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E.I.Aliskenderov, Ho Trung Dung, L.Knöll*

EFFECTS OF ATOMIC COHERENCES IN THE JAYNES-CUMMINGS MODEL: PHOTON STATISTICS AND ENTROPY

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*Friedrich-Schiller-Universität Jena,
Theoretisch-Physikalisches Institut,
Max-Wien-Platz 1, O-6900 Jena, Germany

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Эффекты атомных когерентностей
в модели Джейнса — Каммингса:
статистика фотонов и энтропия

Q-фактор Мандела, эффект антигруппировки и энтропия вычислены для модели Джейнса — Каммингса, когда атом первоначально находится в когерентной суперпозиции нижнего и верхнего уровней. Найдено значение *Q*-фактора Мандела в области коллапса и приведено сравнение с поведением энтропии поля.

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Effects of Atomic Coherences in the Jaynes-Cummings model:
Photon Statistics and Entropy

The Mandel *Q*-factor, antibunching effect and the entropy are calculated for the Jaynes-Cumminigs model with the atom being initially in a coherent superposition of the upper and lower states. The value of Mandel's *Q*-factor in the collapse region is found and compared with the behaviour of the field entropy.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1 Introduction

The Jaynes-Cummings model (JCM) of a single-mode quantized field coupled to a two-level atom in a lossless cavity [1]–[5] is an important fundamental theoretical model of the interaction between two quantum systems. In addition to being exactly solvable it has recently become experimentally realizable with Rydberg atoms in high-Q microwave cavities [6]. The model predicts a lot of interesting effects, among them one finds the so-called "collapses" and "revivals" of the Rabi oscillations [7] which give a clear signature of the quantum nature of the interacting field. The evolution of the field and atomic states in the JCM has recently attracted a lot of attention [8]-[12]. Phoenix and Knight [8] and Gea-Banacloche [9] have shown that the atom and field most closely return to pure states during the collapse region but not at the peak of revivals as may be expected and have found the explicit forms for the atomic and field states at this time [9]-[10]. The pure atomic state at half revival time can be generated even from an initial mixed atomic state [12].

Recently, Agarwal and Puri [13], Zaheer and Zubairy [14] have considered a two-level atom injected into the cavity in a coherent superposition of the upper and lower levels. They have shown that for a certain choice of the relative phase between the atomic dipole and the coherent field for which the initial atomic state is an eigenstate of the semiclassical Hamiltonian, the population inversion essentially remains unaffected. Exactly coherent trapping in two-level atoms has been found to occur [15] when the initial state of the field is an eigenstate of the Susskind-Glogower phase operator [16] and the phases of the field and dipole moment are identical. The effect of phases on the reduction of the fluctuations in atomic variables, and on the quadrature and amplitude-squared squeezing in the JCM has also been investigated [17], [18]. In this paper we study the time behaviour of the Mandel's Q-factor [19] and the antibunching effects using the definition based on the sign of $(d/d\tau)g^{(2)}(t,t+\tau)$ at $\tau = 0$ [20], [21]. We calculate the quasi-steady value which Q takes in the collapse region and find that Q is larger than zero when the atom is initially in an eigenstate of the semiclassical Hamiltonian and is equal to zero for another initial atomic state. On the other hand, the entropy in the first case is lower than that in the second one. Thus, we give one more example illustrating another important result of [8] stating that the entropy, rather than the variance, is a reliable parameter to characterize the fluctuations of the field.

2 Field Statistics

The model Hamiltonian of the JCM in the electric dipole and rotating wave approximations is given by ($\hbar = 1$)

$$H = \omega_0 R^z + \omega a^\dagger a + g(R^+ a + a^\dagger R^-), \qquad (1)$$

where the operators R^z and $R^\pm$ describe the atom with transition frequency ω_0 while $a^\dagger$ and a are creation and annihilation operators of photons with frequency ω; g is the atom-field coupling constant which may be treated as real and positive without any loss of generality.

Let the atom at the initial time $t = 0$ be prepared in a coherent superposition of the excited and ground states [17]

$$|\psi_{atom}(t=0)\rangle = \cos\frac{\theta}{2}|e\rangle + e^{i\phi}\sin\frac{\theta}{2}|g\rangle, \qquad (2)$$

and the field be in a coherent state

$$|\psi_{field}(t=0)\rangle = \sum_n q_n|n\rangle, \qquad q_n = \exp(-\bar{n}/2)\frac{\bar{n}^{\frac{n}{2}}}{\sqrt{n!}}. \qquad (3)$$

Assume that at $t = 0$ the atom and the field are decoupled, then we can write for the initial atom-field state

$$|\psi(t=0)\rangle = \sum_n q_n|n\rangle|\psi_{atom}(t=0)\rangle. \qquad (4)$$

The wave function of the total system at time t is found from the Hamiltonian (1) to be

$$|\psi(t)\rangle = \sum_n q_n\left\{\cos\frac{\theta}{2}\left[A_{ne}(t)|e,n\rangle + B_{ne}(t)|g,n+1\rangle\right] + e^{i\phi}\sin\frac{\theta}{2}\left[A_{n,g}(t)|g,n\rangle + B_{n,g}(t)|e,n-1\rangle\right]\right\}, \qquad (5)$$

where

$$\begin{aligned}
A_{n,e}(t) &= \exp\left[-i\omega\left(n+\frac{1}{2}\right)t\right]\left(\cos f_n t - i\frac{\Delta}{2f_n}\sin f_n t\right),\\
A_{n+1,g}(t) &= \exp\left[-i\omega\left(n+\frac{1}{2}\right)t\right]\left(\cos f_n t + i\frac{\Delta}{2f_n}\sin f_n t\right),\\
A_{0,g}(t) &= \exp\left(\frac{i\omega_0 t}{2}\right), \quad B_{0,g}(t) = 0, \qquad (6)\\
B_{n,e}(t) &= B_{n+1,g}(t) = \exp\left[-i\omega\left(n+\frac{1}{2}\right)t\right]\left(-i\frac{g\sqrt{n+1}}{f_n}\sin f_n t\right),
\end{aligned}$$

with f_n being the generalized Rabi frequency

$$f_n = \sqrt{g^2(n+1) + \Delta^2/4} \qquad (7)$$

and Δ being the detuning parameter

$$\Delta = \omega_0 - \omega. \qquad (8)$$

A good measure of the extend to which the photon statistics of a state is sub-Poissonian is the Q-factor

$$Q \equiv \frac{\langle(\Delta n)^2\rangle}{\langle n\rangle} - 1 \qquad (9)$$

introduced by Mandel [19]. The more negative Q is, the more sub-Poissonian is the field statistics. Another important characteristics of the field statistics is the quantum degree of second-order coherence. It is obtained by evaluating the transition rate for a joint absorption of photons at two space-time points [22]. For a single-mode radiation field, if purely temporal correlations are of interest, the relevant correlation function in the normalized form is

$$g^{(2)}(t,t+\tau) = \frac{\langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle}{\langle a^\dagger(t)a(t)\rangle\langle a^\dagger(t+\tau)a(t+\tau)\rangle}. \qquad (10)$$

The coincidence rate $g^{(2)}(t,t+\tau)$ plays the central role in the definition of photon-antibunching. Two definitions are commonly used in the literature [23]-[24]: the value of $g^{(2)}(t,t+\tau)$ at $\tau=0$ is less than unity or the derivative of $g^{(2)}(t,t+\tau)$ as a function of the delay time τ at $\tau=0$ is positive. If under antibunching one means the tendency of photons to distribute themselves separately rather than in bunches so that when a light beam falls on a photo-detector fewer photon pairs are detected close together than further apart, one must use the latter definition [20].

Making use of (5) and (6) we can rewrite the Mandel's Q-factor as

$$Q = \frac{\langle a^\dagger(t)a^\dagger(t)a(t)a(t)\rangle - \langle a^\dagger(t)a(t)\rangle^2}{\langle a^\dagger(t)a(t)\rangle} \qquad (11)$$

and the derivative of $g^{(2)}(t,t+\tau)$ as

$$\frac{d}{d\tau}g^{(2)}(t,t+\tau)\mid_{\tau=0}=$$

$$\Big\{\frac{d}{d\tau}\langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle\mid_{\tau=0}\langle a^\dagger(t)a(t)\rangle$$
$$-\langle a^\dagger(t)a^\dagger(t)a(t)a(t)\rangle\frac{d}{d\tau}\langle a^\dagger(t+\tau)a(t+\tau)\rangle\mid_{\tau=0}\Big\}\langle a^\dagger(t)a(t)\rangle^{-3}, \qquad (12)$$

where

$$\begin{aligned}
\langle a^\dagger(t)a(t)\rangle &= \bar{n} + \sum_n Q_n\sin^2 f_n t \\
&- \frac{1}{2}\sin\theta\sum_n q_nq_{n+1}F_n(t), \\
\langle a^\dagger(t)a^\dagger(t)a(t)a(t)\rangle &= \bar{n^2} - \bar{n} + 2\sum_n nQ_n\sin^2 f_n t \\
&- \sin\theta\sum_n q_nq_{n+1}nF_n(t), \qquad (13) \\
\frac{d}{d\tau}\langle a^\dagger(t+\tau)a(t+\tau)\rangle\mid_{\tau=0} &= \sum_n f_nQ_n\sin 2f_nt \\
&- \sin\theta\sum_n q_nq_{n+1}J_n(t), \\
\frac{d}{d\tau}\langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle\mid_{\tau=0} &= \sum_n nf_nQ_n\sin 2f_nt \\
&- \sin\theta\sum_n q_nq_{n+1}nJ_n(t).
\end{aligned}$$

In Eq. (13) for simplicity the notation

$$\begin{aligned}
Q_n &= \frac{g^2(n+1)}{f_n^2}\left[q_n^2\cos^2\frac{\theta}{2} - q_{n+1}^2\sin^2\frac{\theta}{2}\right], \\
F_n(t) &= \frac{g\sqrt{n+1}}{f_n}\left[\sin\phi\sin 2f_nt + \frac{\Delta}{f_n}\cos\phi\sin^2 f_nt\right], \qquad (14) \\
J_n(t) &= g\sqrt{n+1}\left[\sin\phi\cos 2f_nt + \frac{\Delta}{2f_n}\cos\phi\sin 2f_nt\right],
\end{aligned}$$

has been introduced.

It can be easily checked that for two limiting situations when the atom is initially purely excited ($\theta/2=0$) or de-excited ($\theta/2=\pi/2$) Eqs. (13) reduce to those obtained earlier by Dung et al. [21]

By substituting expressions (13) and (14) into (11) and (12) one easily obtains explicit results for Mandel's Q-factor and the derivative $[g^{(2)}(t,t+\tau)]'_\tau$ at $\tau=0$. Recall that light for which $Q>0$ (<0) has fluctuations larger (smaller) than those of a Poissonian process and is said to exhibit super- (sub-) Poissonian behaviour. However, as will be shown below, $Q=0$ is not enough to state the Poissonian character of the field. The photon bunching and antibunching are defined by the behaviour of the normalized coincidence rate $g^{(2)}(t,t+\tau)$ in the vicinity of $\tau=0$. If $g^{(2)}(t,t+\tau)$ falls with increasing τ from $\tau=0$ (negative derivative), the light is said to be bunched. If $g^{(2)}(t,t+\tau)$ rises as τ increases from $\tau=0$ (positive derivative), the light is said to be antibunched.

In the collapse region Q is nearly constant and is found to be, at exact resonance ($\Delta=0$)

$$Q_{quasi-steady} \simeq \frac{2-2\cos\theta-\cos^2\theta}{4\bar{n}+2\cos\theta}, \tag{15}$$

where we have put the relative phase between the field and the atomic dipole ϕ equal to zero. For non-resonant case (15) is very complicated and we have no such simple dependence for $Q_{quasi-steady}$ from θ and $\bar{n}$. From (15) we easily obtain the value of θ for which $Q_{quasi-steady}$ vanishes

$$\theta = \arccos(\sqrt{3}-1). \tag{16}$$

Equation (15) can be easily generalized to the case of m-photon JCM with the Hamiltonian [25]–[26] (without taking into account the ac Stark shifts)

$$H = \omega_0 R^z + \omega_0 a^\dagger a + g(R^+ a^m + a^{\dagger m} R^-). \tag{17}$$

to be

$$Q_{quasi-steady} \simeq \frac{2m^2-2m\cos\theta-m^2\cos^2\theta}{4\bar{n}+2m\cos\theta}. \tag{18}$$

Then the condition for $Q_{quasi-steady}=0$ reads

$$\cos\theta = \frac{-1\pm\sqrt{1+2m^2}}{m}. \tag{19}$$

For the two-photon JCM ($m=2$) equation (19) means $\cos\theta=1$, i.e. $Q_{quasi-steady}=0$ for an initially inverted atom. For m larger than 2 Eq. (19) has no real solution θ, and the field shows only super-Poissonian statistics in the collapse region. Note that in this region $[g^{(2)}(t,t+\tau)]'_\tau\,|_{\tau=0}$ is equal to zero for any values of m, θ and $\bar{n}$, which is in contradiction to the first definition of antibunching.

It does not appear possible be express the sums in equation (13) in closed form. But for not too large $\bar{n}$, the direct numerical evaluations can be performed. The results show that, in general, Q and $[g^{(2)}(t,t+\tau)]'_\tau\,|_{\tau=0}$ oscillate near the initial time $t=0$ and in the revival region. The field can be sub-Poissonian or super-Poissonian when the photons exhibit antibunching and alternatively, the photons can be bunched or antibunched when the cavity field shows sub-Poissonian photon statistics (see Fig.1.). By changing θ we can reach positive and negative values or zero for Mandel's Q-factor. The numerical calculations of Mandel's Q-factor for the initially coherent field are presented in Fig 2. for (a) $\cos\theta=0$ and (b) $\cos\theta=\sqrt{3}-1$. As can be seen from Fig 2.a., for the case (a) the interacting field shows super-Poissonian statistics, during the whole time. In the case (b) (Fig 2.b.) in accordance with Eq. (19) the Mandel's Q-factor is equal to zero in the collapse region. But as we shall see from the analytical and numerical results presented below it does not mean that in this region we have the Poissonian photon statistics. This indicates only that the variance, which is sensitive to the moment of the second order, in the case (a) is larger than that in the case (b).

Further, we calculate the photon number distribution in the given model. The density matrix is defined as

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|. \tag{20}$$

Then, the photon number distribution at later time t is given by

$$P(n,t) = \mathrm{Tr}_A\langle n|\rho(t)|n\rangle. \tag{21}$$

By using the expression (5) for the wave function $|\psi(t)\rangle$ of the total system, we obtain the following formula for the photon number distribution

$$P(n,t) = q_n^2\left[\cos^2\frac{\theta}{2}I_n(t) + \sin^2\frac{\theta}{2}I_{n-1}(t)\right] + q_{n-1}^2\cos^2\frac{\theta}{2}L_{n-1}(t)$$
$$+q_{n+1}^2\sin^2\frac{\theta}{2}L_n(t) + \frac{1}{2}q_nq_{n+1}\sin\theta F_n(t) - \frac{1}{2}q_nq_{n-1}\sin\theta F_{n-1}(t), \tag{22}$$

where

$$\begin{aligned} I_n(t) &= \cos^2 f_n t + \frac{\Delta^2}{4f_n^2}\sin^2 f_n t, \\ L_n(t) &= \frac{g^2(n+1)}{f_n^2}\sin^2 f_n t \end{aligned} \tag{23}$$

and $F_n(t)$ is defined by (14).

The evolution of the photon number distribution is presented in Fig. 3. for the initially coherent field and for cases (a) $\cos\theta = 0$ and (b) $\cos\theta = \sqrt{3}-1$. In the case of $\cos\theta = \sqrt{3}-1$ though $Q_{quasi-steady} = 0$ the photon number distribution at time t has a multipeaked structure, i.e., differs from being a Poissonian distribution. But for $\cos\theta = 0$ the curves $P(n,t)$ are similar to Poissonian one. As can be seen from Eq. (15) and Fig. 2.a. in this case $Q_{quasi-steady} > 0$ and the photon number distribution has the super-Poissonian photon statistics.

3 Entropy

In this section we compare the time evolution of the entropy with that of the Mandel's Q-factor. Since for systems in which both the atom and field start from pure states, the atomic and field entropies are equal [8], [27] and the calculation of the atomic entropy is more transparent [8], [12], [28], below we treat the time behaviour of the atomic entropy. The atomic density matrix can be obtained by tracing over the field variables and can be written as

$$\rho_{atom}(t) = \begin{bmatrix} \alpha(t) & \gamma(t) \\ \gamma^*(t) & \beta(t) \end{bmatrix} \tag{24}$$

where $\alpha(t)$, $\gamma(t)$, $\beta(t)$ can be easily found by using equations (5) and (20).

The atomic entropy is given by

$$S_{atom} = -\mathrm{Tr}(\rho_{atom}\ln\rho_{atom}). \tag{25}$$

Since the trace is invariant under a similarity transformation, we can go to a basis in which the atomic density matrix is diagonal and write Eq. (25) in the form

$$S_{atom} = -\sum_k(\lambda_k\ln\lambda_k). \tag{26}$$

The λ_k can be derived from equation (24) in a straightforward manner

$$\lambda_{1,2} = \frac{1}{2}\left\{1 \pm \sqrt{1-4\left[\alpha(t)\beta(t) - |\gamma(t)|^2\right]}\right\}, \tag{27}$$

with the elements of the atomic density matrix being

$$\alpha(t) = \frac{1}{2} + \frac{1}{2}\cos\theta\sum_n \rho_{nn}(0)\cos 2gt\sqrt{n+1}$$

$$+ \frac{1}{2}\sin\theta\sin\phi\sum_n \rho_{nn+1}(0)\sin 2gt\sqrt{n+1}, \tag{28}$$

$$\begin{aligned}\gamma(t) &= \cos^2\frac{\theta}{2}\sum_n i\rho_{nn-1}(0)\cos gt\sqrt{n+1}\sin gt\sqrt{n}\\ &- \sin^2\frac{\theta}{2}\sum_n i\rho_{n+1,n}(0)\sin gt\sqrt{n+1}\cos gt\sqrt{n}\\ &+ \frac{1}{2}\exp(-i\phi)\sin\theta\sum_n \rho_{nn}(0)\cos gt\sqrt{n+1}\cos gt\sqrt{n}\\ &+ \frac{1}{2}\exp(i\phi)\sin\theta\sum_n \rho_{n+1,n-1}(0)\sin gt\sqrt{n+1}\sin gt\sqrt{n},\end{aligned} \tag{29}$$

$$\beta(t) = 1-\alpha(t). \tag{30}$$

where for initial coherent field we can write $\rho_{nm}(0) = q_n q_m$ and q_n are defined by (3).

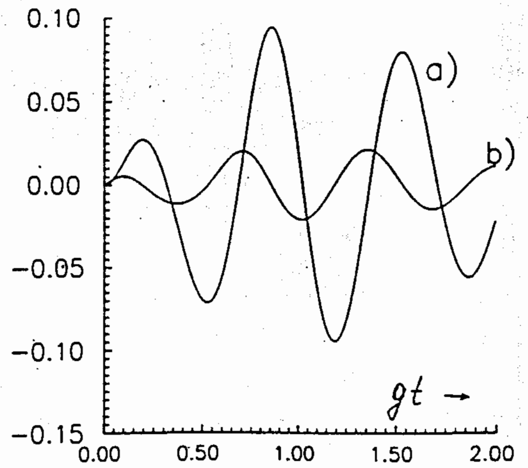
In Fig. 4 we have plotted the entropy for the two values of $\cos\theta$ (*a*) $\cos\theta = 0$ and (*b*) $\cos\theta = \sqrt{3}-1$. In the case (b), as we see from the figure, when the interaction is turned on, the entropy increases rapidly from the initial value zero, but in the middle of the collapse region, it decreases significantly indicating that the atomic and field subsystems roughly return to pure states. This is in agreement with the results of [8]–[12]. In the case (a), which corresponds to a trapping state [13]-[15], the atomic and field subsystems go away from their initial pure states very slowly.

Even more interesting is that in the first case (a) the entropy is always smaller than that in the second case (b), which allows us to conclude (somewhat arbitrarily) that in the collapse region the field state, when $\cos\theta = 0$, is closer to a Poissonian one than when $\cos\theta = \sqrt{3}-1$. Figure 3 of the photon number distributions also supports this conclusion. On the other hand, as has been shown in equations (15), (16), Mandel's factor $Q_{quasi-steady} = 0$ for $\cos\theta = \sqrt{3}-1$ and $Q_{quasi-steady} > 0$ for $\cos\theta = 0$.

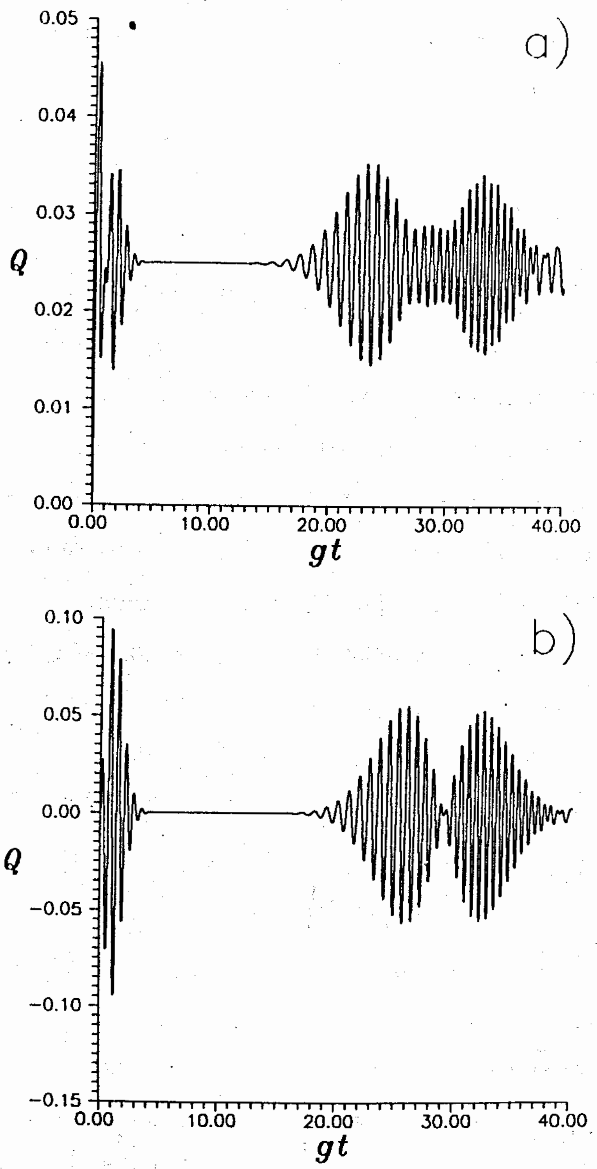
Thus, relying on the Q-factor alone can lead to some misinterpretations on the fluctuations of the field.

4 Conclusion

It is shown for JCM that by varying the weights of the upper and lower states in the initial coherent superposition atomic state, one can change the cavity field from being sub-Poissonian to super-Poissonian. Moreover, sub-Poissonian photon statistics does not imply photon anti-bunching and can be accompanied by photon bunching. It is found that the investigation of the Q-factor, which is equivalent to an investigation of the variance of the photon number distribution, is not enough to conclude about the behaviour of the field fluctuations. This is also confirmed

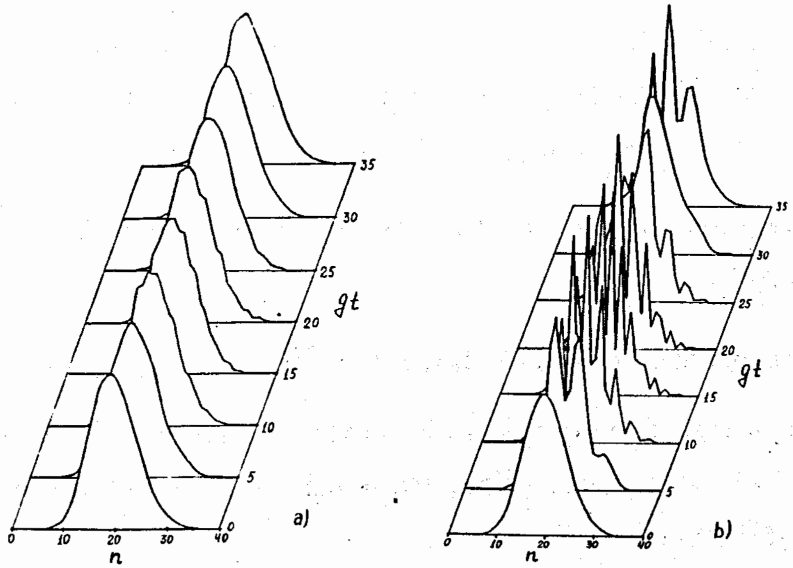


• Figure 1. The time evolution of (a) the Mandel's Q-factor and (b) derivative $[g^{(2)}(t,t+\tau)]'_\tau\,|_{\tau=0}$ for $\bar{n} = 20$, $\phi = 0$, $\Delta = 0$, $\cos\theta = 0$.

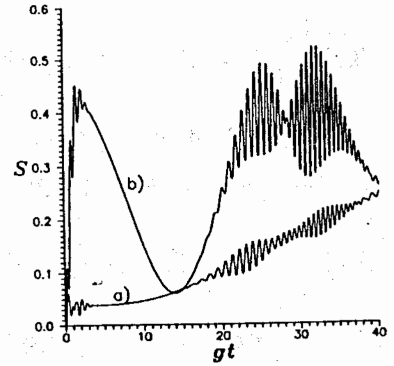


• Figure 2. Mandel's Q-factor for $\bar{n} = 20$, $\phi = 0$, $\Delta = 0$ and (a) $\cos\theta = 0$ and (b) $\cos\theta = \sqrt{3} - 1$.

by the results concerning the photon number distribution and the atomic entropy. The explicit expression for Mandel's Q-factor and the derivative $\left[g^{(2)}(t, t+\tau)\right]'_{\tau}\Big|_{\tau=0}$ in the collapse region has been found.



• Figure 3. Photon number distribution for $\bar{n} = 20$, $\phi = 0$, $\Delta = 0$ and (a) $\cos\theta = 0$ and (b) $\cos\theta = \sqrt{3} - 1$.



• Figure 4. The atomic entropy for $\bar{n} = 20$, $\phi = 0$, $\Delta = 0$ and (a) $\cos\theta = 0$ and (b) $\cos\theta = \sqrt{3} - 1$.

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