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COLLAPSES AND REVIVALS OF PHASE FLUCTUATIONS IN PARAMETRIC DOWN-CONVERSION WITH QUANTUM PUMP

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A degenerate parametric down-conversion process is known to produce optical fields with nonclassical properties [1]. In this process a pump photon of frequency 2ω is down-converted into two highly correlated photons at the subharmonic frequency ω. The quantum theory of this process is usually treated in the so-called parametric approximation in which the pump mode is assumed as classical and nondepleted. In the parametric approximation the time evolution of the subharmonic field can be found analytically and is described by a Bogoliubov transformation that maps the initial vacuum state into an ideal squeezed state [1]. The parametric down-conversion process turned out to be very effective in producing squeezed states in practice [2]–[6].

The parametric approximation which ignores the quantum fluctuations and depletion of the pump mode, is not applicable if a considerable amount of power is transferred from the pump mode into the signal mode. In such situations the pump mode must be treated dynamically and its quantum mechanical evolution must be taken into account. Since no close form solutions are known in this case, some approximations or numerical calculations are needed to find the field evolution. Owing to the energy conservation the intensity of the signal mode cannot grow infinitely, and the solutions become oscillatory. The field states of the signal mode are no longer the ideal squeezed states, and their properties become different.

In a recent series of papers [7]–[9] we studied the effect of the quantum fluctuations and depletion of the pump on the phase properties of the fields produced in the down-conversion process. We have shown that the quantum character of the pump mode essentially changes phase properties of the field at later stages of the evolution, while at earlier stages of the evolution the signal mode properties are very close to those of ideal squeezed states. The phase distribution of such states has two sharp peaks at the initial stages of the evolution that reflect the two-photon character of the process. Because of the quantum fluctuations and depletion of the pump mode the two peaks of the signal mode are broadened, and at later time the phase distribution becomes uniform.

The present paper extends these earlier works by calculating the long-time evolution of phase fluctuations of the field produced in the down-conversion process with quantum pump. The fully quantum mechanical approach using the method of numerical diagonalization of the interaction Hamiltonian [10] is employed for getting the evolution of the system. The Hermitian phase formalism of Pegg and Barnett [11]–[13] is used to calculate the quantum phase fluctuations.

The two-photon down-conversion process is described by the following model Hamiltonian:

$$H = H_0 + H_I = \hbar\omega a^\dagger a + 2\hbar\omega b^\dagger b + \hbar g(b^\dagger a^2 + b a^{\dagger 2}), \tag{1}$$

where a ($a^\dagger$) and b ($b^\dagger$) are the annihilation (creation) operators of the signal mode at frequency ω and the pump mode at frequency 2ω, respectively. The coupling constant g, which is assumed real, proportional to the second order nonlinear polarizability coefficient of the crystal.

Since H_0 and H_I commute, there are two constants of motion, H_0 and H_I. H_0 determines the total energy stored in both modes which is conserved by the interaction H_I. This allows us to factor out $\exp(-iH_0t/\hbar)$ from the evolution operator and, in fact, to drop it altogether. In effect, the resulting state of the field can be written as

$$|\psi(t)\rangle = \exp(-iH_It/\hbar)|\psi(0)\rangle, \tag{2}$$

where $|\psi(0)\rangle$ is the initial state of the field. If the Fock states are used as basis states, the interaction Hamiltonian H_I is not diagonal in such a basis. To find the state evolution, we apply the numerical method of diagonalization of H_I [10].

Let the signal mode be initailly in the vacuum state and the pump mode in an arbitrary state so that the initial state of the field is given by

$$|\psi(0)\rangle = \sum_{n=0}^{\infty} b_n e^{in\varphi_b}|0,n\rangle, \tag{3}$$

where b_n is real. With these initial conditions the resulting state (2) can be written as

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} b_n e^{in\varphi_b} \sum_{k=0}^{n} c_{2n,k}(t)|2k, n-k\rangle, \tag{4}$$

where the coefficients $c_{2n,k}(t)$ are the matrix elements of the evolution operator

$$c_{2n,k}(t) = \langle 2k, n-k|\exp(-iH_It/\hbar)|0,n\rangle, \tag{5}$$

and they are calculated numerically by diagonalizing the interaction Hamiltonian. This allows us to find the evolution of the state (4) and, in effect, its phase properties.

According to Pegg and Barnett [11]–[13], the Hermitian phase operator can be constructed in an $(s+1)$–dimensional state space Ψ spanned either by the number states, $|n\rangle$, or $(s+1)$ orthonormal phase states, $|\theta_m\rangle$. The phase states can be expanded in terms of the number states as

$$|\theta_m\rangle \equiv \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \exp(in\theta_m)|n\rangle, \qquad m = 0, 1, ..., s, \tag{6}$$

where

$$\theta_m \equiv \theta_0 + \frac{2\pi m}{s+1}. \tag{7}$$

The value of θ_0 is arbitrary and defines a particular basis set of $(s+1)$ mutually orthogonal phase states. The Hermitian phase operator is defined as

$$\hat{\phi}_\theta \equiv \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m|. \tag{8}$$

Physical results are obtained in the limit $s \to \infty$, and according to Pegg and Barnett this limit has to be taken only after c numbers, such as the expectation value and variance of the phase, have been calculated in the finite basis (6). The failure of earlier attempts to construct a Hermitian phase operator result from taking this limit at a premature stage [14].

By means of Eq. (6) we now calculate the joint phase probability amplitude

$$\langle\theta_{m_a}|\langle\theta_{m_b}|\psi(t)\rangle = (s_a+1)^{-1/2}(s_b+1)^{-1/2} \times \sum_{n=0}^{s_a} b_n e^{in\varphi_b} \sum_{k=0}^{n} \exp\{-i[2k\theta_{m_a} + (n-k)\theta_{m_b}]\} c_{2n,k}(t) \tag{9}$$

of a state (4). We use the indices a and b to distinguish between the signal (a) and pump (b) modes. There is still a freedom of choice in (9) of the values of $\theta_0^{a,b}$ which define the phase values window. We can choose these values at will, so we take them as

$$\theta_0^{\mathrm{a,b}} = \varphi_{\mathrm{a,b}} - \frac{\pi s_{\mathrm{a,b}}}{s_{\mathrm{a,b}}+1}, \tag{10}$$

and we introduce the new phase values

$$\theta_{\mu_{\mathrm{a,b}}} = \theta_{m_{\mathrm{a,b}}} - \varphi_{\mathrm{a,b}}, \tag{11}$$

where the new phase labels $\mu_{\mathrm{a,b}}$ run in unit step between the values $-s_{\mathrm{a,b}}/2$ and $s_{\mathrm{a,b}}/2$. This means that we symmetrize the phase windows for the signal and pump modes with respect to the phases φ_{a} and φ_{b}, respectively. We are free to choose the parameters $s_{\mathrm{a,b}}$ as large as they are needed, and for physical states, according to their definition by Pegg and Barnett [12, 13], it is always possible to choose $s_{\mathrm{a,b}}$ much larger than the contributing number states. In this case, the parameters $s_{\mathrm{a,b}}$ in the sum of Eq. (9) can be replaced to any desired degree of accuracy by the infinity. On inserting (10) and (11) into (9), taking the modulus squared of (9), and performing the continuum limit transition, we arrive at the continuous joint probability distribution for the continuous variables θ_{a} and θ_{b}, which has the form

$$\begin{aligned} P(\theta_{\mathrm{a}},\theta_{\mathrm{b}}) &= \frac{1}{(2\pi)^2}\Bigg|\sum_{n=0}^{\infty} b_n \sum_{k=0}^{n} c_{2n,k}(t) \\ &\times \exp\Big\{-i\big[2k\theta_{\mathrm{a}} + (n-k)\theta_{\mathrm{b}} + k(2\varphi_{\mathrm{a}}-\varphi_{\mathrm{b}})\big]\Big\}\Bigg|^2. \end{aligned} \tag{12}$$

The distribution (12) is normalized so as

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} P(\theta_{\mathrm{a}},\theta_{\mathrm{b}})\mathrm{d}\theta_{\mathrm{a}}\mathrm{d}\theta_{\mathrm{b}} = 1. \tag{13}$$

To choose the phase windows for θ_{a} and θ_{b}, we have to assign to φ_{a} and φ_{b} particular values. It is interesting to note that the distribution $P(\theta_{\mathrm{a}},\theta_{\mathrm{b}})$ given by (12) depends on the phase difference $2\varphi_{\mathrm{a}}-\varphi_{\mathrm{b}}$ only. This reproduces the classical phase relation for the parametric amplifier, and to minimize the phase variance we choose this value equal to $2\varphi_{\mathrm{a}}-\varphi_{\mathrm{b}} = \pi/2$.

Once the joint phase distribution $P(\theta_{\mathrm{a}},\theta_{\mathrm{b}})$ is known, all quantum mechanical phase expectation values can be calculated with this function in a classical-like manner by performing appropriate integrations over θ_{a} and θ_{b}. In partucular, the phase variance for the signal mode can be calculated according to the formula

$$\begin{aligned} (\Delta\hat{\phi}_{\theta_{\mathrm{a}}})^2 &= \langle\hat{\phi}^2_{\theta_{\mathrm{a}}}\rangle - \langle\hat{\phi}_{\theta_{\mathrm{a}}}\rangle^2 \\ &= \int_{-\pi}^{\pi}\mathrm{d}\theta_{\mathrm{a}}\,\theta_{\mathrm{a}}^2\int_{-\pi}^{\pi}\mathrm{d}\theta_{\mathrm{b}}\,P(\theta_{\mathrm{a}},\theta_{\mathrm{b}}) \\ &= \frac{\pi^2}{3} + \mathrm{Re}\sum_{n>n'} b_n b_{n'}\frac{\exp[-i(n-n')(2\varphi_{\mathrm{a}}-\varphi_{\mathrm{b}})]}{(n-n')^2} \\ &\times \sum_{k=0}^{n'} c_{2n,k+n-n'}(t)c^*_{2n',k}(t), \end{aligned} \tag{14}$$

and for the pump mode we have

$$\begin{aligned} (\Delta\hat{\phi}_{\theta_{\mathrm{b}}})^2 &= \int_{-\pi}^{\pi}\mathrm{d}\theta_{\mathrm{b}}\,\theta_{\mathrm{b}}^2\int_{-\pi}^{\pi}\mathrm{d}\theta_{\mathrm{a}}\,P(\theta_{\mathrm{a}},\theta_{\mathrm{b}}) \\ &= \frac{\pi^2}{3} + 4\mathrm{Re}\sum_{n>n'} b_n b_{n'}\frac{(-1)^{n-n'}}{(n-n')^2}\sum_{k=0}^{n'} c_{2n,k}(t)c^*_{2n',k}(t), \end{aligned} \tag{15}$$

where we have used (12), and we take $2\varphi_a - \varphi_b = \pi/2$. So far we have derived exact analytical formulae for the phase variances. The time evolution of the phase variances can be calculated numerically using these expressions for given initial field states.

We consider the case in which the pump mode is initially in the coherent state $|\beta\rangle$ with the mean photon number $\bar{n}_{\mathrm{b}}$. For the coherent states the expansion coefficients b_n are given by

$$b_n = \exp(-|\beta|^2/2)|\beta|^n/\sqrt{n!}, \tag{16}$$

where $\beta = |\beta|\exp(i\varphi_{\mathrm{b}})$ and $\bar{n}_{\mathrm{b}} = |\beta|^2$. The dynamical behaviour of the phase variances calculated from Eqs. (14) and (15) with the coefficients (16) is illustrated in Figs. 1 and 2, respectively, for various values of $\bar{n}_{\mathrm{b}}$. The dashed line $\pi^2/3$ marks the variance for the state with random distribution of phase. From the figures we observe that the

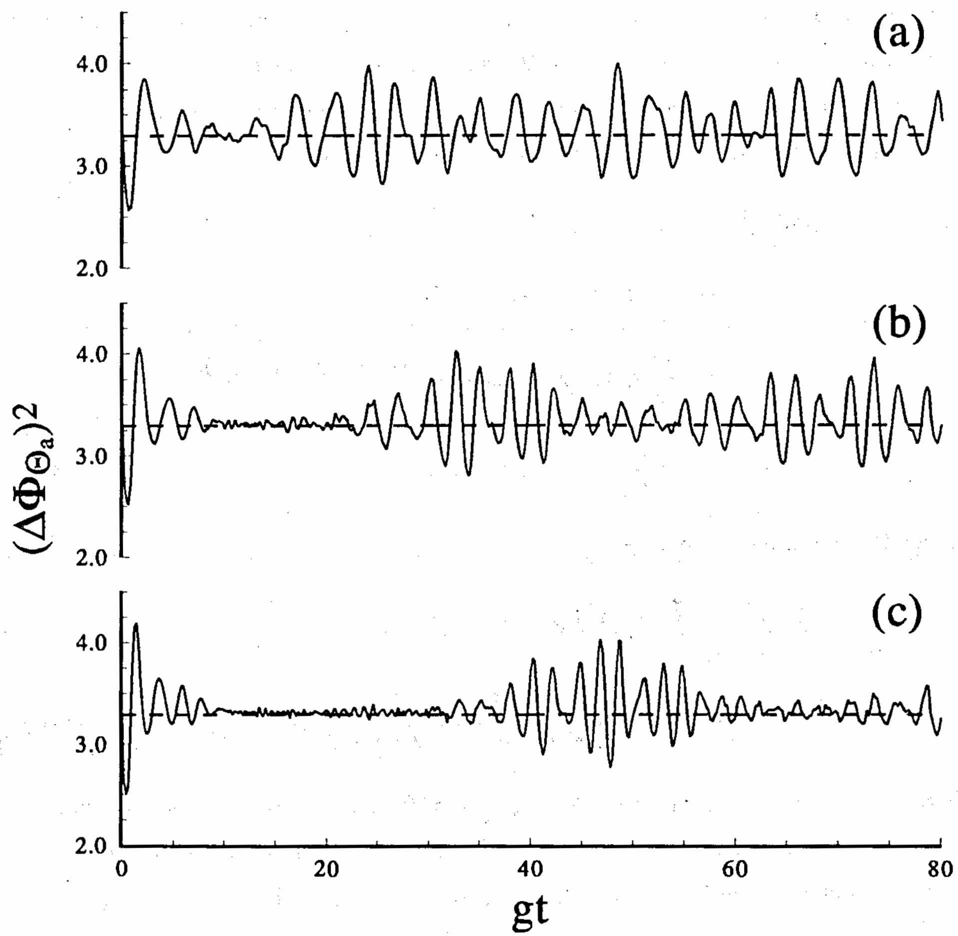

Figure 1: Phase variance of the signal mode $(\Delta\hat{\phi}_{\theta_a})^2$ as a function of gt for the pump initially in the coherent state $|\beta\rangle$ and the signal in the vacuum state $|0\rangle$. The dashed line marks the value $\pi^2/3$ of a randomly distributed phase. The initial mean number of photons in the pump mode $\bar{n}_b = |\beta|^2$ is: (a) 4, (b) 9, and (c) 16.

phase variances oscillate irregularly around the value $\pi^2/3$, and there is a collapse and revival of these oscillations. It is also evident that the revivals occur on a longer time scale with increasing initial mean number of photons in the pump field. In other words, the stronger the initial pump field is, the clearer the collapses and revivals become.

In order to set properly the time scale on which the essential changes of phase properties take place, we have plotted in Fig. 3 the evolution of the mean number of photons in the signal mode, $\langle a^\dagger a\rangle$, for various values of $\bar{n}_b$. In direct contrast with the parametric approximation, in which the mean number of photons in the signal mode is a monotonic function of the time, $\langle a^\dagger a\rangle = \sinh^2(2|\beta|gt)$, the oscillatory behaviour of the quantum solution and, the collapses and revivals of oscillations are clearly visible. The time evolution of the mean number of photons in the pump mode can be easily derived by using the conservation law $\langle a^\dagger a\rangle + 2\langle b^\dagger b\rangle = 2|\beta|^2$. Comparing Figs. 1, and 2 with Fig. 3 one can conclude that the revival time of the oscillations of the phase fluctuations does not coincide with that of the mean photon number; the first revival of the phase fluctuations takes place when the second revival of the oscillations of the mean photon numbers occurs. Collapse and revival phenomenon in the energy exchange between two modes in the process of $k-$photon down conversion with quantum pump has recently been discussed by Drobný and Jex [15].

Note that the collapse and revival phenomenon of phase fluctuations is sensitive to the chosen initial state of the field. As an example, let us consider the situation where the pump mode is initially in the number state $|n\rangle$ for which only one $b_n = 1$ is nonzero. In this case the last terms in Eqs. (14) and (15) vanish and as a result we get

$$(\Delta\hat{\phi}_{\theta_a})^2 = (\Delta\hat{\phi}_{\theta_b})^2 = \frac{\pi^2}{3}. \tag{17}$$

This result is also true for a more general case in which both the pump mode and the signal mode are initially in the number states. So even though the oscillations of the mean number of photons in the fields which are initially in the number states show the collapse and revival [15], the phase variances in such fields do not exhibit the oscillations.

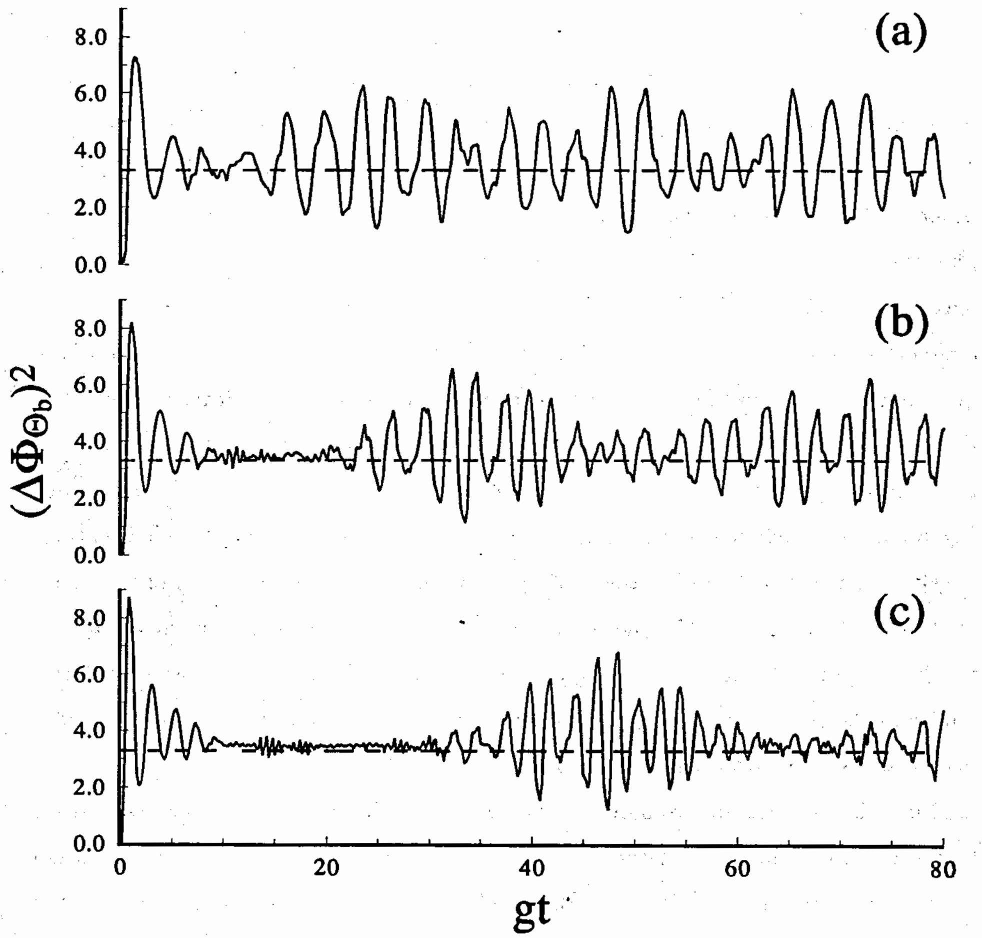

Figure 2: Phase variance of the pump mode $(\Delta\hat{\phi}_{\theta_b})^2$ as a function of gt. The initial conditions and the parameters are the same as in Fig. 1.

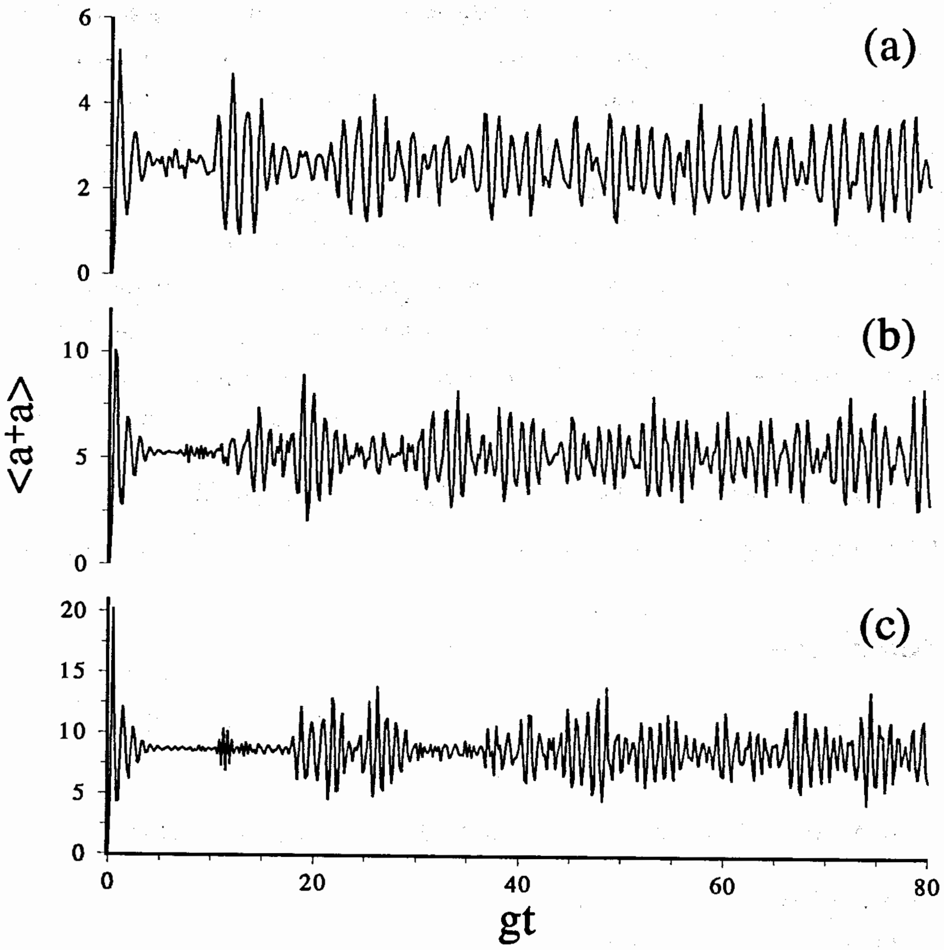

Figure 3: Mean number of photons in the signal mode $\langle a^{\dagger}a\rangle$ as a function of gt. The initial conditions and the parameters are the same as in Fig. 1.

In conclusion, we have studied the long-time behaviour of the phase quantum fluctuations in the two-photon parametric down-conversion process with quantum pump. Our numerical calculations show that if the pump mode is initially in a coherent state, the phase variances show the irregular oscillations around the value $\pi^2/3$ of a randomly distributed phase and the oscillations exhibit the collapses and revivals. We also have shown that if the pump mode is initially in a number state, the phase fluctuations do not oscillate and are equal to $\pi^2/3$. It should be mentioned that the collapse and revival phenomenon of the phase fluctuations considered in this paper has recently been shown to exist also for the field in the M–photon Jaynes–Cummings model [16].

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Ганцог Ц. E17-92-311

Затухание и возобновление осцилляций фазовых флуктуаций в параметрической даун-конверсии с квантовой накачкой

Изучается долговременная эволюция квантовых флуктуаций фазы для полей, генерируемых в процессе параметрической даун-конверсии с квантовой накачкой. Показано, что,если мода накачки в начальный момент времени находится в когерентном состоянии, то для обеих мод, сигнальной моды и моды накачки, появляется эффект затухания и возобновления осцилляции дисперсии фазы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Gantsog Ts. E17-92-311

Collapses and Revivals of Phase Fluctuations in Parametric Down-Conversion with Quantum Pump

The long-time behaviour of the phase quantum fluctuations in the field produced by the parametric down-conversion with quantum pump is studied. It is shown that if the pump is initially in a coherent state the phase variances for both the signal and the pump modes show the collapses and revivals in their long-time evolution.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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