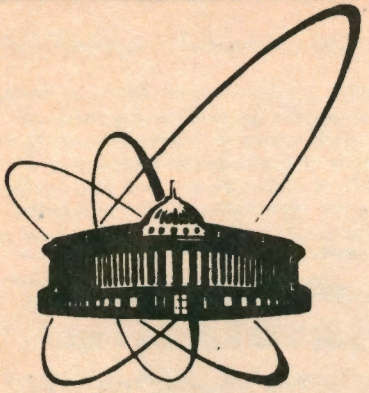

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E17-92-419

Ho Trung Dung, Nguyen Dinh Huyen

TWO-ATOM SINGLE-MODE RADIATION FIELD INTERACTION: STATE EVOLUTION, LEVEL OCCUPATION PROBABILITIES AND EMISSION SPECTRA

Submitted to "Journal of Modern Optics"

1992

1 Introduction

The Jaynes-Cummings model (JCM) [1] describing the interaction of a two-level atom with a single-mode quantized radiation field is of fundamental importance in quantum optics [2]-[4], and is realizable to a very good approximation in experiments with Rydberg atoms in high-Q superconducting cavities [5]. The model predicts a variety of interesting phenomena, such as quantum collapse and revival [6], vacuum-field Rabi splittings [7]-[9]. A generalization of the JCM to the case where N Rydberg atoms interact resonantly with a cavity field mode has been treated by Barnett and Knight [10], and Haroche and Raimond [11] (see also [12] for a review). The two-atom single-mode field system is also of considerable interest. Deng has studied quantum collapses and revivals for this case and shown that when the two atoms are initially in the lower state, the probabilities for the occupancies of the ground and excited states of the system manifest two series of revivals [13]. The situation where only one of the two atoms is initially in the upper state has been treated by Kozierowski et al [14] and Bužek [15]. Two nonidentical atoms have been considered by Mahmood et al [16] and by Iqbal et al [17]. The squeezing [18], emission spectra [19]-[21], properties of the field phase [22], and the effects of photon statistics [23] and cavity damping [24] have also been studied.

Recently, Gea-Banacloche has derived an asymptotic result for the JCM which is valid when the field is initially in a coherent state with a large average photon number [25, 26]. It is shown that the atom is to a good approximation in a pure state in the middle of the collapse region (this has been first noticed by Phoenix and Knight [27] by using the entropy concepts to examine the fluctuations in the quantum evolution of the JCM) and the most remarkable feature of this state is that it is reached regardless of the initial atomic state. In this paper, we analyze the atomic and field state evolution in the two-atom JCM supposing, as in [25, 26], that the field is initially in a coherent state with a large average photon number. The eigenstates

of the semiclassical Hamiltonian are found. It is shown that if the atoms are initially prepared in one of these states, the system evolves in such a way that atomic and field parts separately remain in a pure state. However, only for certain initial atomic states, the crossing of the "trajectories" in the Hilbert space of atomic states at half-revival time is observed. The initial states where both atoms are excited or de-excited, for example, do not belong to this group.

Further, we turn our attention to some "old" problems, namely, the effects of cavity detuning on atomic level occupation probabilities and emission spectra. Though some accounts on the system dynamics with nonresonant coupling can be found in [22], where phase properties of the interacting field have been considered, and in [28], where the author has extended the coherent state representation formalism proposed by Papadopoulous [29] to the N-atom case, most discussions in the available literature are restricted to on resonance. We show, in particular, that far off-resonance, if the initial state of the atomic system finds atom 1 excited and atom 2 unexcited (nonsymmetrical excitation), the initial excitation is transferred from atom 1 to atom 2 and back, but not to the field. Nevertheless, the field itself becomes essentially a two-state quantity in contrast with the case of two initially excited, de-excited atoms and symmetrical excitation where the field remains a one-state quantity with changing phase.

As for the emission spectra, it is shown that nonzero detunings give rise to a nine-peaked spectrum instead of an eight-peaked one obtained at exact resonance [20]. Comparing the emission spectra in the two situations – symmetrical and nonsymmetrical excitations, we find that whereas the numbers and positions of the spectral lines are the same, their heights in the first case are two times larger than those in the latter case.

The paper is organized as follows. In Section 2 the model Hamiltonian and its solution are presented. In Section 3 we investigate the atomic and field state evolution for large initial fields. The effects of cavity detuning on the time evolution of level oc-

cupation probabilities are examined in Section 4, and in Section 5 we use the definition of "physical spectrum" introduced by Eberly and Wodkiewicz [31] to study stimulated and spontaneous emission spectra.

2 Model Hamiltonian and its solution

The hamiltonian describing the system of two two-level atoms coupled to a single-mode radiation field in the rotating wave approximation is given by ($\hbar = 1$)

$$H = \omega a^+ a + \omega_0 \sum_{i=1}^{2} R_i^z + \sum_{i=1}^{2} g_i (a R_i^+ + a^+ R_i^-), \tag{1}$$

where a^+ and a are the creation and annihilation operators of the field mode with the frequency ω, ω_0 is the atomic transition frequency, R_i^z, $R_i^\pm$ are the pseudospin operators describing the ith atom and g_i is the atom-field coupling constant for the ith atom which may be treated as real without any loss of generality.

Following [2] , we separate the Hamiltonian (1) into two mutually commuting parts

$$\begin{aligned} H &= H_0 + H_1, \qquad [H_0, H_1] = 0, \\ H_0 &= \omega(a^+ a + \sum_{i=1}^{2} R_i^z), \\ H_1 &= \Delta \sum_{i=1}^{2} R_i^z + \sum_{i=1}^{2} g_i (a R_i^+ + R_i^- a^+), \qquad (\Delta = \omega_0 - \omega), \end{aligned} \tag{2}$$

so that the time translation operator $U(t) = \exp(-iHt)$ factors

$$\begin{aligned} U(t) &= U_0(t) U_1(t), \\ U_0(t) &= \exp(-iH_0 t), \qquad U_1(t) = \exp(-iH_1 t). \end{aligned} \tag{3}$$

For the model in question only four eigenstates of the free-atom-free-field Hamiltonian H_0

$$|1\rangle \equiv |e,e,n\rangle, \quad |2\rangle \equiv |e,g,n+1\rangle, \quad |3\rangle \equiv |g,e,n+1\rangle, \quad |4\rangle \equiv |g,g,n+2\rangle \tag{4}$$

are needed. Here $|e\rangle, |g\rangle$ represent the upper and lower atomic states, respectively, and $|n\rangle$ is the Fock state of the radiation field. The states (4) form a basis which gives the following matrix representation of H_0 and H_1

$$H_0 = (n+1)\omega I, \tag{5a}$$

$$H_1 = \begin{pmatrix} \Delta & g_2\sqrt{n+1} & g_1\sqrt{n+1} & 0 \\ g_2\sqrt{n+1} & 0 & 0 & g_1\sqrt{n+2} \\ g_1\sqrt{n+1} & 0 & 0 & g_2\sqrt{n+2} \\ 0 & g_1\sqrt{n+2} & g_2\sqrt{n+2} & -\Delta \end{pmatrix}, \tag{5b}$$

where I is a 4×4 unit matrix. By expanding the state vector of the atom-field system as

$$|\psi(t)\rangle = \exp[-i(n+1)\omega t]\,[C_1(t)|1\rangle + C_2(t)|2\rangle + C_3(t)|3\rangle + C_4(t)|4\rangle], \tag{6}$$

and using the Schrödinger equation, one gets

$$\dot{C}(t) = -iH_1 C(t), \tag{7}$$

where

$$C(t) = \begin{pmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \\ C_4(t) \end{pmatrix}, \tag{8}$$

and H_1 is defined by Eq.(5b). A solution of equation (7) is

$$\begin{aligned} C_i(t) &= \sum_{j=1}^{4} A_{ij}(t) C_j(0), \\ A_{ij}(t) &= \sum_{l=1}^{4} \alpha_i^l \, (\alpha_j^l)^* \exp(-i\lambda_l t). \end{aligned} \tag{9}$$

Here λ_l are the eigenvalues of the matrix H_1, and α_i^l is the ith element of the lth eigenvector. The eigenvalues λ_l are to be found from the fourth-order secular equation

$$\lambda^4 - \lambda^2[(g_1^2 + g_2^2)(2n+3) + \Delta^2] + \lambda\Delta(g_1^2 + g_2^2) + (g_1^2 - g_2^2)^2 (n+1)(n+2) = 0. \tag{10}$$

For nonidentical atoms and exact resonance, equation (10) reduces to a second-order equation with respect to λ^2, the solution of which has been given in [18]. For identical

atoms ($g_1 = g_2 \equiv g$) and arbitrary detuning, equation (10) factors into a third-order equation and a first-order one with the roots

$$\begin{aligned}
\lambda_1 &= \frac{2}{\sqrt{3}}[2g^2(2n+3)+\Delta^2]^{1/2}\cos(\theta), \\
\lambda_2 &= \frac{2}{\sqrt{3}}[2g^2(2n+3)+\Delta^2]^{1/2}\cos(\theta+\frac{2\pi}{3}), \\
\lambda_3 &= \frac{2}{\sqrt{3}}[2g^2(2n+3)+\Delta^2]^{1/2}\cos(\theta+\frac{4\pi}{3}), \\
\lambda_4 &= 0,
\end{aligned} \tag{11}$$

where

$$\theta = \frac{1}{3}\arccos\left[\frac{-27\Delta g^2}{\{3[2g^2(2n+3)+\Delta^2]\}^{3/2}}\right]. \tag{12}$$

For convenience, we introduce the following notation

$$\begin{aligned}
\beta_1 &= \{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)[\lambda_1(\lambda_1+\Delta)-2g^2(n+2)]\}^{1/2}, \\
\beta_2 &= \{(\lambda_2-\lambda_3)(\lambda_2-\lambda_1)[\lambda_2(\lambda_2+\Delta)-2g^2(n+2)]\}^{1/2}, \\
\beta_3 &= \{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)[\lambda_3(\lambda_3+\Delta)-2g^2(n+2)]\}^{1/2}.
\end{aligned} \tag{13}$$

Hence, the elements α_i^l can be written as

$$\begin{aligned}
\alpha_1^l &= [\lambda_l(\lambda_l+\Delta)-2g^2(n+2)]/\beta_l, \\
\alpha_2^l &= \alpha_3^l = g\sqrt{n+1}(\lambda_l+\Delta)/\beta_l, \\
\alpha_4^l &= 2g^2\sqrt{(n+1)(n+2)}/\beta_l
\end{aligned} \tag{14}$$

for $l = 1, 2, 3$, and

$$\alpha_1^4 = \alpha_4^4 = 0, \qquad \alpha_2^4 = \frac{1}{\sqrt{2}}, \qquad \alpha_3^4 = -\frac{1}{\sqrt{2}} \tag{15}$$

for $l = 4$. The eigenvectors of the atom-field system are defined in terms of the basis states (4) as follows

$$|\psi^l\rangle = \sum_i \alpha_i^l |i\rangle, \tag{16}$$

and vice versa

$$|i\rangle = \sum_l (\alpha_i^l)^* |\psi^l\rangle. \tag{17}$$

On resonance, by putting $\Delta = 0$ into Eqs. (11)-(15), one obtains

$$\lambda_{1,2} = \pm\sqrt{2g^2(2n+3)}, \qquad \lambda_{3,4} = 0, \tag{18}$$

and

$$\begin{aligned}
|\psi^1\rangle &= \sqrt{\frac{n+1}{4n+6}}|1\rangle + \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle + \sqrt{\frac{n+2}{4n+6}}|4\rangle, \\
|\psi^2\rangle &= \sqrt{\frac{n+1}{4n+6}}|1\rangle - \frac{1}{2}|2\rangle - \frac{1}{2}|3\rangle + \sqrt{\frac{n+2}{4n+6}}|4\rangle, \\
|\psi^3\rangle &= -\sqrt{\frac{n+2}{2n+3}}|1\rangle + \sqrt{\frac{n+1}{2n+3}}|4\rangle, \\
|\psi^4\rangle &= \frac{1}{\sqrt{2}}|2\rangle - \frac{1}{\sqrt{2}}|3\rangle.
\end{aligned} \tag{19}$$

When the total excitation number of the atom-field system is equal to unity, only three basis states

$$|1\rangle \equiv |e,g,0\rangle, \qquad |2\rangle \equiv |g,e,0\rangle, \qquad |3\rangle \equiv |g,g,1\rangle \tag{20}$$

are possible. In this case, the solution can be written in a simple form for nonidentical atoms and arbitrary detuning. In the interaction picture it reads

$$\begin{aligned}
\lambda_{1,2} &= -\frac{\Delta}{2} \pm f, \qquad \lambda_3 = 0, \\
f &= \sqrt{g_1^2+g_2^2+\Delta^2/4},
\end{aligned} \tag{21}$$

and

$$\begin{aligned}
|\psi^1\rangle &= g_1/\sqrt{2f(f-\Delta/2)}|1\rangle + g_2/\sqrt{2f(f-\Delta/2)}|2\rangle + \sqrt{(f-\Delta/2)/(2f)}|3\rangle, \\
|\psi^2\rangle &= g_1/\sqrt{2f(f+\Delta/2)}|1\rangle + g_2/\sqrt{2f(f+\Delta/2)}|2\rangle - \sqrt{(f+\Delta/2)/(2f)}|3\rangle, \\
|\psi^3\rangle &= -g_2/\sqrt{g_1^2+g_2^2}|1\rangle + g_1/\sqrt{g_1^2+g_2^2}|2\rangle.
\end{aligned} \tag{22}$$

Once the complete exact solution is known, various quantities characterizing the atomic and field systems can be evaluated subject to certain initial conditions. For

instance, the level occupation probabilities in the case of two atoms initially being excited and the field being in a Fock state are given by

$$P_{ee}(t) = |C_{1,n}(t)|^2, \quad P_{eg}(t) = |C_{2,n}(t)|^2, \quad P_{ge}(t) = |C_{3,n}(t)|^2, \quad P_{gg}(t) = |C_{4,n}(t)|^2, \tag{23}$$

where P_{ee} and P_{gg} are the probabilities that both atoms are in the states $|e\rangle$ and $|g\rangle$, respectively, while P_{eg} (P_{ge}) is the probability that the first atom is in the state $|e\rangle$ ($|g\rangle$) and the other is in the state $|g\rangle$ ($|e\rangle$). Also we have for the average number of photons

$$\langle a^+ a\rangle_t = n|C_{1,n}(t)|^2 + (n+1)\left[|C_{2,n}(t)|^2 + |C_{3,n}(t)|^2\right] + (n+2)|C_{4,n}(t)|^2. \tag{24}$$

In Eqs. (23), (24) the lower index n indicates the dependence of the corresponding quantities on the photon number. If initially only one atom is in the upper state, or both atoms are in the lower state, n must be replaced by $(n-1)$ and $(n-2)$, respectively. When the cavity field is initially prepared in a superposition of the number states, Eqs. (23) and (24) become useful after averaging the right-hand sides over the photon number distribution.

3 Atomic and field state evolution for large initial fields

For a large number of photons, it turns out to be more convenient to use the eigenstates of the semiclassical interaction Hamiltonian, rather than the energy eigenstates, as the basis of atomic states. The semiclassical Hamiltonian corresponding to H_1 in Eq. (2) is obtained by replacing the annihilation operator a by a complex number v, i.e.,

$$H_{SC} = \sum_{i=1}^{2} g_i \left(v R_i^+ + v^* R_i^-\right). \tag{25}$$

Here the exact resonance ($\Delta = 0$) is assumed. The eigenvalues and eigenvectors of (25) are found to be

$$\lambda_{1,2} = \pm 2g|v|, \qquad \lambda_{3,4} = 0, \tag{26}$$

and

$$\begin{aligned}
|\phi^1\rangle &= \frac{1}{2}\left[\exp(2i\varphi)|e,e\rangle + \exp(i\varphi)(|e,g\rangle + |g,e\rangle) + |g,g\rangle\right], \\
|\phi^2\rangle &= \frac{1}{2}\left[\exp(2i\varphi)|e,e\rangle - \exp(i\varphi)(|e,g\rangle + |g,e\rangle) + |g,g\rangle\right], \\
|\phi^3\rangle &= \frac{1}{\sqrt{2}}\left[-\exp(2i\varphi)|e,e\rangle + |g,g\rangle\right], \\
|\phi^4\rangle &= \frac{1}{\sqrt{2}}\left(|e,g\rangle - |g,e\rangle\right),
\end{aligned} \tag{27}$$

where φ denotes the phase of the field $v = |v|\exp(i\varphi)$. It can be seen from Eqs. (27) that $|\varphi^4\rangle$ is nothing but the singlet state, and $|\varphi^1\rangle$, $|\varphi^2\rangle$, $|\varphi^3\rangle$ are composed completely of the triplet states. Except the state $|\varphi^4\rangle$, which is a true trapping state in both semiclassical and fully quantized theories, we can expect that in the latter case, if the atomic state is prepared in one of the states $|\varphi^1\rangle$, $|\varphi^2\rangle$, $|\varphi^3\rangle$ and the quantized-field intensity is large, the system would still show some Rabi oscillations but with their amplitudes being strongly suppressed [26, 32, 33].

We consider now the state evolution of the system having states (27) as a starting point. Suppose that the cavity field is initially in a coherent state

$$|\psi_{field}(0)\rangle = |v\rangle = \exp(-|v|^2/2)\sum_{n=0}^{\infty}\frac{|v|^n}{\sqrt{n!}}\exp(in\varphi)|n\rangle \tag{28}$$

with a large average photon number $n \gg 1$ ($n = |v|^2$). Then, in the interaction picture one gets asymptotically

$$\begin{aligned}
|\phi^1\rangle|v\rangle\Big|_{t=0} \longrightarrow \frac{1}{2}\Big[&\exp(-i2gt/\sqrt{n})\exp(2i\varphi)|e,e\rangle + \exp(-igt/\sqrt{n})\exp(i\varphi)(|e,g\rangle + |g,e\rangle) \\
&+|g,g\rangle\Big]\exp(-n/2)\sum_{n=0}^{\infty}\frac{n^{n/2}}{\sqrt{n!}}\exp(in\varphi)\exp(-i2gt\sqrt{n})|n\rangle,
\end{aligned} \tag{29a}$$

$$\begin{aligned}
|\phi^2\rangle|v\rangle\Big|_{t=0} \longrightarrow \frac{1}{2}\Big[&\exp(i2gt/\sqrt{n})\exp(2i\varphi)|e,e\rangle - \exp(igt/\sqrt{n})\exp(i\varphi)(|e,g\rangle + |g,e\rangle) \\
&+|g,g\rangle\Big]\exp(-n/2)\sum_{n=0}^{\infty}\frac{n^{n/2}}{\sqrt{n!}}\exp(in\varphi)\exp(i2gt\sqrt{n})|n\rangle,
\end{aligned} \tag{29b}$$

$$|\phi^3\rangle|v\rangle\Big|_{t=0} \longrightarrow |\phi^3\rangle|v\rangle, \tag{29c}$$

$$|\phi^4\rangle|v\rangle_t = |\phi^4\rangle|v\rangle. \tag{29d}$$

Equations (29a)–(29c) are asymptotic in the sense that the differences of their right-hand sides and the exact solution are vectors whose norm tends to zero (as fast as $t/\bar{n}$) in the limit $\bar{n} \to \infty$. Since the revival time scale is of the order of $\sqrt{\bar{n}}/g$ [13], Eqs. (29) hold even over a large number of revivals, provided $\bar{n}$ is large enough.

The result (29) shows that the atomic and field parts separately remain in a pure state in the course of time. However, there is no time at which the atomic systems prepared initially in any of the four basis states (27) are found in the same pure state. Of particular interest are the states $|\phi^1\rangle$ and $|\phi^2\rangle$. At half of the revival time $t_0 = \pi\sqrt{\bar{n}}/(2g)$ (note that there are two series of revivals in the system under consideration [13] with the revival times $\pi\sqrt{\bar{n}}/g$ and $2\pi\sqrt{\bar{n}}/g$; under half-revival time here we mean that of the first series), we do find them in the same pure state which is equal to

$$\frac{1}{2}\left[-\exp(2i\varphi)|e,e\rangle - i\exp(i\varphi)(|e,g\rangle + |g,e\rangle) + |g,g\rangle\right]. \tag{30}$$

Thus, the crossing of the atomic "trajectories" occurs only if the atomic system is initially prepared in a linear superposition of two basis states $|\phi^1\rangle$ and $|\phi^2\rangle$, for example, in the states

$$(|e,g\rangle + |g,e\rangle)/\sqrt{2} = \exp(-i\varphi)\left(|\phi^1\rangle - |\phi^2\rangle\right)/\sqrt{2} \tag{31}$$

(symmetrical excitation), or

$$\left[\exp(2i\varphi)|e,e\rangle + |g,g\rangle\right]/\sqrt{2} = \left(|\phi^1\rangle + |\phi^2\rangle\right)/\sqrt{2}. \tag{32}$$

For these, the field state at t_0 is a coherent superposition of macroscopically distinct states

$$|\Phi_\pm(t)\rangle = \exp(-\bar{n}/2)\sum_{n=0}^{\infty}\frac{\bar{n}^{n/2}}{\sqrt{n!}}\exp(in\varphi)\exp(\mp i2gt\sqrt{n})|n\rangle. \tag{33}$$

In other words, we have what is usually called a "Schrödinger cat". It also follows from Eqs. (29) that an initial atomic state, which is a linear combination of $|\phi^1\rangle$ (and/or

$|\phi^2\rangle$) and $|\phi^3\rangle$ (and/or $|\phi^4\rangle$), as time goes on, no longer becomes pure again and the Schrödinger cat, then, does not appear.

In Fig. 1, we have plotted $\mathrm{Tr}(\rho_{at}^2)$ versus gt for the average photon number $\bar{n} = 50$ and for the initial atomic states (a) $(|e,g\rangle + |g,e\rangle)/\sqrt{2}$, (b) $[\exp(2i\varphi)|e,e\rangle + |g,g\rangle]/\sqrt{2}$, (c) $|e,e\rangle$, and (d) $[-\exp(2i\varphi)|e,e\rangle + |g,g\rangle]/\sqrt{2}$ $(= |\phi^3\rangle)$. The curves a and b represent the time evolution of $\mathrm{Tr}(\rho_{at}^2)$ for initial atomic states being linear combinations of $|\phi^1\rangle$ and $|\phi^2\rangle$. They show apparently the recreation of the state vector in the middle of the collapse region. The curve c corresponding to the linear combination

$$|e,e\rangle = \exp(-2i\varphi)/\sqrt{2}\left[(|\phi^1\rangle + |\phi^2\rangle)/\sqrt{2} - |\phi^3\rangle\right]$$

shows that though the system reorders to a great extent at t_0, the reordering is not complete. This is clearly due to the presence of $|\phi^3\rangle$ in the initial state. The same presence (and absence) of $|\phi^3\rangle$ in the initial state leads to the fact that in the long-time region (not shown in the figure), the curve c oscillates around a value larger than that around which the curves a and b do. It is interesting to note that both these values are larger than 1/4 – the value of $\mathrm{Tr}(\rho_{at}^2)$ for a maximally mixed two-two-level-atom system, i.e., the system does not become completely unpolarized under the influence of cooperative interaction.

The result (29) can also be useful to predict some properties of the quasiprobability distributions in the limit of large $\bar{n}$ (see Refs. [34, 35] for recent studies regarding the quasiprobability distributions for the field in the one-atom JCM). Indeed, supposing that the two atoms are initially in the upper state, from the fact that $|e,e\rangle$ is a linear superposition of $|\phi^1\rangle$, $|\phi^2\rangle$ and $|\phi^3\rangle$, each being a phasor in the phase plane, it follows that as the interaction is switched on, the Q function (or the Wigner function) is caused to split into three peaks. The one connected with $|\phi^3\rangle$ is unmoved whereas the two connected with $|\phi^1\rangle$ and $|\phi^2\rangle$ rotate clockwise and counterclockwise, respectively, in the phase plane. This is consistent with the study of phase properties of the field [22].

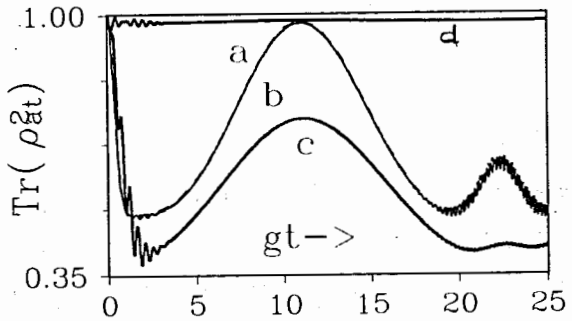


Fig. 1

Time evolution of $\mathrm{Tr}\rho_{at}^2$ for initial atomic states (a) $(|e,g\rangle + |g,e\rangle)/\sqrt{2}$ (dashed curve), (b) $[\exp(2i\varphi)|e,e\rangle + |g,g\rangle]/\sqrt{2}$ (curves a and b are almost indistinguishable), (c) $|e,e\rangle$, and (d) $[-\exp(2i\varphi)|e,e\rangle + |g,g\rangle]/\sqrt{2}$. The field is initially in a coherent state with $\bar{n} = 50$.

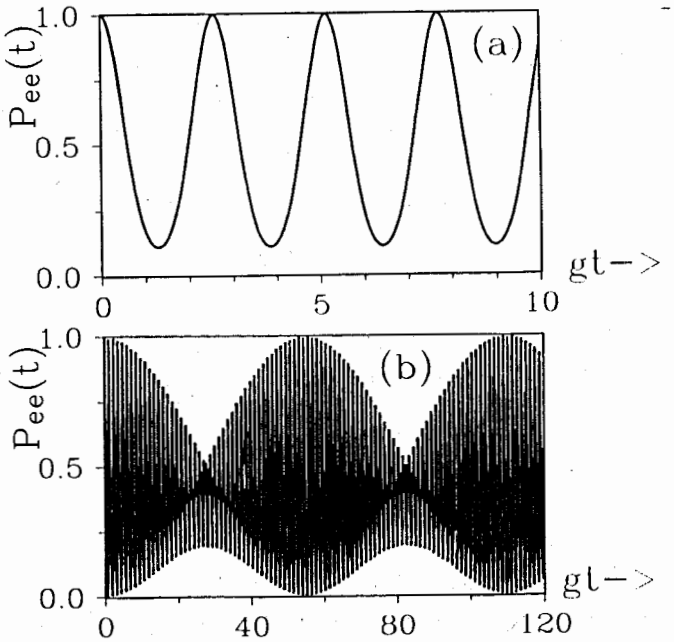


Fig. 2

Probability of finding both atoms in the upper state P_{ee} versus gt for the field initially in a Fock state with (a) $\Delta = 0$, $n = 0$, (b) $\Delta = 0.5g$, $n = 5$. The atoms are initially excited.

4 Atomic level occupation probabilities: Effects of nonzero detuning

As has been mentioned previously, earlier discussions of the two-atom systems have usually been restricted to the exact resonance. In this section, we use the solution obtained in Section 2 to investigate the effects of cavity detuning on the time behavior of the atomic level occupation probabilities. We show that, in some cases, they are not as trivial as they may seem to be at the first glance.

Consider first the initial condition where the atoms are in the upper state and the field is in a Fock state

$$|\psi(0)\rangle = |e,e;n\rangle, \qquad (C_1(0) = 1). \tag{34}$$

Then on resonance, equations (9), (18), (19), and (23) yield

$$P_{ee}(t) = \left(\frac{n+1}{2n+3}\right)^2 \cos^2(gt\sqrt{4n+6}) + 2\frac{(n+1)(n+2)}{(2n+3)^2}\cos(gt\sqrt{4n+6}) + \left(\frac{n+2}{2n+3}\right)^2. \tag{35}$$

Equation (35) shows that $P_{ee}(t)$ oscillates with two commensurate Rabi frequencies, and has a minimum value equal to $1/(2n+3)^2$, which means some trapping of the total system energy in the atomic sub-system (this trapping is most transparent in the limit of weak fields). This is in accordance with the so-called cooperative inhibition of the average radiated energy reported by Bužek [15] and Seke et al [36]. As $\Delta \neq 0$, three distinct nonzero roots of Eq. (10) give rise to three noncommensurate Rabi frequencies. Owing to the beating between these, the Rabi oscillations begin to exhibit a tendency towards collapses and revivals, as can be seen in Fig. 2b. Note that the collapses and revivals in a Fock state field occurring here have the same root as those taking place in single-atom systems with prepared atomic coherence [37], and in cascade three-level systems with arbitrary detunings [38]. Though in two-atom systems the Rabi oscillations do not collapse completely, they do when the number of atoms increases [12].

The time behavior of $P_{ee}(t)$ is shown in Fig. 2a for $\Delta = 0$, $n = 0$, and Fig. 2b for

$\Delta = 0.5g$, $n = 5$, where the partial trapping of the energy in the atomic system and the Fock state field collapses and revivals are clearly visible.

Next, we consider the field being initially in a coherent state. Then $P_{ee}(t)$ is given by

$$P_{ee}(t) = \exp(-\bar{n}) \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \left\{ \sum_{l=1}^{3} |\alpha_{1,n}^l|^4 + 2 \sum_{\substack{l,m=1 \\ (l \neq m)}}^{3} |\alpha_{1,n}^l|^2 |\alpha_{1,n}^m|^2 \cos\left[(\lambda_{l,n} - \lambda_{m,n})t\right] \right\}. \quad (36)$$

The above equation confirms again the conclusion of Deng [13] that each revival series corresponds to a beat frequency. If a system has N eigenvalues whose absolute values are not equal, there will be $N(N-1)/2$ nonzero beat frequencies and consequently, $N(N-1)/2$ series of revivals. In our case, since $\alpha_{1,n}^4 = 0\ \forall n$, it follows from Eq. (36) that there are only three series of revivals. We call them (12), (23), and (13) series. On resonance ($\Delta = 0$) $\lambda_{3,n} = 0\ \forall n$, therefore, $|\lambda_{1,n} - \lambda_{3,n}| = |\lambda_{2,n} - \lambda_{3,n}| = \lambda_{1,n}$, $|\lambda_{1,n} - \lambda_{2,n}| = 2\lambda_{1,n}$ and the number of revival series reduces to two, which agrees with the result of [13].

In Fig. 3 we have plotted the time evolution of $P_{ee}(t)$ for a coherent initial field with $\bar{n} = 10$ and for various values of the detuning parameter. The effects of $\Delta \neq 0$ can be seen to have three aspects: the time average of $P_{ee}(t)$ is shifted to its initial value, the heights of revival signals decrease and the revival times delay. Taking into account the sharp peak of the photon distribution around its mean $\bar{n}$ when $\bar{n} \gg 1$, the revival periods can be evaluated analytically in some extreme cases. For example, when the near resonance condition

$$\delta^2 = \frac{\Delta^2}{4g^2\bar{n}} \ll 1 \quad (37)$$

is met, one gets

$$\begin{aligned} T_{12} &= 2\pi \left[(\lambda_{1,\bar{n}} - \lambda_{2,\bar{n}}) - (\lambda_{1,\bar{n}-1} - \lambda_{2,\bar{n}-1})\right]^{-1} \\ &\simeq \frac{\pi\sqrt{\bar{n}}}{g}\left(1 + \frac{\delta^2}{2}\right), \\ T_{23} &= 2\pi \left[(\lambda_{2,\bar{n}} - \lambda_{3,\bar{n}}) - (\lambda_{2,\bar{n}-1} - \lambda_{3,\bar{n}-1})\right]^{-1} \end{aligned}$$

$$\begin{aligned} &\simeq \frac{2\pi\sqrt{\bar{n}}}{g}\left(1 + \frac{\delta^2}{2} + \frac{\delta}{\bar{n}}\right), \quad (38) \\ T_{13} &= 2\pi \left[(\lambda_{1,\bar{n}} - \lambda_{3,\bar{n}}) - (\lambda_{1,\bar{n}-1} - \lambda_{3,\bar{n}-1})\right]^{-1} \\ &\simeq \frac{2\pi\sqrt{\bar{n}}}{g}\left(1 + \frac{\delta^2}{2} - \frac{\delta}{\bar{n}}\right). \end{aligned}$$

In general, it is difficult to follow analytically the dependence of T_{lm} on the detuning parameter since λ_l are expressed in terms of Δ in a rather complicated way. Therefore, we have performed some numerical calculations showing that $|\lambda_3|$, though does not vanish when Δ takes nonzero values, is very small as compared with $|\lambda_1|$ and $|\lambda_2|$. As a result, $T_{(23)}$ and $T_{(13)}$ hardly differ from each other and in the figure, only two series of revivals are seen. The computational calculations also reveal that the height of the revival signals of series (12) and (23) drop rapidly with increasing Δ and the only series remaining in Fig. 3d is connected with the beating between λ_1 and λ_3. The revival signals of this series, contrary to the results for the single-atom case [6], almost do not spread in time as Δ increases. To explain this fact, one needs a more detailed investigation, possibly, in a way similar to that used in [6] employing the saddle point technique. However, we will not pursue this problem here, but proceed to treat the far-off-resonance limit

$$\delta^2 = \frac{\Delta^2}{4g^2\bar{n}} \gg 1, \quad (39)$$

supposing that the field itself is large. Then, using the result presented in Section 2, in the interaction picture one gets approximately for

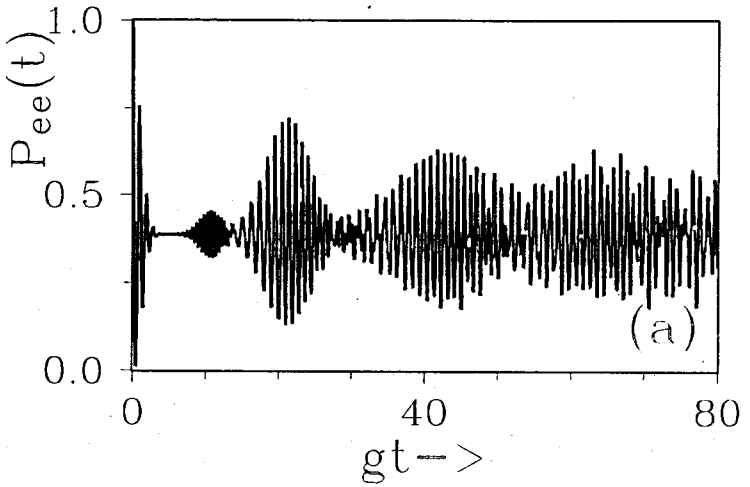
– two initially excited atoms

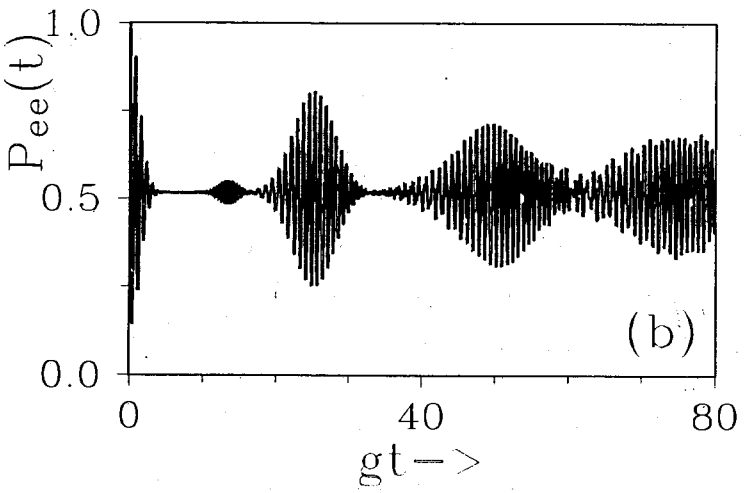
$$P_{ee}(t) \simeq 1, \qquad P_{eg}(t) \simeq P_{ge}(t) \simeq P_{gg}(t) \simeq 0, \quad (40a)$$

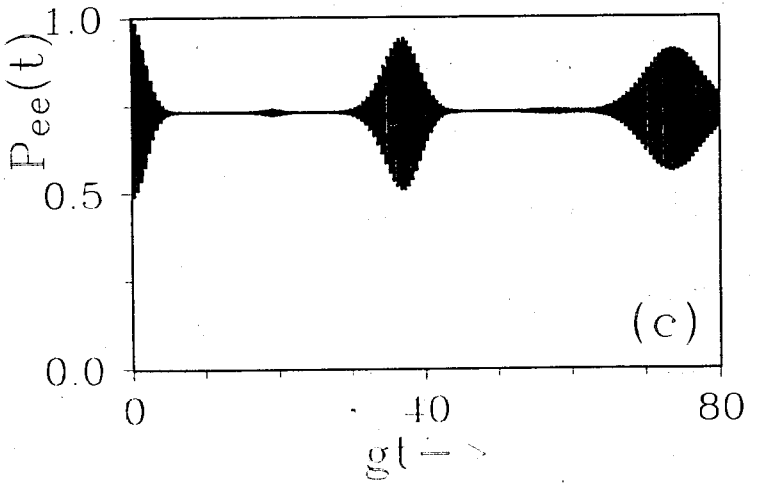
$$|e,e\rangle|v\rangle\Big|_{t=0} \longrightarrow \exp(-i\Delta t)|e,e\rangle|v\exp(-i2g^2t/\Delta)\rangle \quad (40b)$$

– symmetrical excitation $\left(C_2(0) = C_3(0) = 1/\sqrt{2}\right)$

$$P_{ee}(t) \simeq 0, \quad P_{eg}(t) = P_{ge}(t) \simeq \frac{1}{2} \quad P_{gg}(t) \simeq 0, \quad (41a)$$







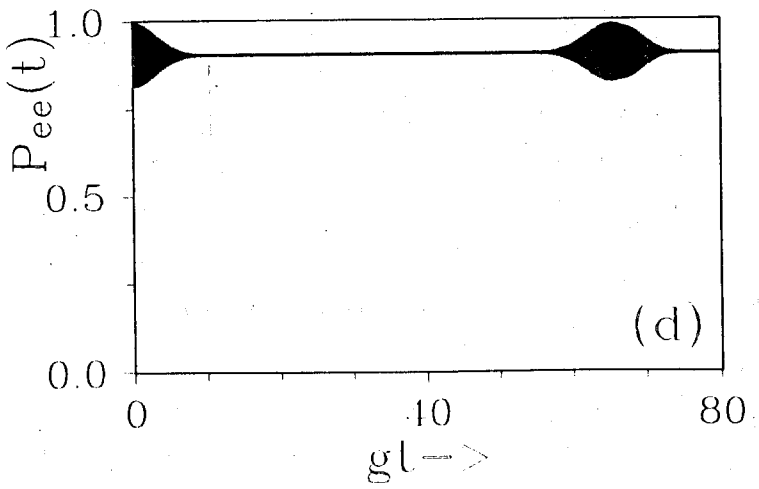


Figure 3

Probability of finding both atoms in the upper state P_{ee} versus gt for various values of detuning (a) $\Delta = 0$, (b) $\Delta = 5g$, (c) $\Delta = 10g$, and (d) $\Delta = 20g$ and for the field initially in a coherent state with $\bar{n} = 10$. The atoms are initially excited.

$$\frac{1}{\sqrt{2}}(|e,g\rangle+|g,e\rangle)|v\rangle\Big|_{t=0} \longrightarrow \exp(-i2g^2t/\Delta)\frac{1}{\sqrt{2}}(|e,g\rangle+|g,e\rangle)|v\exp(i8g^4t/\Delta^3)\rangle \quad (41b)$$

– and for nonsymmetrical excitation ($C_2(0)=1$)

$$P_{ee}(t)\simeq P_{gg}(t)\simeq 0,$$

$$P_{eg}(t)\simeq \frac{1}{2}+\frac{1}{2}\exp(-\bar{n})\sum_{n=0}^{\infty}\frac{\bar{n}^n}{n!}\cos\left[\frac{2\Delta g^2t}{2g^2(2n+1)+\Delta^2}\right],$$

$$P_{ge}(t)\simeq \frac{1}{2}-\frac{1}{2}\exp(-\bar{n})\sum_{n=0}^{\infty}\frac{\bar{n}^n}{n!}\cos\left[\frac{2\Delta g^2t}{2g^2(2n+1)+\Delta^2}\right], \quad (42a)$$

$$|e,g\rangle|v\rangle\Big|_{t=0} \longrightarrow \frac{1}{2}(|e,g\rangle-|g,e\rangle)|v\rangle+\exp(-i2g^2t/\Delta)\frac{1}{2}(|e,g\rangle+|g,e\rangle)|v\exp(i8g^4t/\Delta^3)\rangle. \quad (42b)$$

In the first two cases, the picture resembles that in the one-atom JCM at far-off-resonance [2, 39]: the excited atoms are reluctant to emit a photon and the state of the system approximately remains factored into an atomic and a field state with the field being simply a phasor rotating in the phase plane. For nonsymmetrical excitation one has quite another behavior. Though, as before, the atomic system does not radiate, the energy initially stored in atom 1 is nearly periodically exchanged with atom 2, despite the fact that no direct coupling between them is included in the Hamiltonian (1) and the field mode is far detuned. Thus, here the radiation field acts like a virtual level through which the excitation is transferred from atom 1 to atom 2 and back. The state of the field itself is also dramatically changed. Equation (42b), which is no longer of a product form, indicates clearly the splitting of the quasiprobability distributions into two peaks, one is unmoved while the other rotates. Unfortunately, a Schrödinger cat cannot be generated using this scheme because the atomic states appearing in Eq. (42b) are all the time orthogonal.

5 Emission spectra

Beginning from the work by Sanchez-Mondragon et al [7], the problem of spectra of light emitted from atoms enclosed in cavities has attracted a great deal of attention. Recently, the vacuum-field Rabi splittings, first predicted in [7, 8], have been observed experimentally [9]. The effects of the field statistics [40, 41], cavity damping [42], atomic coherence [33], and the correlations between the spectrum sidebands [43] have been theoretically studied. In the system in question, the results for exact resonance and two initially excited atoms have been reported in [19]–[21]. Here, we discuss some aspects of the emission spectra for nonzero detuning and various initial atomic states.

For simplicity, we restrict ourselves to the case of the initial Fock state field, i.e., the initial state of the total atom-field system can be written as

$$|\psi(0)\rangle = C_1(0)|e,e;n\rangle + C_2(0)|e,g;n\rangle + C_3(0)|g,e;n\rangle + C_4(0)|g,g;n\rangle. \quad (43)$$

To calculate the spectrum, we need to know the two-time dipole correlation function

$$\begin{aligned}\mathcal{D}(t,\tau) &\equiv \langle R^+(t+\tau)R^-(t)\rangle \\ &= \langle\psi(0)|\exp[iH(t+\tau)]R^+\exp(-iH\tau)R^-\exp(-iHt)|\psi(0)\rangle, \quad (44)\end{aligned}$$

The solution (9) and (14)–(15) together with the initial condition (43) give

$$\begin{aligned}\mathcal{D}(t,\tau)=\exp(i\omega\tau)\sum_{l,m,k=1}^{3}\Big\{&|C_1(0)|^2\mathcal{D}_{l,m,k}(1)\exp[i(\lambda_l-\lambda_m)t]\exp[i(\lambda_l-\lambda_m')\tau] \\ &+|C_2(0)+C_3(0)|^2\mathcal{D}'_{l,m,k}(2)\exp[i(\lambda_l'-\lambda_m')t]\exp[i(\lambda_l'-\lambda_k'')\tau] \\ &+|C_4(0)|^2\mathcal{D}''_{l,m,k}(4)\exp[i(\lambda_l''-\lambda_m'')t]\exp[i(\lambda_l''-\lambda_k''')\tau]\Big\}, \quad (45)\end{aligned}$$

where

$$\mathcal{D}_{l,m,k}(j)=4\alpha_j^l\alpha_j^m\left[\alpha_1^l(\alpha_2^k)'+(\alpha_4^k)'\alpha_2^l\right]\left[\alpha_1^m(\alpha_2^k)'+(\alpha_4^k)'\alpha_2^m\right], \quad (46)$$

and the prime mark indicates the replacement of n by $(n-1)$ in the corresponding quantities. The physical transient spectrum [31] is given in terms of the correlation

function (44) by

$$S(\nu,T)=2\Gamma\mathrm{Re}\int_0^T d\tau\exp[(\Gamma-i\nu)\tau]\int_0^{T-\tau}dt\exp[-2\Gamma(T-t)]\mathcal{D}(t,\tau). \qquad (47)$$

Here Γ is the detector bandwidth and T is the time at which the measurement takes place. Under a definite initial condition, by substituting $\mathcal{D}(t,\tau)$ from Eq. (45) with appropriate coefficients $C_j(0)$ into Eq. (47), after integration over t and τ, the explicit expressions for the emission spectra can be easily obtained.

(i) *Two atoms initially in the upper state*

Then one finds

$$S(\nu,T;|e,e\rangle)=2\Gamma\mathrm{Re}\sum_{l,m,k=1}^{3}\frac{\mathcal{D}_{l,m,k}(1)}{2\Gamma+i(\lambda_l-\lambda_m)}$$
$$\times\left(\frac{1}{\Gamma+i(\nu-\omega-\lambda_m+\lambda'_k)}\Big\{\exp[i(\lambda_l-\lambda_m)T]-\exp[-\Gamma T-i(\nu-\omega-\lambda_l+\lambda'_k)T]\Big\}\right.$$
$$\left.-\frac{1}{\Gamma-i(\nu-\omega-\lambda_l+\lambda'_k)}\Big\{\exp[-\Gamma T-i(\nu-\omega-\lambda_l+\lambda'_k)T]-\exp(-2\Gamma T)\Big\}\right). \qquad (48)$$

If ΓT is large, and if we ignore small terms connected with $l\neq m$ [42], formula (48) is transformed into

$$S(\nu,T;|e,e\rangle)\longrightarrow\sum_{l,k=1}^{3}\mathcal{D}_{l,l,k}(1)\frac{\Gamma}{\Gamma^2+[\nu-\omega-(\lambda_l-\lambda'_k)]^2}. \qquad (49)$$

In this limit, the spectrum of emitted light consists of nine lines whose positions and heights are determined by $\nu=\omega+(\lambda_l-\lambda'_k)$, $\mathcal{D}_{l,l,k}(1)$. When $\Delta=0$, it follows from Eqs. (19) and (46) that $\mathcal{D}_{3,3,3}(1)=0$, which, in turn, implies the reduction of the number of lines from nine to eight. This is in accordance with the earlier results of Kien et al and Chai et al [20]. For nonidentical atoms, the upper limit of the sums appearing in Eqs. (45), (48) and (49) must be replaced by 4; consequently, the number of spectral lines increases to 16 [21]. In the dressed-state representation [44], this sixteen-peaked

spectrum arises naturally from the transitions $|\psi_n^l\rangle\longrightarrow|\psi_{n-1}^l\rangle$, $(l=1,2,3,4)$, where the dressed states $|\psi_n^l\rangle$ are nothing but the eigenstates of the system Hamiltonian. When the two atoms are identical, one of the states $|\psi_n^l\rangle$ ($|\psi_n^4\rangle$ in our notation) becomes subradiant, which results in a nine-peaked structure of the emission spectrum mentioned above. In the same way, one can explain the quantity of vacuum-field Rabi splittings in the case of the two initially excited atoms, which is equal to 6 for identical atoms [19, 20] and 12 for nonidentical ones [21]. We can also predict, for example, the number of spectral lines of light spontaneously emitted from an excited atom in the presence of another unexcited one. Such a system has only three dressed states (see Eqs. (21), (22)). The decay of these into the unique ground state $|g,g;0\rangle$ will obviously lead to a three-peaked structure of the spontaneous emission spectrum. A closer observation of the expressions (22) reveals that for identical atoms, the number of peaks reduces to two since then one of the dressed states becomes subradiant. These conclusions are confirmed by more detailed analyses below.

(ii) *Symmetrical and nonsymmetrical excitations*

From Eqs. (45), (47) one gets, in the limit of long times,

$$S\big[\nu,T;(|e,g\rangle+|g,e\rangle)/\sqrt{2}\big]\longrightarrow 2\sum_{l,k=1}^{3}\mathcal{D}'_{l,l,k}(2)\frac{\Gamma}{\Gamma^2+[\nu-\omega-(\lambda'_l-\lambda''_k)]^2}, \qquad (50)$$

$$S(\nu,T;|e,g\rangle)=\frac{1}{2}S\big[\nu,T;(|e,g\rangle+|g,e\rangle)/\sqrt{2}\big]. \qquad (51)$$

Equation (51) indicates that the system of the two symmetrically excited atoms emits light two times stronger than that of the two nonsymmetrically excited ones does. This, clearly, stems from the fact that the nonsymmetrical state is a linear superposition of the symmetrical and antisymmetrical states while the antisymmetrical state is subradiant.

When $\Delta\neq0$, the spectra defined by Eqs. (50) and (51) consist of nine lines, which are asymmetric both in height and position. The symmetry reappears as the detuning

vanishes but then, instead of eight peaks as in the case of two initially excited atoms, only six peaks at the frequencies

$$\begin{aligned} \nu_{\pm 1} &= \omega \pm g(\sqrt{4n+2} - \sqrt{4n-2}), \\ \nu_{\pm 2} &= \omega \pm g\sqrt{4n+2}, \\ \nu_{\pm 3} &= \omega \pm g(\sqrt{4n+2} + \sqrt{4n-2}) \end{aligned} \tag{52}$$

survive. Note that in Eqs. (52) $\nu_{\pm 3}$ represent the quantum electrodynamic analogue of the so-called cooperative additional sidebands [45]. The relations between the heights of the peaks are

$$I_{\pm 1} : I_{\pm 2} : I_{\pm 3} = \left(\sqrt{\frac{n}{4n+2}} + \sqrt{\frac{n}{4n-2}}\right)^2 : \frac{n-1}{2n-1} : \left(\sqrt{\frac{n}{4n+2}} - \sqrt{\frac{n}{4n-2}}\right)^2 . \tag{53}$$

As n increases, the two central peaks $\nu_{\pm 1}$ draw closer to each other and eventually emerge into one peak at ν_0 while the heights of the two extreme side peaks $\nu_{\pm 3}$ go to zero as fast as $1/n^2$. Therefore, the spectra will have three peaks at ω and $\omega \pm 2g\sqrt{n}$ for large photon numbers with the height of the sideband peaks being 1/4 that of the central peak. Recall that this ratio is 3/4 for both atoms initially excited [20] and 1/2 for a one-atom system [8].

We now consider the vacuum-field Rabi splittings for the system consisting of one initially excited atom and another unexcited one. We suppose for a while that the atoms can be different ($g_1 \neq g_2$). Then, using the solution (21), (22) in (44), (47) one gets, for long times,

$$S(\nu, T; |j\rangle) \longrightarrow \sum_{l=1}^{3} |\alpha_j^l|^2 (\alpha_1^l + \alpha_2^l)^2 \frac{\Gamma}{\Gamma^2 + (\nu - \omega_0 - \lambda_l)^2}, \tag{54}$$

where the initial state $|j\rangle$ is one of the states (20). Equation (54) shows that the emission spectrum in this case has three peaks at frequencies $\omega_0 + \lambda_l$ with their heights proportional to $|\alpha_j^l|^2(\alpha_1^l + \alpha_2^l)^2$. When the atoms are identical, since $\alpha_1^3 = -\alpha_2^3$ (see

Eq (22)), the central peak connected with λ_3 disappears resulting in a two-peaked spectrum. Similarly as for the stimulated emission, the intensity of spontaneous-emission spectrum in the case of symmetrical excitation is twice larger than that in the case of nonsymmetrical excitation, which can be interpreted as another indication of the superradiance.

Finally, we emphasize that for a more general field state, which may be an arbitrary superposition of the number states, all the spectral characteristics will essentially depend on the photon statistics.

Acknowledgment

We are thankful to Dr. V.Bužek for many helpful suggestions.

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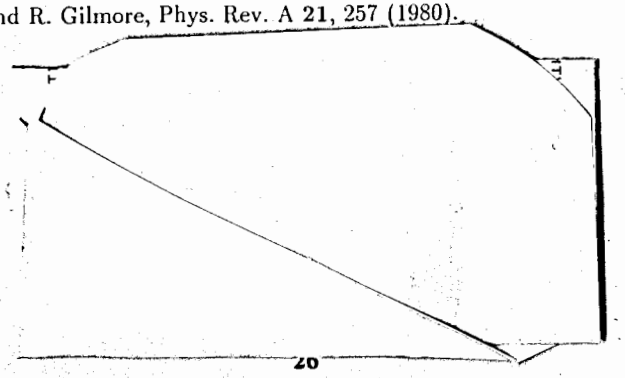

Хо Чунг Зунг, Нгуен Динь Хуен E17-92-419

Взаимодействие двух атомов с одной модой поля излучения:
эволюция состояния, вероятности населенностей уровней
и спектры флюоресценции

Представлено точное решение для задачи взаимодействия двух двухуровневых атомов с одномодовым полем излучения при ненулевой расстройке. При точном резонансе приведен асимптотический результат для первоначально сильных когерентных полей. Показано, что если в начальный момент времени атомы находятся в одном из полуклассических собственных состояний, волновая функция системы атом + поле остается почти факторизированной на атомную и полевую часть в течение взаимодействия. При определенных условиях, атомная часть эволюционирует в единственное чистое состояние. Изучено влияние расстройки резонатора на динамику вероятностей населенностей атомных уровней и спектры флюоресценции. Найдено, что при несимметричном возбуждении, вдалеке от резонанса, поле действует как виртуальный уровень, путем которого энергия передается между атомами. Из-за ненулевой расстройки появляются девятиликовые спектры флюоресценции. Дается ясное объяснение с точки зрения формализма одетых состояний. Установлено, что в случае симметричного возбуждения, интенсивность спектра флюоресценции в два раза сильнее, чем в случае симметричного возбуждения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1992

Ho Trung Dung, Nguyen Dinh Huyen E17-92-419

Two-Atom Single-Mode Redlation Field Interaction: State Evolution,
Level Occupation Probabilities and Emission Spectra

Exact solution for the problem of two two-level atoms interacting with a single-mode detuned quantized radiation field is presented. On resonance, an asymptotic result is derived for large coherent initial fields. It is shown that when the atoms are initially prepared in one of the semiclassical eigenstates, the atom-plus-field wave function almost remains factored into an atomic and a field part throughout the interaction. Under certain conditions, the atomic part evolves into a unique pure state at half-revival time. The effects of the cavity detuning on the dynamics of the atomic level occupation probabilities and emission spectra ate studied. It is found, for nonsymmetrical excitation, that far off-resonance the field acts like a virtual level by means of which the energy is transferred between the atoms. Due to nonzero detuning, nine-peaked spectra are observed and are given a clear explanation from the dressed-state viewpoint. It is established that in the case of symmetrical excitation, the intensity of the emission spectrum is two times larger than that in the case of nonsymmetrical excitation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1992