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ANTIBUNCHING AND SUB-POISSONIAN PHOTON STATISTICS IN THE JAYNES-CUMMINGS MODEL

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Антигруппировка и субпуассоновская статистика фотонов в модели Джейнса-Каммингса

Исследованы субпуассоновская статистика и антигруппировка фотонов с помощью определения, основанного на положительности производной $[g^{(2)}(t,t+\tau)]'_{\tau}|_{\tau=0} > 0$ при различных начальных условиях. Показано, что антигруппировка не сопровождается субпуассоновской статистикой фотонов. Обнаружено, в частности, что антигруппировка фотонов появляется и при сильном хаотическом поле, в то время как субпуассоновское поведение исчезает, а сжатие не появляется вообще.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Antibunching and Sub-Poissonian Photon Statistics in the Jaynes-Cummings Model

Sub-Poissonian photon statistics and antibunching by the definition based on the positive derivative of $g^{(2)}(t,t+\tau)$ as a function of the delay time τ at $\tau=0$ are treated in a single-mode single-two-level-atom model. It is shown, in particular, that sub-Poissonian behaviour may be accompanied by bunching or antibunching and photon-antibunching may be accompanied by sub-Poissonian or super-Poissonian field statistics. It is also shown that the photon-antibunching persists with increasing mean photon number of an initial chaotic state while the sub-Poisson behaviour disappears.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1 Introduction

As the experimental techniques become ever more sophisticated, it is now possible to investigate in the laboratories new forms of light that have never been realized before. Three most striking examples are squeezed light, sub-Poisson light and antibunched light. Light with quantum fluctuations in one quadrature smaller than those associated with coherent light is said to be squeezed [1-3]. Light whose photon number fluctuations are smaller than those of the Poisson distribution is called sub-Poisson or alternatively, photon-number-squeezed light [4-7]. A good measure of the extent to which the photon statistics of a state are sub-Poissonian is the Q-parameter

$$Q \equiv \frac{<(\Delta n)^2>}{<n>} - 1 \qquad (1)$$

introduced by Mandel [8]. The more negative Q is, the more sub-Poissonian are the field statistics. Another important characteristic of the field statistics is the quantum degree of second-order coherence. It is obtained by evaluating the transition rate for a joint absorption of photons at the two space-time points [9]. For a single-mode radiation field, if purely temporal correlations are of interest, the relevant correlation function in the normalized form is

$$g^{(2)}(t,t+\tau) = \frac{<a^+(t)a^+(t+\tau)a(t+\tau)a(t)>}{<a^+(t)a(t)><a^+(t+\tau)a(t+\tau)>}. \qquad (2)$$

The coincidence rate $g^{(2)}(t,t+\tau)$ plays the central role in the definition of photon-antibunching . Two definitions are commonly used in the

literature [6,7]: the value of $g^{(2)}(t,t+\tau)$ at $\tau = 0$ is less than unity or the derivative of $g^{(2)}(t,t+\tau)$ as a function of the delay time τ at $\tau = 0$ is positive. If under antibunching one means the tendency of photons to distribute themselves separately rather than in bunches so that when a light beam falls on a photo-detector more photon pairs are detected close to each other than apart, one must use the latter definition [10].

In this paper, we study the sub-Poissonian photon statistics and antibunching in the Jaynes-Cummings model (JCM) of a single two-level atom coupled to a single mode of the cavity radiation field [11]. We understand antibunching by the definition based on the positive derivative of $g^{(2)}(t,t+\tau)$ as a function of τ at $\tau = 0$. It is shown that sub-Poissonian photon statistics is not associated with photon-antibunching : when the cavity field exhibits sub-Poissonian photon statistics, the photons can be either bunched or antibunched and oppositely, photon-antibunching can be accompanied by super- or sub-Poissonian field statistics. Examining the influence of initial atomic conditions on the evolution of Q and $[g^{(2)}(t,t+\tau)]'_{\tau}|_{\tau=0}$, we find that at exact resonance, when the atom is injected into the cavity in its ground state, an initially coherent field first becomes sub-Poisson. However, at later times, the super-Poisson behaviour dominates. By contrast with this, when the atom is injected into the cavity in the excited state, an initially coherent field first exhibits super-Poisson behaviour, but for longer times it turns up to be sub-Poisson rather than super-Poisson. Thus, the ground state atom case proves to be more effective in producing sub-Poisson light in the short-time region but is inferior to the excited atom case at later times. We also find that right after turning the interaction on, the excited atom case shows antibunching while the ground state atom case does not. Further, we compare the time behaviour of Q and $[g^{(2)}(t,t+\tau)]'_\tau|_{\tau=0}$ for the field being initially in coherent and chaotic states. It is found that for both these initial field states photon-antibunching occurs. Finally, the effects of cavity detuning on statistical properties of the radiation field are discussed.

2 Basic equations

We consider a system of a single two-level atom interacting with a single-mode of the cavity radiation field. The model Hamiltonian in the electric dipole and rotating wave approximations is given by ($\hbar = 1$)

$$H = \omega_0 R^z + \omega a^+ a + g(R^+ a + R^- a^+), \tag{3}$$

where the operators R^z and $R^\pm$ describe the atom with the transition frequency ω_0 while a^+ and a are creation and annihilation operators of photons with frequency ω; g is the atom-field coupling constant which may be treated as real and positive without any loss of generality.

Let us denote by $|e>$ and $|g>$ the excited and ground states of the two-level atom and $|n>$ the field in the Fock state with n photons. We

assume that the atom and the field are initially decoupled and the cavity field is in an arbitrary state $\rho_F = \sum_{n,n'} p_{n,n'}|n><n'|$, then the initial density matrix is

$$\rho(0) = \sum_{n,n'} p_{n,n'}|n;e><n';e| \tag{4.a}$$

for the initially excited atom and

$$\rho(0) = \sum_{n,n'} p_{n,n'}|n;g><n';g| \tag{4.b}$$

for the initially unexcited atom. By solving the corresponding equations of motion one finds

$$\begin{aligned} e^{-iHt}|n;e> &= A_{n,e}(t)|n;e> + B_{n,e}(t)|n+1;g>, \\ e^{-iHt}|n+1;g> &= A_{n+1,g}(t)|n+1;g> + B_{n+1,g}(t)|n;e>, \end{aligned} \tag{5}$$

where

$$\begin{aligned} A_{n,e}(t) &= \exp\left[-i\omega\left(n+\frac{1}{2}\right)t\right]\left(\cos f_n t - i\frac{\Delta}{2f_n}\sin f_n t\right), \\ A_{n+1,g}(t) &= \exp\left[-i\omega\left(n+\frac{1}{2}\right)t\right]\left(\cos f_n t + i\frac{\Delta}{2f_n}\sin f_n t\right), \\ A_{0,g}(t) &= \exp\left(\frac{i\omega_0 t}{2}\right), \qquad B_{0,g}(t) = 0, \\ B_{n,e}(t) = B_{n+1,g}(t) &= \exp\left[-i\omega\left(n+\frac{1}{2}\right)t\right]\left(-i\frac{g\sqrt{n+1}}{f_n}\sin f_n t\right) \end{aligned} \tag{6}$$

with f_n being the generalized Rabi frequency

$$f_n = \sqrt{g^2(n+1) + \Delta^2/4}, \tag{7}$$

and Δ being the detuning parameter

$$\Delta = \omega_0 - \omega. \tag{8}$$

The quantities we are interested in are Mandel's Q-parameter (1) which can be rewritten as

$$Q = \frac{< a^+(t)a^+(t)a(t)a(t) > - < a^+(t)a(t) >^2}{< a^+(t)a(t) >}, \tag{9}$$

and the derivative of $g^{(2)}(t, t+\tau)$ as a function of τ at $\tau = 0$

$$\begin{aligned} &[g^{(2)}(t,t+\tau)]'_\tau|_{\tau=0} = \\ &\Big\{[< a^+(t)a^+(t+\tau)a(t+\tau)a(t) >]'_\tau\,|_{\tau=0} < a^+(t)a(t) > \\ &- < a^+(t)a^+(t)a(t)a(t) > [< a^+(t+\tau)a(t+\tau) >]'_\tau\,|_{\tau=0}\Big\} \\ &\times < a^+(t)a(t) >^{-3}. \end{aligned} \tag{10}$$

Making use of (5) and (6) one gets

$$\begin{aligned} &< a^+(t)a(t) >= \bar{n} + \sum_{n=0}^{\infty} p_n \frac{g^2(n+1)}{f_n^2} \sin^2 f_n t, \\ &< a^+(t)a^+(t)a(t)a(t) >= \overline{n^2} - \bar{n} + \sum_{n=0}^{\infty} p_n \frac{2g^2(n+1)n}{f_n^2} \sin^2 f_n t, \\ &[< a^+(t+\tau)a(t+\tau) >]'_\tau\,|_{\tau=0} = \sum_{n=0}^{\infty} p_n \frac{g^2(n+1)}{f_n} \sin 2f_n t, \\ &[< a^+(t)a^+(t+\tau)a(t+\tau)a(t) >]'_\tau\,|_{\tau=0} = \sum_{n=0}^{\infty} p_n \frac{g^2(n+1)n}{f_n} \sin 2f_n t \end{aligned} \tag{11.a}$$

for the initial condition (4.a) and

$$\begin{aligned} &< a^+(t)a(t) >= \bar{n} - \sum_{n=0}^{\infty} p_n \frac{g^2 n}{f_{n-1}^2} \sin^2 f_{n-1} t, \\ &< a^+(t)a^+(t)a(t)a(t) >= \overline{n^2} - \bar{n} - \sum_{n=0}^{\infty} p_n \frac{2g^2 n(n-1)}{f_{n-1}^2} \sin^2 f_{n-1} t, \\ &[< a^+(t+\tau)a(t+\tau) >]'_\tau\,|_{\tau=0} = -\sum_{n=0}^{\infty} p_n \frac{g^2 n}{f_{n-1}} \sin 2f_{n-1} t, \\ &[< a^+(t)a^+(t+\tau)a(t+\tau)a(t) >]'_\tau\,|_{\tau=0} = -\sum_{n=0}^{\infty} p_n \frac{g^2 n(n-1)}{f_{n-1}} \sin 2f_{n-1} t \end{aligned} \tag{11.b}$$

for the initial condition (4.b). Here for brevity we have used the notation p_n instead of p_{nn}. By substituting expressions (11) into Eqs.(9) and (10) one easily obtains explicit results for Mandel's Q-parameter and the derivative $[g^{(2)}(t, t+\tau)]'_\tau$ at $\tau = 0$. Recall that the value $Q = 0$ corresponds to the Poisson distribution with its variance exactly equal to the mean. Light for which $Q > 0$ (< 0) has fluctuations larger (smaller) than those of the Poisson and is said to exhibit super-(sub-)Poisson behaviour. Q has a lower bound of -1 corresponding to a pure number state. The photon bunching and antibunching are defined by the behaviour of the normalized coincidence rate $g^{(2)}(t, t+\tau)$ in the vicinity of $\tau = 0$. If $g^{(2)}(t, t+\tau)$ falls with increasing τ from $\tau = 0$ (negative derivative), the light is said to be bunched. If $g^{(2)}(t, t+\tau)$ rises as τ increases from $\tau = 0$ (positive derivative), the light is said to be antibunched.

3 Effects of initial conditions

In this section we consider the evolution of the Q-parameter and the derivative $[g^{(2)}(t,t+\tau)]'_\tau$ at $\tau = 0$ for the cavity field initially prepared in the coherent and chaotic states. Cases of the initially excited and unexcited atom are compared for the initial coherent field. We suppose for the time being that the atom-field interaction takes place at exact resonance $\Delta = 0$.

A coherent state field

The coherent state field has a Poisson photon number distribution

$$p_n = \exp(-\bar{n})\frac{\bar{n}^n}{n!} \tag{12}$$

with $<(\Delta n)^2>= \bar{n}$ and $Q = 0$. After inserting this weight function into Eqs.(11), direct numerical calculations can be performed for not too large $\bar{n}$. The results for the atom being initially in the ground state are shown in Figs.1 where we have plotted the Q-parameter (solid curves) and the derivative $[g^{(2)}(t,t+\tau)]'_\tau$ at $\tau = 0$ (dashed curves) against the dimensionless time gt for two values of the mean $\bar{n} = 1$ (Fig.1a) and $\bar{n} = 10$ (Fig.1b). As the interaction is switched on, the curves representing Q go down indicating that the field statistics become sub-Poissonian . After some times, these curves go up and Q reaches positive values, which mean super-Poissonian photon statistics. In general, Q oscillates near the initial zero in the course of time and the field statistics changes correspondingly between being the sub-Poissonian and super-Poissonian. However, as can be seen from the figures, the interacting field spends more its time

in the state with super-Poissonian photon statistics. When the field intensity increases, the magnitude of the oscillations of Q decreases. This supports the conclusion of Hillery [12] that the larger the amplitude of the initial coherent state, the less sub-Poissonian (or super-Poissonian) the photon statistics can become.

As soon as $t > 0$, the value of $[g^{(2)}(t,t+\tau)]'_\tau|_{\tau=0}$, which is initially equal to zero, decreases, i.e. the cavity field exhibits photon bunching. As time goes on, $[g^{(2)}(t,t+\tau)]'_\tau|_{\tau=0}$ starts oscillating around zero and the cavity field undergoes a regime of bunching-antibunching oscillatory transition. The field can be sub-Poisson or super-Poisson when the photons exhibit antibunching and alternatively, the photons can be bunched or antibunched when the cavity field shows sub-Poissonian photon statistics. From the figures one can also see that while the interacting field spends more its time in the state with super-Poissonian statistics, it does not show such overall time preference with respect to photon-bunching or antibunching.

The results for the initially excited atom are presented in Figs.2. Just after the interaction begins, Q increases which amounts to that the cavity field becomes super-Poisson. Thus in the short time region, an unexcited atom is more effective in producing a field with sub-Poisson statistics than an excited atom. At later times, however, there is a reversal of roles: in the case of the excited atom, the maximal extent to which the cavity field may become sub-Poisson is deeper, and the time in which the cavity field is found to be sub-Poisson is longer than those in the case of the unexcited atom. The quantity $[g^{(2)}(t,t+\tau)]'_{\tau}|_{\tau=0}$ for the initially excited atom also alternates between positive and negative values in a fashion similar to that for the initially unexcited atom. Significant discrepancy appears near $t=0$: the excited atom case shows antibunching while the unexcited atom case shows photon bunching.

When the field intensity gets stronger, collapses and revivals [13] are observed in the time behaviour of Q and $[g^{(2)}(t,t+\tau)]'_{\tau}|_{\tau=0}$. Taking into account the assumption that $\bar{n} >> 1$, the quasi-steady value of Q which is reached in the time regions between collapse and revival can be found to be

$$Q_{quasi-steady} \sim \frac{3}{4\bar{n}}$$

for the initial atomic state $|i>=|g>$ and

$$Q_{quasi-steady} \sim -\frac{1}{4\bar{n}}$$

for the initial atomic state $|i>=|e>$. In these time regions, $[g^{(2)}(t,t+\tau)]'_{\tau}|_{\tau=0}$ oscillates around zero with amplitude smaller than that in times of collapses and revivals which means a less pronounced character of the bunching and antibunching effects. It is understandable since in the quasi-steady regime the average photon number almost remains unchanged.

A chaotic state field

A chaotic state field has a diagonal field density matrix with

$$p_n = \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}, \tag{13}$$

$<(\Delta n)^2>=\bar{n}(\bar{n}+1)$ and Mandel's Q-parameter $Q=\bar{n}$. Since $Q>0$, the chaotic state is super-Poisson and is more and more so as its photon number increases. It is natural then to expect that a chaotic field may become sub-Poisson owing to the interaction with the two-level atom at very low photon number only, and that is the case [12]. In figures 3, where we have plotted Q and $[g^{(2)}(t,t+\tau)]'_{\tau}|_{\tau=0}$ as functions of the dimensionless time gt for the cavity field being initially in the chaotic state and the atom in the excited state, one sees that for $\bar{n}=1$ (Fig.3a) sub-Poisson statistics still occurs but for $\bar{n}=3$ (Fig.3b) it does not. The next point one can notice from figures 3 is that while the sub-Poisson behaviour disappears with increasing the mean photon number, $[g^{(2)}(t,t+\tau)]'_{\tau}|_{\tau=0}$ stays to alternate between positive and negative values implying that the cavity field displays oscillatory transition between antibunching and bunching. The fact that photon-antibunching persists despite the broadening of the photon number distribution is not surprising because in defining antibunching according to the positive derivative at $\tau=0$, it is important for us to know how the coincidence rate $g^{(2)}(t,t+\tau)$ changes in the vicinity of $\tau=0$ but not the value of $g^{(2)}(t,t+\tau)$ at $\tau=0$ by itself. Clearly, the changes of $g^{(2)}(t,t+\tau)$ as a function of τ are determined by the atom which absorbs and emits photons rather than by the width of the photon distribution. Of course, the photon distribution also exercises its influence on the time behaviour of $[g^{(2)}(t, t+\tau)]'_{\tau}|_{\tau=0}$. For example, one can find from the comparison of Figs.2 and Figs.3 (both have been made for the initially excited atom) that after switching the interaction on, the chaotic state field gets bunched instead of antibunched as in the case of the initially coherent state.

4 Effects of cavity detuning

As is said above, when the atom is sent into the cavity in its excited state and the exact resonance condition is met, an initially coherent field becomes sub-Poisson in the large part of the evolution time. However, it is not always the case if the detuning parameter given in (8) has a non-vanishing value. Indeed, from Fig.4 where the dependence of Q on the detuning parameter is graphically illustrated for the cavity field being initially in the coherent state and the atom in the excited state, one observes that when the increasing Δ exceeds a certain value, the super-Poisson behaviour becomes to predominate over the sub-Poisson behaviour. At far-off-resonance, the field is effectively decoupled from the atom and in consequence of that, the initial value of Q almost remains unaffected. The case of the initially unexcited atom is presented in Fig.5. In this case it is likely that the super-Poisson behaviour dominates for all values of Δ.

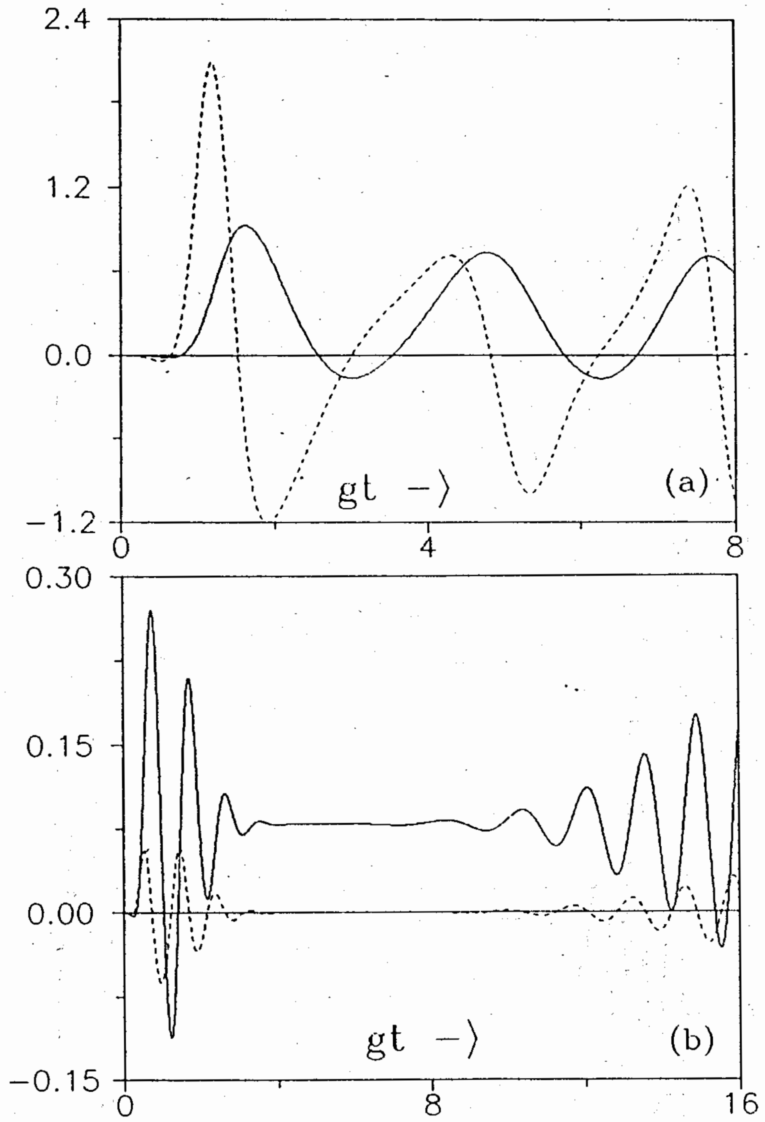

Figure 1

Evolution of Q (solid curves) and $[g^{(2)}(t, t+\tau)]'_{\tau}|_{\tau=0}$ measured in units of the coupling constant g (dashed curves) for the initially coherent state field and initially unexcited atom. Exact resonance is assumed. The mean photon number is: (a) $\bar{n} = 1$, (b) $\bar{n} = 10$.

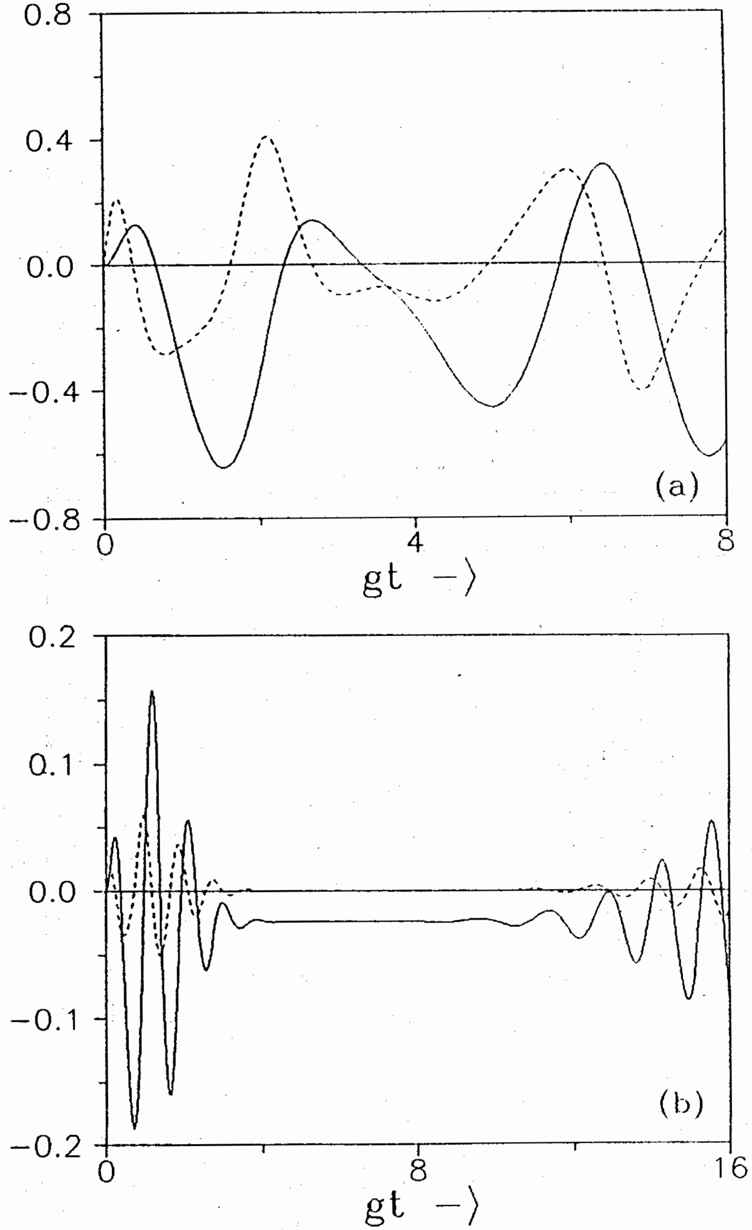

Figure 2

The same as in Fig.1, except for the initially excited atom.

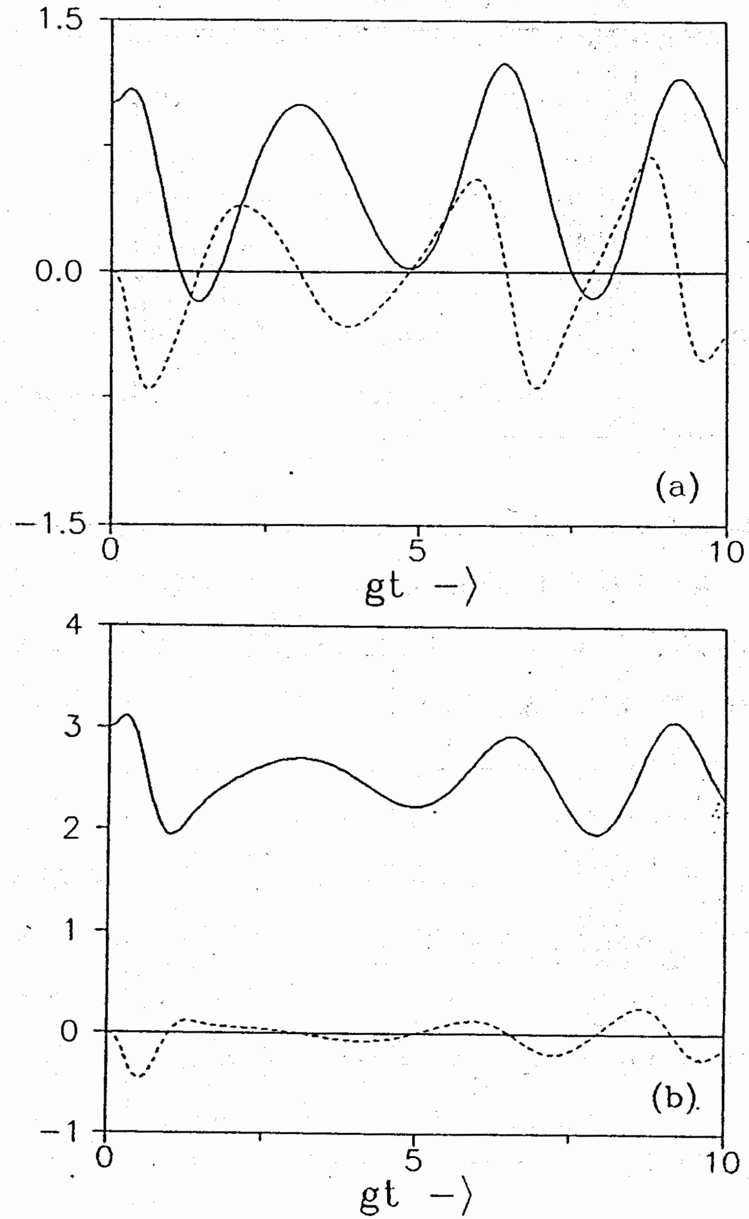

Figure 3

The same as in Fig.2, but now the cavity field is initially prepared in the chaotic state with the mean (a) $\bar{n} = 1$, (b) $\bar{n} = 3$.

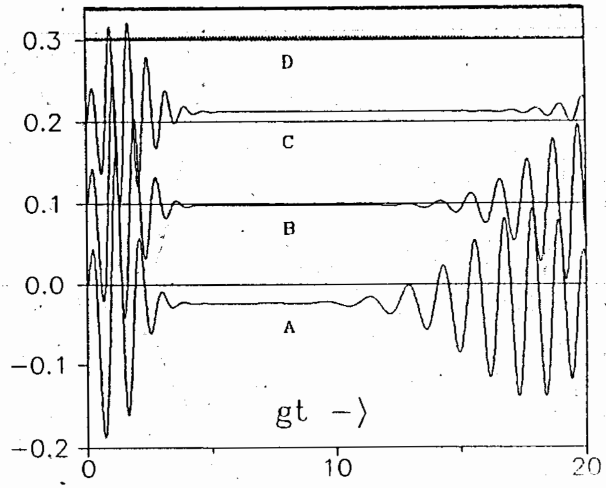

Figure 4

Evolution of Q for various values of the cavity detuning. The field is initially in the coherent state with $\bar{n} = 10$ and the atom is in the excited state. The curves shown are for: A. $\Delta = 0[Q]$, B. $\Delta = 3g[Q + 0.1]$, C. $\Delta = 5g[Q + 0.2]$, D. $\Delta = 50g[Q + 0.3]$.

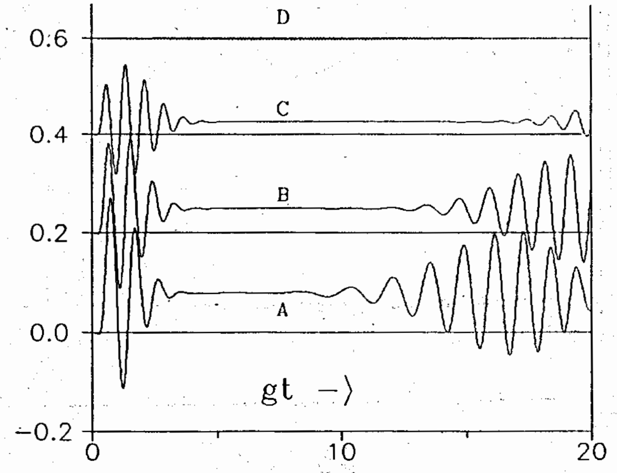

Figure 5

The same as in Fig.4 but for the initially unexcited atom. The curves shown are for:

A. $\Delta = 0[Q]$, B. $\Delta = 3g[Q+0.2]$, C. $\Delta = 5g[Q+0.4]$, D. $\Delta = 50g[Q+0.6]$.

We have also calculated numerically the time evolution of $[g^{(2)}(t, t+\tau)]'_{\tau}|_{\tau=0}$ for different values of the detuning parameter. The results show that nonzero values of Δ lead to changes of the Rabi-type oscillations in a way similar to that for the atomic inversion [13] but do not shift $[g^{(2)}(t, t+\tau)]'_{\tau}|_{\tau=0}$ from oscillations around zero. This means that the deviation of Δ from 0 does not cause the cavity field to exhibit more bunching or antibunching in the course of time.

5 Conclusions

We have investigated the appearance of the sub-Poissonian photon statistics and photon-antibunching in the JCM under different initial conditions. The property $Q < 0$ has been used to define sub-Poisson statistics and the property $[g^{(2)}(t, t+\tau)]'_{\tau}|_{\tau=0} > 0$ has been used to define antibunching . In particular, we have discovered that the photon antibunching, which is a non-classical effect, occurs even when the cavity field is prepared initially in a chaotic state with large mean photon number. This is worth noting in view of the fact that in the JCM driven by a chaotic state, the sub-Poisson behaviour occurs at a very low photon number only and the squeezing does not occur at all [14].

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