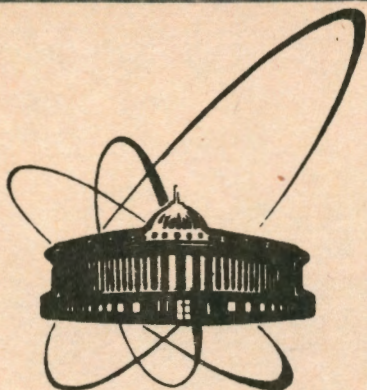


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FLUCTUATIONAL THEORY OF EQUILIBRIUM  
INHOMOGENEOUS STATES  
USING PATH INTEGRALS

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## 1. FROM STOCHASTIC MODELS TO DETERMINISM OR QUITE THE REVERSE?

The path integral formalism has mainly been developed for the use in quantum mechanics and stochastic irreversible thermodynamics with physical time  $t$  as the only independent variable. The distribution function  $f(q^j, t)$  ( $j = 1, 2, \dots, n$ ) of nonequilibrium states satisfies a Fokker — Planck equation

$$\frac{\partial f(q^j, t)}{\partial t} = \left[ -\frac{\partial v^j(q^j)}{\partial q^j} + \frac{1}{2} \frac{\partial^2 a^{jk}(q^j)}{\partial q^j \partial q^k} \right] f(q^j, t) \quad (1.1)$$

This equation is equivalent (only formally but not physically) to the Schrödinger equation with «imaginary time»

$$\frac{\partial f(q^j, t)}{\partial t} = -H(q^j) f(q^j, t), \quad (1.2)$$

where  $H$  is the «Hamiltonian» from (1.1). Then, the distribution function at a moment of time  $t$

$$f(q^j, t) = \int F(q^j, t | q_0^j, t_0) f(q_0^j, t_0) dq_0^j, \quad (1.3)$$

in which the conditional probability  $F$  is fulfilled by two exact equalities

$$\int F(q^j, t | q_0^j, t_0) dq^j = 1, \quad (1.4)$$

$$F(q^j, t | q_0^j, t_0) = \int F(q^j, t | q_1^j, t_1) F(q_1^j, t_1 | q_0^j, t_0) dq_1^j, \quad (1.5)$$

has, in accordance with (1.2), the following form:

$$f(q^j, t) = \int \langle q^j | \exp[-H \cdot (t - t_0)] | q_0^j \rangle f(q_0^j, t_0) dq_0^j, \quad (1.6)$$

where

$$F(q^j, t | q_0^j, t_0) = \langle q^j | \exp[-H \cdot (t - t_0)] | q_0^j \rangle. \quad (1.7)$$

The last equality gives a formal opportunity to use the well-known results of quantum mechanics for calculating each individual multiplier  $F(q'_k, t_k | q'_{k-1}, t_{k-1})$  in the path integral

$$F(q', t | q'_0, t_0) = \int \left( \prod_{k=1}^{k=N} F(q'_k, t_k | q'_{k-1}, t_{k-1}) \right) dq'_1 \dots dq'_{N-1}, \quad (1.8)$$

which corresponds to the limit  $N \rightarrow \infty$  in (1.8). This approach, useful from a practical point of view, is open to criticism on some basic points when we are interested in many-particle systems. Quantum mechanics has mainly been developed to deal with closed dynamical systems. For the investigation of open dynamical systems interacting with the external world it is usually suggested that the total Hamiltonian  $H$  splits into three parts

$$H = H_{\text{sys.}} + H_{\text{int.}} + H_{\text{ext.}}, \quad (1.9)$$

the first  $H_{\text{sys.}}$  depending only on the system, the second  $H_{\text{int.}}$  representing the interaction between the system and the external world, and the third  $H_{\text{ext.}}$  depending only on the external world. When the system in the macroscopic volume has an enormous (but finite) number of degrees of freedom, it is impossible to specify rigorously the part  $H_{\text{int.}}$  in (1.9). The only way, apparently, to avoid this difficulty in the framework of quantum mechanics is adoption of the phenomenological approach of irreversible thermodynamics. Quantum mechanics of non-conservative open dynamical systems, interacting with chaotic thermal environment, has been developed in the Nelson method of stochastic quantization on the basis of the Newton — Langevin equation from classical mechanics

$$m \frac{d^2 q^j}{dt^2} = -l \frac{dq^j}{dt} - \frac{\partial U(q^j, t)}{\partial q^j} + A^j(t), \quad (1.10)$$

where  $m$  denotes «mass»,  $l$  is the «friction coefficient»,  $U$  is the usual potential function, and  $A^j(t)$  the Gaussian white noise in the generalized coordinate space  $\{q^j\}$ . To open systems such as (1.10), the canonical quantization procedure demanding the knowledge of the Hamiltonian  $H$  is not applicable. To open systems such as (1.10), the path integral quantization procedure demanding the knowledge of the Lagrangian  $L$  is not applicable too.

Nelson's quantization procedure uses the Newton — Langevin equation (1.10) (without any arguments) in the generalized but not in the usual physical coordinate space where it is well-founded. In the procedure [1] the probability

density  $f(q', t)$  satisfies the Fokker — Planck equation (1.1) in which the diffusion coefficient is assumed to be constant

$$a = \frac{\hbar}{m}, \quad (1.11)$$

a priori introducing the supposition about the deterministic nature of the classical path for irreversible processes in the open dynamical systems. Finally, it is assumed that the wave function  $\Psi(q', t)$  is defined as in the quasi-classical approximation

$$\Psi(q', t) = [f(q', t)]^{1/2} \exp \left[ \frac{i}{\hbar} W(q', t) \right] \quad (1.12)$$

and hence it follows that the function (1.12) satisfies the quantum-mechanical Schrödinger — Langevin equation

$$i\hbar \frac{\partial \Psi(q', t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial q^j \partial q^k} + U(q', t) - q^j A^j(t) + \frac{l\hbar}{m} W(q', t) \right] \Psi(q', t), \quad (1.13)$$

where  $W(q', t)$  denotes the mechanical action with the introduction by (1.12) making the Hamiltonian operator from (1.13) nonlinear.

From the above discussion of the relation between quantum mechanics and stochastic irreversible thermodynamics it is obvious that the problem of transition from macroscopic deterministic equations to macroscopic stochastic models is important. This problem has been discussed in the well-known works by Grabert, Graham and Green [2,3] where the authors wrote: «It seems to be clear that the deterministic description comes out of the stochastic description in a deterministic limit in which fluctuations become negligible. It is less clear what exactly is involved in that limit and, furthermore, whether the stochastic description can be reconstructed in a unique way if only the deterministic description is known». Since physically different stochastic models are obtained from the same deterministic equations if the widespread procedure, consisting in the requirements of the detailed balance and the canonical form of the equilibrium distribution, is applied in different frames of coordinate in state space, the authors [2,3] have proposed another way. This way consists in using the deterministic equation in Onsager's form:

$$j^\alpha = \frac{dq^\alpha}{dt} = \left( \frac{dq^\alpha}{dt} \right)_{\text{rev.}} + \left( \frac{dq^\alpha}{dt} \right)_{\text{irrev.}} = d^k \chi_k^\alpha \quad (\alpha + i = n), \quad (1.14)$$



where  $\chi_k$  are the thermodynamical forces

$$\chi_j = \partial S(q^j) / \partial q^j, \quad (1.15)$$

$S(q^j)$  is the total entropy ( $S$  is maximal for the equilibrium state), and  $dq^j/dt$  are the fluxes that have been caused by the forces (1.15). Since the deterministic drift  $v^j$  in (1.14) splits into reversible and irreversible parts, the diffusion matrix  $a^{jk}$  of the transport coefficients in (1.14) and, as a result, in the Fokker — Planck equation (1.1) is singular. In other words, the differences between the reversible  $\{q^\alpha\}$  and irreversible  $\{q^i\}$   $\alpha + i = n$  variables appear in vanishing of the symmetric parts  $\bar{a}^{jk}$  of some transport coefficients

$$\bar{a}^{\alpha\beta} = \bar{a}^{\alpha i} = \bar{a}^{i\alpha} = 0. \quad (1.16)$$

The second law of thermodynamics requires that the remaining transport coefficients  $\bar{a}^{ik}$  be positively definite. Then, the authors [2,3] making a hypothesis about the conditional probability  $F$  for a small time interval and the author [4], representing the «Hamiltonian»  $H$  (1.2) from (1.1) as

$$H = \left[ \frac{\partial v^\alpha}{\partial q^\alpha} + \frac{\partial v^j}{\partial q^j} - \frac{1}{2} \frac{\partial^2 a^{jk}}{\partial q^j \partial q^k} \right], \quad (1.17)$$

have obtained the same result — the function  $F$  in the form of a path integral (1.8).

As we quite agree with the assertion from [2] about the need «to put the discussed problem in a physical rather than a mathematical context because the physical context is interesting in itself but it also has the advantage of putting the very complicated calculations in a framework that helps in their understanding», we must point out some questions in this context. The discussion in the following part will be represented in the form of some questions from the works [1—4] and possible new answers to them. As a more detailed knowledge of the proposed approach can be gained from the other works by the authors [5] and forthcoming publications on this topic, we shall present only the basic steps. Because in dependence upon admitting determination of the characteristic of differential (or pseudodifferential) operator from the same Schrödinger equation not only the reversible deterministic Hamilton — Jacobi equation may be obtained but also the irreversible equation of heat conductivity, for example, particular attention should be given to the well-founded choice of deterministic dynamical equations in the generalized coordinate space. Let us try to formulate the problem of transition to the deterministic description of nonequilibrium thermodynamics as faithfully as possible.

## 2. QUESTIONS AND POSSIBLE ANSWERS

The first question: are the linear (or quasi-linear) relations between fluxes and forces in Onsager's form (1.14) real deterministic equations of motion in nonequilibrium thermodynamics?

The second question: what physical sense do the generalized coordinates  $\{q^j\}$  have in this task?

One of the possible answers to the first question has been given by the phrase [2]: «The fluctuations are supposed to come from the underlying molecular nature of the system, which primarily manifests itself in the form of the deterministic equations of motion as a linear relation between fluxes and forces». Then, the purely reversible (1.16) parts of the equations (1.14) are treated [3] as constraints in the formulation of these deterministic equations according to a variational principle by introducing the Lagrangian  $L$  in the form

$$L \left( q^j, \frac{dq^j}{dt} \right) = \frac{1}{2} \bar{a}_{ik} \left( \frac{dq^j}{dt} - v^j \right) \left( \frac{dq^k}{dt} - v^k \right) \quad (2.1)$$

as well as the action functional

$$W \left( q^j(t), t_1 \leq t \leq t_2 \right) = \int_{t_1}^{t_2} L \left( q^j(t), \frac{dq^j}{dt} \right) dt. \quad (2.2)$$

The Euler — Lagrange equations

$$\frac{dP_\alpha}{dt} = \frac{\partial L}{\partial q^\alpha} - p_\beta \frac{\partial v^\beta}{\partial q^\alpha}, \quad (2.3)$$

$$\frac{dP_i}{dt} = \frac{\partial L}{\partial q^i} - p_\alpha \frac{\partial v^\alpha}{\partial q^i} \quad (2.4)$$

have been obtained [2,3] by the method of Lagrange multipliers:

$$\int_{t_1}^{t_2} \left\{ L \left[ q^j(t), \frac{dq^j}{dt} \right] - p_\alpha(t) \left[ \frac{dq^\alpha(t)}{dt} - q^\alpha(t) \right] \right\} dt, \quad (2.5)$$

which leads to the additional requirements

$$p_\alpha = 0, \quad \partial L / \partial \left( \frac{dq^j}{dt} \right) = 0 \quad (2.6)$$

in the final state ( $t$ ) and where  $p_i$  denotes

$$p_i = \partial L / \partial \left( \frac{dq^i}{dt} \right) = \bar{a}_{ik} \left( \frac{dq^k}{dt} - v^k \right). \quad (2.7)$$

We propose the following variant for the above phrase from [2]: "The fluctuations are supposed to come from the underlying molecular nature of the system, which mainly manifests itself in the inhomogeneity of equilibrium and nonequilibrium (i.e. nonstationary) states". Another possible answer to the first question may be as follows. The real deterministic equations of motion in a field theoretic treatment are the balance equations for the main extensive properties of the many-particle systems. For a pure substance in the equilibrium state in agreement with the fundamental differential Gibbs form

$$dE = -PdV + \mu dN + TdS, \quad (2.8)$$

such main properties are extensive values  $E, V, N$  for the equilibrium thermodynamical entropy

$$S = S(E, V, N). \quad (2.9)$$

The fixed scale, which defines the system, is usually taken to be either the volume  $V$  or the particle number  $N$  (and the mass:  $m = Nm_0$ ). Then we have, respectively:

$$d\varepsilon = \mu d\rho + Td\sigma, \quad (2.10)$$

$$\sigma = \sigma(\varepsilon, \rho) \quad (2.11)$$

for the extensive densities ( $\varepsilon, \rho$ ), and

$$de = -Pdv + Tds, \quad (2.12)$$

$$s = s(e, v) \quad (2.13)$$

for the specific values ( $e, v = 1/\rho$ ). On the basis of the function (2.9) there was constructed Einstein's thermodynamical fluctuation theory in close vicinity with the equilibrium state, where it has been postulated for the conditional probability distribution

$$F(q^i | q_0^i) dq^i = B \exp \left[ \frac{1}{k} S(q^i | q_0^i) \right] dq^i. \quad (2.14)$$

$B$  is the normalization factor and  $S(q^i | q_0^i)$  is the total entropy of the external world having the coordinates  $\{q_0^i\}$  and the fluctuating system  $\{q^i\}$ . Near the equilibrium point  $\{q_0^i\}$  the fluctuations

$$\delta q^j = (q^j - q_0^j) \quad (2.15)$$

in the macroscopic volume  $V_0$  reflect either the part of the interaction  $H_{\text{int}}$  (1.9) between the system and the external world (for an open system), or the inhomogeneity of the macroscopic system. The part of the external world  $H_{\text{ext}}$  (1.9) in equilibrium thermodynamics is characterized by the constant values of thermodynamical fields ( $\mu$  — chemical potential and  $T$  — temperature) which form the differential Gibbs — Duhem equation:

$$dP = \rho d\mu + \sigma dT, \quad (2.16)$$

$$P = P(\mu, T). \quad (2.17)$$

Now we can formulate a possible answer to the second question. In the case that the entropy  $S$  is used in the equation (2.14) for the conditional probability  $F$ , the values of natural independent variables from the functions (2.9), (2.11), (2.13) must be the generalized coordinates of the task. Hence, for the function (2.11), for instance, the real deterministic equations of motion in a field theoretical treatment of variables

$$q^i = q^i(x_i, t), \quad i = 1, 2, 3 \quad (2.18)$$

are the balance equation for the internal energy  $E$  (in the usual notation)

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial(\varepsilon v_i)}{\partial x_i} - P_{ik} v_{ik} + \frac{\partial y_i}{\partial x_i} = 0 \quad (2.19)$$

and the balance equation of the particle number  $N$  (and the mass)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_i)}{\partial x_i} = 0. \quad (2.20)$$

These equations may be easily transformed into a simpler form if one admits the condition of isotropicity of the system (i.e., the viscosity is absent):

$$P_{ik} = -P\delta_{ik}. \quad (2.21)$$

This admission gives us the opportunity to deal only with self-diffusion and heat conductivity having the microscopic, molecular nature of the transport processes. Taking account of (2.21) we have from (2.19):

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial(\varepsilon v_i)}{\partial x_i} + P \frac{\partial v_i}{\partial x_i} + \frac{\partial y_i}{\partial x_i} = 0. \quad (2.22)$$

Then, we must take into account the existence of the function (2.13) (and the differential form (2.12)) incorporating the balance equation for the total entropy  $S$ :

$$\rho T \frac{ds}{dt} + \frac{\partial y_i}{\partial x_i} - d = 0. \quad (2.23)$$

As the viscosity is absent, we really obtain for the dissipation function  $d$  the condition

$$d = 0, \quad (2.24)$$

and two equations: for the entropy

$$\rho \frac{ds}{dt} = - \frac{1}{T} \frac{\partial y_i}{\partial x_i} \quad (2.25)$$

and for the internal energy

$$\rho \frac{de}{dt} + \frac{P}{T} \frac{\partial v_i}{\partial x_i} = - \frac{1}{T} \frac{\partial y_i}{\partial x_i}, \quad (2.26)$$

having the equal right-hand sides including the divergence of a heat flux.

The third question: is the way of dividing the drift (1.14) from [3] into the reversible and irreversible components the only possible way? In the field theoretical approach [6], the way of dividing the drift  $v_{dr}$  in the physical space, i.e. the field of the velocity of any physical property of the system, into the convection component  $v = \{v_i\}$  and the diffusion component  $v_{dif}$  is the usual way

$$v_{dr} = v + v_{dif} \quad (2.27)$$

which is completely analogous to the way from [3] for the generalized space of coordinates. If we consider the field of the particles velocity of the system (or mass):

$$v_{dr} = v, \quad v_{dif} = 0, \quad (2.28)$$

these equalities are a deterministic equivalent of the equations (1.16) In statistical mechanics there are own natural macroscopic variables  $\{a^j\}$  [3], which «are algebraic sums of molecular variables that depend on few microscopic degrees of freedom» of course; these variables are thermodynamical densities ( $\epsilon, \rho$ ) (2.11), or specific values ( $e, v$ ) (2.13), or, at last, extensive values ( $E, V, N$ ) (2.9), where internal energy  $E$  is connected with the Hamiltonian of the system  $H_{\text{sys}}$  (1.9). In the field theoretical approach, this selection of natural variables brings some serious difficulties in the formulation of the irreversible thermodynamics tasks. The internal energy ( $\epsilon, e$ ), in a strict sense, is not a local-conserving variable. In nonequilibrium, inhomogeneous

systems, the local states strongly correlate with the same local states from their close surroundings. In that case the separation (1.9) of the total Hamiltonian into three solitary parts is impossible. This cause provokes strong fluctuations of the densities ( $\epsilon, \rho$ ), specific values ( $e, v$ ) and the entropy ( $\sigma, s$ ) in such nonequilibrium systems. In equilibrium systems, the same variables are undetermined in phase transition states. Based on this discussion we make two conclusions. First, we refuse to use the natural macroscopic variables of statistical mechanics as generalized coordinates, i.e.,

$$\{a^j\} = \{a^1 = \rho, a^2 = \epsilon\} = \{q^j\}, \quad (2.29)$$

and propose using in nonequilibrium tasks the thermodynamical fields (2.16), (2.17)

$$\{b^l\} = \{b^1 = \mu, b^2 = T\} = \{q^l\} \quad (2.30)$$

as in the theory of equilibrium phase transitions. These variables may be considered as continuous quantities in inhomogeneous systems in contrast to the densities (2.29). This crucial proposition of our work results in the following claims. The real deterministic equations of motion are: the balance equation for  $N$  (2.20) and the balance equation for  $S$  (2.25) in the form

$$\frac{\partial \sigma}{\partial t} + \frac{\partial(\sigma v_i)}{\partial x_i} = - \frac{1}{T} \frac{\partial y_i}{\partial x_i} = - \left( \frac{\partial j_i^s}{\partial x_i} \right)_{dif}, \quad \sigma = sp \quad (2.31)$$

instead of the equation for  $E$  (2.26). In the last equality (2.31) we admit a more general form for the irreversible component

$$j_{dr}^s = \sigma v + j_{dif}^s \quad (2.32)$$

in accordance with (2.27). Now a possible answer to the third question is that the left sides of the equations (2.20) and (2.31) correspond to the irreversible motion. This claim is analogous in the physical space to that for the generalized space of coordinates [3].

The fourth question: is the entropy  $S$  unique function which can be used in foundation of fluctuation theory as in Einstein's equality (2.14)?

Since the entropy is a function of state, it transforms as a scalar, but the transformation of the surface element  $\prod_j dq^j = dq^1 dq^2$  in (2.14) introduces as a multiplicative factor the Jacobian of the coordinate transformation [9,7]. Generally, it is not constant and cannot simply be absorbed in the normalization factor  $B$ . Hence, equation (2.14) cannot retain the same form in all coordinate systems [7]. Since we propose using only field variables (2.30) in any tasks of equilibrium or nonequilibrium theories as generalized coordinates, we must



specify the metric  $g^{ik}$  on the thermodynamical surface (2.17) in field state space. Usually, a thermodynamical treatment lacks a meaningful metric structure, an expression for the distance between equilibrium point states. In some interesting works by Ruppeiner [7,8] it has been proposed in the framework of equilibrium fluctuational theory to represent thermodynamical systems by Riemannian manifolds. In [5] we discuss the problem of building in the field space a global equation of the thermodynamical surface of equilibrium states. The important difference of our work from all fluctuational theories of nonequilibrium thermodynamics lies in using instead of entropy  $S$  the thermodynamical potential for an open system ( $-PV$ ). It is not only supposition of such a possibility: this is a strict result of the above admissions.

Since the definition of the metric is intimately connected with the meaning of the deterministic trajectory [3], we consider, at first, only reversible motion, i.e. the left sides of the equations (2.20), (2.31) which are equivalent to the resulting equation

$$\rho \frac{ds}{dt} = 0 \quad (2.33)$$

instead of (2.25). This is the well-known model of ideal fluid. If, then, we put these reversible deterministic equations of the motion in correspondence with the Euler — Lagrange equations in field representation (2.30):

$$\frac{\partial \alpha}{\partial q^j} - \frac{\partial}{\partial t} \left( \frac{\partial \alpha}{\partial \dot{q}^j} \right) - \frac{\partial}{\partial x_i} \left( \frac{\partial \alpha}{\partial x_i} \right) = 0, \quad (2.34)$$

for variational principle in the field theoretical approach:

$$\delta \int_t \int_{V_0} \alpha dt dV = 0, \quad (2.35)$$

where the density of Lagrangian depends on  $q^j, \frac{\partial q^j}{\partial t}, \frac{\partial q^j}{\partial x_i}, x_i, t$ :

$$\alpha = \alpha \left( q^j, \frac{\partial q^j}{\partial t}, \frac{\partial q^j}{\partial x_i}, x_i, t \right), \quad (2.36)$$

we can obtain a new density of mechanical action ( $P - P_0$ ) for the reversible motion:

$$\delta \int_t \int_{V_0} \alpha dt dV = \delta \int_{V_0} (p_j dq^j) dV = \delta \int_{V_0} (P - P_0) dV = 0. \quad (2.37)$$

This result is of crucial importance for the new theory of fluctuations in equilibrium and nonequilibrium thermodynamics.

In equilibrium thermodynamics, almost simultaneously, one of us [9] and Ruppeiner [7] have proposed a new method of investigation of nonhomogeneous systems by continuous procedure of decreasing the scale of volume. Ruppeiner has used the representation (2.29) and the traditional entropy approach (without an account of the drift) for path-integral formalism, whereas in [10] the following covariant forms for reversible deterministic trajectories on thermodynamical surface (2.17) were found:

$$\bar{P} = C \bar{\mu} + D \bar{\mu}^{3/2} + \dots, \quad (2.38)$$

$$\bar{P} = A \bar{T} + B \bar{T}^{3/2} + \dots, \quad (2.39)$$

where  $(\bar{P}, \bar{\mu}, \bar{T}) = (b_1^j - b^j)/b_0^j$  and  $(A, B), (C, D)$  are constants on every isoentrop:

$$S = \text{const}; \left( \frac{\sigma}{\rho} = S = \text{Const} \right). \quad (2.40)$$

It is obvious that these equations of reversible motion are similar to the result for the action (2.2) of irreversible motion from [2,3]:

$$W = \alpha \tau + \beta \tau^{3/2} + \dots, \quad (2.41)$$

where  $\tau$  is a small interval of physical time. The cause of this similarity consists in formally common forms of irreversible Lagrangian  $L$  (2.1) and the density of reversible Lagrangian (2.37) in which the impulses:

$$p_j = \frac{\partial P}{\partial q^j}; \quad \rho = \frac{\partial P}{\partial \mu}, \quad \sigma = \frac{\partial P}{\partial T} \quad (2.42)$$

are formally equivalent to the variables  $p_i$  (2.7) from [2]. The volume element of the canonical conjugate pair  $(q^j, p_j)$  is invariant under arbitrary nonlinear transformations of the state variables.

In our approach to problems of nonequilibrium thermodynamics we do not consider the components of reversible motion as constraints (2.5); quite the reverse — the components of irreversible motion are treated as a small addition to deterministic trajectory of reversible motion for every small interval of physical time  $t$ . We substitute the usual equations of coordinate transformations

$$da^j = p^{jk} db^k \quad (2.43)$$

for the transport equations (1.14). The more general form of (2.43) is obvious because we admit the following relation between operators, acting on deterministic and stochastic variables, too:

$$\delta \leftrightarrow d \leftrightarrow \frac{d}{dt} = \frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i} \quad (2.44)$$

Then the density of irreversible Lagrangian is represented in the form:

$$\alpha = \alpha_0 + \bar{\Gamma} \alpha_1 = \delta P + \frac{\bar{\Gamma}}{2} \delta^2 P, \quad (2.45)$$

where  $\bar{\Gamma}$  is a small parameter relaxation to reversible motion. Finally, we have obtained for the irreversibility following from the right side of equation (2.31) of minimum production of entropy:

$$\rho \frac{ds}{dt} = Z_0 \exp\left(-\frac{2t}{\bar{\Gamma}}\right) \geq 0, \quad (2.46)$$

which is a physical confirmation of the theory proposed.

It is possible that the main results of this approach will be useful from practical point of view in solutions of the complicated tasks of irreversible thermodynamics by path-integral techniques.

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Флуктуационная теория равновесных неоднородных состояний, использующая интегралы по путям

Для функции преобразования получена формула в виде интеграла по путям. Соответствующая функция распределения подчиняется уравнению Фоккера — Планка, в котором учет необратимости обуславливает сингулярность диффузионной матрицы. В качестве фазового пространства берется пространство полей (химических потенциалов  $\mu$  и температур  $T$ ); роль «потенциала» играет давление  $P(\mu, T)$ , при этом поведение неоднородных систем может быть исследовано путем использования непрерывного уменьшения масштаба объема: параллельно с физическим временем « $t$ » вводится термодинамическое время  $\tau = \frac{V_0}{V_\tau}$ ,  $V_\tau \rightarrow (V_c)_\tau$ .

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Fluctuational Theory of Equilibrium Inhomogeneous States  
Using Path Integrals

The formula as a path integral is derived for the transformation function. The corresponding distribution function obeys the Fokker — Planck equation in which the irreversibility specifies the diffusion matrix singularity. As a phase space we take the space of fields (chemical potentials and temperatures); the role of the «potential» is played by the pressure  $P(\mu, T)$ . In this case, the behaviour of an inhomogeneous system can be studied by continuous decrease of the volume scale; simultaneously with the physical time « $t$ » we introduce the thermodynamic time  $\tau = \frac{V_0}{V_\tau}$ ,  $V_\tau \rightarrow (V_c)_\tau$ .

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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