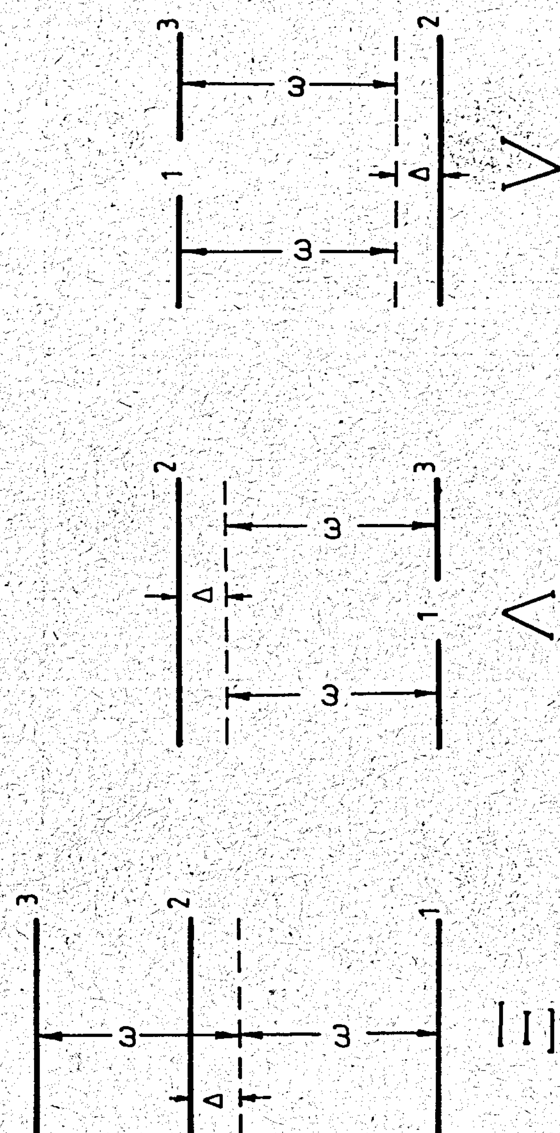

FIG. 1 The three possible energy level configurations for a three–level atom under the two–photon resonance condition.

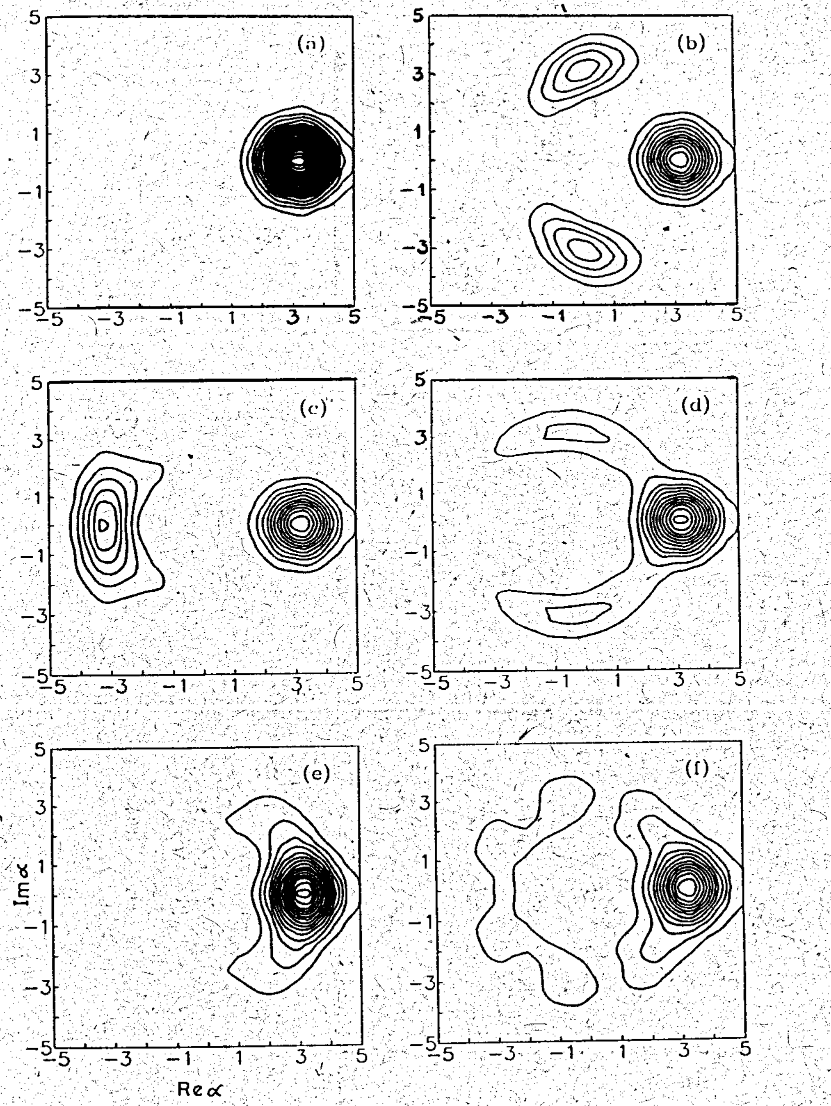

FIG. 2

The quasiprobability distribution $Q(\alpha, t)$ for the $\Xi = \Lambda$–type three–level atom plotted in the complex α–plane for $\Delta = 0$, $\xi = \eta \equiv \kappa$, $|\alpha_0|^2 = 10$, $\varphi = 0$, and for various times: (a) $T = 0.$, (b) $T = .5$, (c) $T = 1.$, (d) $T = 1.5$, (e) $T = 2.$, (f) $T = 6.$. The scaled time $T = \kappa t/(\pi|\alpha_0|\sqrt{2})$.

splittings of the quasiprobability distribution Q into contributing components have been explained from the point of view of the eigenstates of the semiclassical Hamiltonian. It has also been shown that for certain initial atomic states, the atom is forced into a unique pure state at precisely half of the revival time and at that time, the field in the cavity represents a coherent superposition of the two macroscopically distinct states.

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