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PHASE DISTRIBUTIONS OF SQUEEZED NUMBER STATES AND SQUEEZED THERMAL STATES

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1 Introduction

The problem of the quantum description of the optical field phase has been the subject of considerable study for many years [1]. This is connected with the difficulty in constructing a linear Hermitian phase operator. Within the past few years the notion of phase variables in quantum systems has been greatly clarified. Pegg and Barnett [2]–[4] have shown how such an operator can be defined for quantized electromagnetic fields. This new formalism makes it possible to describe the quantum properties of optical phase in a direct way within quantum mechanics on the basis of the Hermitian phase operator and its eigenstates.

A quite different approach to the concepts of the phase variable has also been widely used in quantum optics [5]– [7] and which involves quantum quasiprobability distributions such as the Glauber-Sudarshan $\mathcal{P}$ function, the Q function and the Wigner function rather than Hermitian operators and their eigenstates. These quasiprobability distributions depend upon the complex eigenvalue α of the non-Hermitian annihilation operator, which can be expressed in terms of a radial variable $|\alpha|$ and a "phase" θ both of which are real. If we integrate over the radius, the resulting distributions are periodic in the phase angle and, for the most of states they satisfy all properties required by a proper phase distribution [5, 6]. In recent papers, we have compared the Pegg-Barnett phase distribution with those distributions obtained from the Wigner and Q functions by integrating them over the radius for the multi-photon down-conversion [8] and displaced number states [9].

In recent years, special attention in quantum optics has been paid to a class of optical field states that are called squeezed states (for a recent review see, for example, special issues of two optical journals [10] devoted to this subject). These states show reduced fluctuations in one quadrature component of the electromagnetic field and enhanced fluctuations in the other. They are manifestations of the quantum nature of the radiation field and have recently been generated by using several experimental setups and optical systems [11]– [17]. There is also good reason to believe that number states of the electromagnetic field will be generated in the near future. Filipowicz, Javanainen and Meystre [18] have shown that if inverted atoms with a well-defined velocity are injected inside a micromaser cavity, it is possible for the field to evolve towards a number state. The number states and the thermal states are defined only by their photon number and average photon number, respectively, and have a completely random phase. However, if we use the number states and/or thermal states as an input field in a squeezing device, such as a parametric amplifier, the properties of these states become phase dependent. So it should be interesting to study their phase properties. The squeezing effect itself is a completely non-classical phenomenon, and it is essential to use quantum theory in the description of its phase properties. Statistical properties of the squeezed number states as well as the squeezed thermal states have been described in detail elsewhere [19, 20]. Higher-order squeezing properties and correlation functions for the squeezed number states have also been studied [21]. The phase fluctuations in the squeezed number states have been considered in the weak-squeezing limit case by Nath and Kumar [22] using the Pegg-Barnett Hermitian phase formalism.

The purpose of this paper is to study the phase properties of the squeezed number states and squeezed thermal states. We use the Pegg-Barnett Hermitian phase formalism to find the phase distribution functions for the squeezed number states and squeezed thermal states. The Pegg-Barnett phase distribution is compared to the phase distributions obtained from the Wigner function, Q-function and the Glauber-Sudarshan $\mathcal{P}$ function by integrating them over the radial variable.

2 Phase distributions

The squeezed number states are defined by acting with the squeeze operator $S(r,\varphi)$ on the number state $|N\rangle$, that is

$$|N\rangle_{(r,\varphi)} = S(r,\varphi)|N\rangle, \tag{1}$$

where

$$S(r,\varphi) \equiv \exp\left[\frac{r}{2}(a^2 e^{-2i\varphi} - a^{\dagger 2} e^{2i\varphi})\right]. \tag{2}$$

The number state decomposition of the squeezed number state (1) can be written as

$$\begin{aligned} |N\rangle_{(r,\varphi)} &= \sum_n |n\rangle\langle n|N\rangle_{(r,\varphi)} = \sum_n |n\rangle\langle n|S(r,\varphi)|N\rangle \\ &= \sum_n b_n e^{i\varphi_n}|n\rangle, \end{aligned} \tag{3}$$

where [19, 20]

$$\begin{aligned} b_n &= \left(\frac{n!N!}{\cosh r}\right)^{1/2} \left(\frac{1}{2}\tanh r\right)^{(n+N)/2} \\ &\times \sum_{i=0}^{\min(n,N)} (-1)^{(N-i)/2} \frac{(2/\sinh r)^i}{i!} \frac{H_{n-i}(0)}{(n-i)!} \frac{H_{N-i}(0)}{(N-i)!}, \end{aligned} \tag{4}$$

and

$$\varphi_n = (n-N)\varphi \tag{5}$$

with φ being an angle which describes the orientation of the quadrature phase uncertainty ellipse, and $H_n(x)$ is the nth order Hermite polynomial.

The squeezed thermal state is the Bose-Einstein weighted sum of the squeezed number states with density matrix [19]

$$\hat{\rho}_{\mathrm{sq.th}} \equiv \frac{1}{1+\bar{n}} \sum_{N=0}^{\infty} \left(\frac{\bar{n}}{1+\bar{n}}\right)^N |N\rangle_{(r,\varphi)(r,\varphi)}\langle N| \tag{6}$$

where $\bar{n}$ is the average photon number of the thermal input field.

Having the number state decomposition (3) of the squeezed number states as well as the density matrix (6) of the squeezed thermal states, we can employ the Pegg-Barnett [2]– [4] Hermitian phase formalism to find the phase distribution functions for such states. The Pegg-Barnett (PB) formalism is based on the observation that the Hermitian phase operator can be defined in a finite $(s+1)$–dimensional state space Ψ spanned by the number states $|0\rangle, |1\rangle, \ldots, |s\rangle$. The main idea of the PB formalism is to evaluate all necessary expectation values on this finite dimensional state space, and only after that the value of s is allowed to tend to infinity. A complete orthonormal basis of $(s+1)$ phase states is defined on Ψ as

$$|\theta_m\rangle \equiv \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \exp(in\theta_m)|n\rangle, \tag{7}$$

where

$$\theta_m \equiv \theta_0 + \frac{2\pi m}{s+1}, \qquad (m = 0, 1, \ldots, s). \tag{8}$$

The value of θ_0 is arbitrary and it defines a particular basis set of $(s+1)$ mutually orthogonal phase states. The Hermitian phase operator is then defined as

$$\hat{\phi}_\theta \equiv \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m|. \tag{9}$$

The phase states (7) are eigenstates of the phase operator (9) with the eigenvalues θ_m restricted to lie within a phase window between θ_0 and $\theta_0 + 2\pi$.

The expectation value of the phase operator (9) in a state $|\psi\rangle$ is given by

$$\langle\psi|\hat{\phi}_\theta|\psi\rangle = \sum_{m=0}^{s} \theta_m |\langle\theta_m|\psi\rangle|^2, \tag{10}$$

where $|\langle\theta_m|\psi\rangle|^2$ gives the probability of being in the phase state $|\theta_m\rangle$. We are free to choose the parameter s as large as it is needed, and for physical states, according to their definition by Pegg and Barnett [3, 4], it is always possible to choose s much larger than the contributing number states. In this case, we can simplify the calculation of the sum in Eq. (10) by replacing it by an integral in the limit as s tends to infinity. Since the density of phase states is $(s+1)/2\pi$, we can write Eq. (10) as

$$\langle\psi|\hat{\phi}_\theta|\psi\rangle = \int_{\theta_0}^{\theta_0+2\pi} \theta P^{(\mathrm{PB})}(\theta) d\theta, \tag{11}$$

where the continuum phase distribution $P^{(\mathrm{PB})}(\theta)$ is introduced by

$$P^{(\mathrm{PB})}(\theta) = \lim_{s\to\infty} \frac{s+1}{2\pi} |\langle\theta_m|\psi\rangle|^2, \tag{12}$$

where θ_m has been replaced by the continuous phase variable θ. Once the phase distribution function $P^{(\mathrm{PB})}(\theta)$ is known, all the quantum mechanical phase expectation values can be calculated with this function in a classical-like manner by integrating over θ. The choice of θ_0 defines the particular window of phase values.

In the case of the squeezed number states we have

$$\begin{aligned} \langle\theta_m|N\rangle_{(r,\varphi)} &= \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} b_n \exp[-i(n\theta_m - \varphi_n)] \\ &= \frac{e^{-iN\varphi}}{\sqrt{s+1}} \sum_{n=0}^{s} b_n \exp[-in(\theta_m - \varphi)]. \end{aligned} \tag{13}$$

We choose θ_0 as to

$$\theta_0 = \varphi - \frac{\pi s}{s+1}, \tag{14}$$

that is, we symmetrize the phase window with respect to the phase φ. On inserting (14) into (13), taking the modulus squared of (13) and taking the continuum limit, we arrive at the continuous phase probability distribution $P_{\mathrm{SN}}^{(\mathrm{PB})}(\theta)$ for the squeezed number states which has the form

$$P_{\mathrm{SN}}^{(\mathrm{PB})}(\theta) = \frac{1}{2\pi}\left\{1 + 2\sum_{n>k}^{\infty} b_n b_k \cos[(n-k)\theta]\right\}, \tag{15}$$

where b_n are given by Eq. (4), and the phase window is now from $-\pi$ to π. This form of the phase distribution is common for the partial phase states [3, 4]. However, due to the particular choice of b_n this phase distribution shows some interesting features that characterize the squeezed number states.

Other phase distributions $P^{(Q)}$ and $P^{(W)}$ can be obtained by integrating the $Q(\alpha)$ and $W(\alpha)$ functions, respectively, over the radial variable $|\alpha|$ [5, 6]. As we have previously shown [8, 9], all three phase distributions can be unified into one analytical formula which has the form

$$P^{(s)}(\theta) = \frac{1}{2\pi}\left\{1 + 2\sum_{n>k} b_n b_k \cos[(n-k)\theta] G^{(s)}(n,k)\right\}. \tag{16}$$

where the coefficients $G^{(s)}(n,k)$ distinguish between three distributions, and they are:

(i) for the Pegg-Barnett phase distribution

$$G^{(\mathrm{PB})}(n,k) \equiv 1, \tag{17}$$

(ii) for the distribution $P^{(Q)}(\theta) = \int_0^\infty Q(\alpha)|\alpha| d|\alpha|$ obtained by integrating the Q function over the radius [8]

$$G^{(Q)}(n,k) = \frac{\Gamma[(n+k)/2+1]}{\sqrt{n!k!}}, \tag{18}$$

(iii) for the distribution $P^{(W)}(\theta) = \int_0^\infty W(\alpha)|\alpha| d|\alpha|$ obtained by integrating the Wigner function over the radius [9]

$$G^{(W)}(n,k) = \sum_{m=0}^{p}(-1)^{p-m}2^{(|n-k|+2m)/2} \times\sqrt{\binom{p}{m}\binom{q}{p-m}}G^{(Q)}(m,|n-k|+m), \quad (19)$$

where $p = \min(n,k)$, $q = \max(n,k)$. All the coefficients $G^{(s)}(n,k)$ are symmetric, $G^{(s)}(n,k) = G^{(s)}(k,n)$, and $G^{(s)}(n,n) = 1$. Relation (16) is quite general and can be applied to any states with known amplitudes b_n. Here, we apply it to the squeezed number states. The phase distributions for the squeezed thermal states can be obtained by summing the phase distributions for the squeezed number states over the photon number N with the Bose-Einstein weighting factor $\bar{n}^N/(1+\bar{n})^{N+1}$.

The Wigner function for the squeezed number states has the following simple analytical form [19]

$$W_{SN}(\alpha) = \frac{2}{\pi}\exp\left[\frac{1}{2}(\alpha-\alpha^*)^2 e^{-2r} - \frac{1}{2}(\alpha+\alpha^*)^2 e^{2r}\right] \times(-1)^N \mathcal{L}_N[(\alpha+\alpha^*)^2 e^{2r} - (\alpha-\alpha^*)^2 e^{-2r}] \quad (20)$$

where $\mathcal{L}_N(x)$ is the Laguerre polynomial of an order of N. The phase distribution $P^{(W)}_{SN}(\theta)$ for the squeezed number states can be calculated in a straightforward way from the Wigner function (20) by integrating it over the radial variable which gives

$$P^{(W)}_{SN}(\theta) = \frac{1}{2\pi(\cosh 2r + \sinh 2r\cos 2\theta)}. \quad (21)$$

It is interesting to note that the phase distribution $P^{(W)}_{SN}(\theta)$ appears to be independent of the photon number N — it depends only on the squeeze parameter r. Asymptotically, in the limit of large squeezing ($r \to \infty$) the distribution (21) becomes a sum of two symmetrically placed delta functions

$$P^{(W)}_{SN}(\theta) = \frac{1}{2}\left[\delta(\theta-\pi/2) + \delta(\theta+\pi/2)\right]. \quad (22)$$

In Fig. 1, we show the plots of the three phase distributions calculated according to formula (16) with the coefficients (17) and (18) and formula (21) for the squeezed number states with $r = 0.5$ and $N = 0, 1, 2, 3$. We see that there is a significant difference in a behaviour of three phase distributions when photon number N is small. When photon

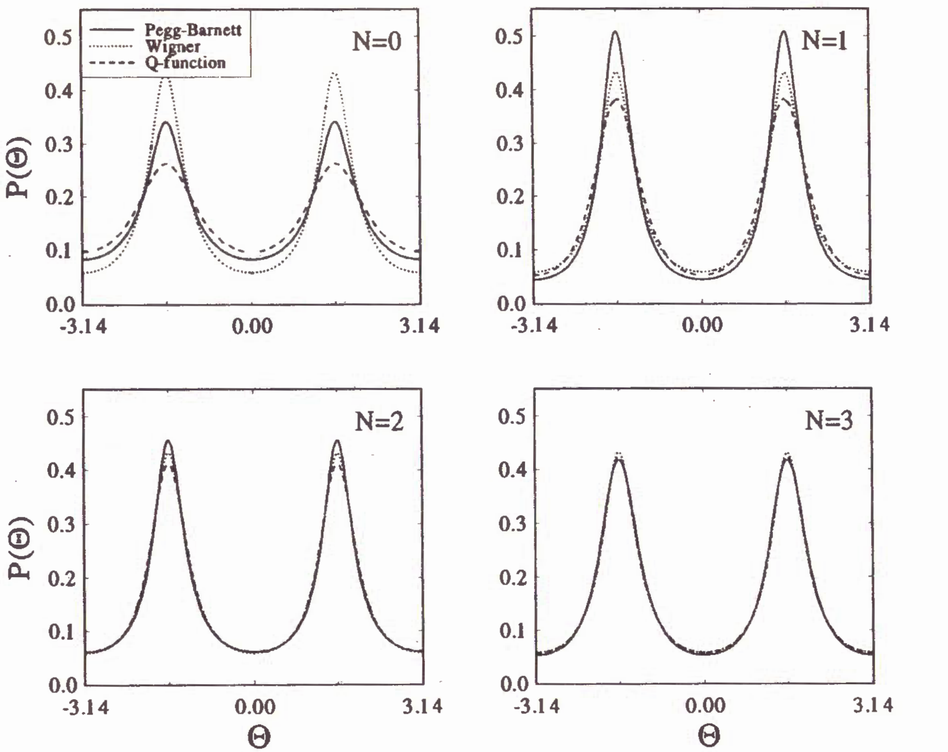

Figure 1: Plots of the phase distributions $P^{(PB)}(\theta)$ (full curves), $P^{(W)}(\theta)$ (dotted curves), and $P^{(Q)}(\theta)$ (dashed curves) for the squeezed number states with $r = 0.5$ and $N = 0, 1, 2, 3$.

number N increases the phase distributions $P^{(PB)}_{SN}(\theta)$ and $P^{(Q)}_{SN}(\theta)$ approach $P^{(W)}_{SN}(\theta)$, and for the values of $N \geq 4$ these three curves become already indistinguishable.

Let us now consider the phase distributions for the squeezed thermal states (6). As we have mentioned above phase distributions for the squeezed thermal states can be obtained from those for the squeezed number states by summing over the photon number N with the Bose-Einstein weighting factor $\bar{n}^N/(1+\bar{n})^{N+1}$. When the input average photon number is small, i. e. $\bar{n} \leq 1$, the only important contribution of the sum comes from the squeezed number state $N = 0$ (a squeezed vacuum). Thus phase distributions for the squeezed thermal state of $\bar{n} \leq 1$ are similar to those for

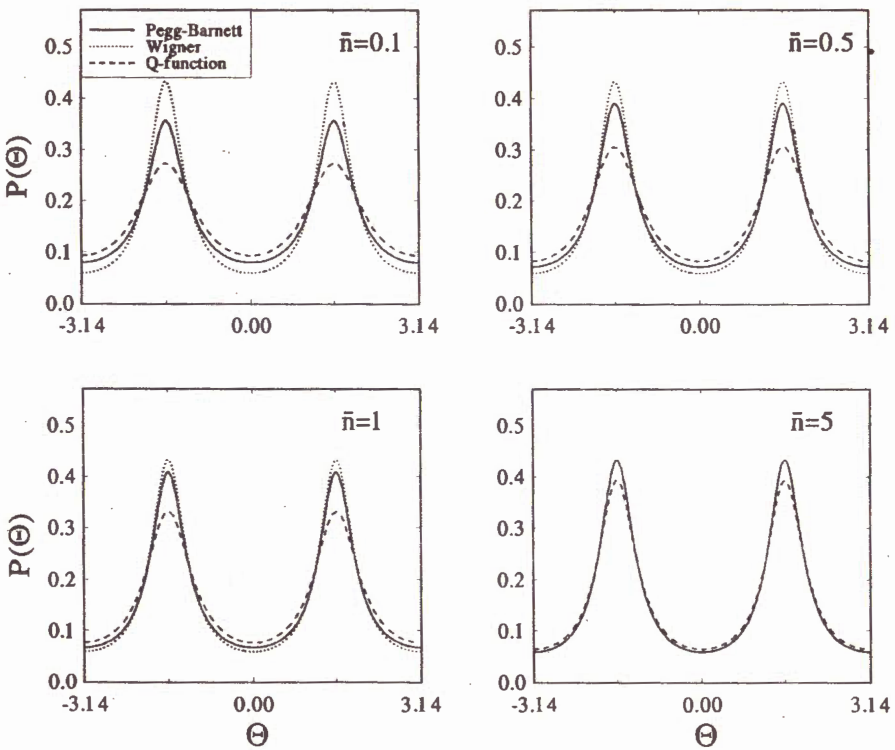

Figure 2: Same as in Fig. 1 but for the squeezed thermal states with $r = 0.5$ and $\bar{n} = 0.1, 0.5, 1, 5$

the squeezed vacuum. However, when $\bar{n}$ is large the contributions from appropriately large photon number states are more important. So, in the case $n > 1$ the difference between three distributions is small, just as for the squeezed number state with large N. Since $P_{\rm SN}^{(W)}(\theta)$ does not depend on the photon number N, the distribution $P_{\rm ST}^{(W)}(\theta)$ for the squeezed thermal state is equal to that of the squeezed number state, which is given by Eq. (21). The Q function has, in the case of squeezed thermal states, the following quite simple analytical form:

$$Q_{\rm ST}(\alpha) = \frac{1}{\pi\bar{n}\cosh r[(1+1/\bar{n})^2 - \tanh^2 r]^{1/2}} \times \exp\left\{-\frac{(1+1/\bar{n}+\tanh^2 r)|\alpha|^2 + (\alpha^2+\alpha^{*2})(1+1/(2\bar{n}))\tanh r}{\bar{n}[(1+1/\bar{n})^2 - \tanh^2 r]}\right\} \quad (23)$$

from which the phase distribution $P_{\rm ST}^{(Q)}(\theta)$ is easily calculated to be

$$P_{\rm ST}^{(Q)}(\theta) = \frac{\cosh r[(1+1/\bar{n})^2 - \tanh^2 r]^{1/2}}{2\pi[\cosh 2r + \cosh^2 r/\bar{n} + (1+1/(2\bar{n}))\sinh 2r\cos 2\theta]} \quad (24)$$

Asymptotically, in the limit of large $\bar{n}$ formula (24) goes over into formula (21) for $P_{\rm ST}^{(W)}(\theta)$. In Fig. 2, the behaviour of the above mentioned phase distributions $P_{\rm ST}^{(\rm PB)}(\theta)$, $P_{\rm ST}^{(W)}(\theta)$ and $P_{\rm ST}^{(Q)}(\theta)$ is shown for $r = 0.5$ and $\bar{n} = 0.1, 0.5, 1, 5$. From the figures we see that the distributions $P_{\rm ST}^{(\rm PB)}(\theta)$ and $P_{\rm ST}^{(Q)}(\theta)$ become narrower as $\bar{n}$ increases and for the large $\bar{n}$ the three curves completely coincide. This means that in the case of squeezed thermal states the distribution $P_{\rm ST}^{(W)}(\theta)$ is the narrowest limiting distribution that can be approached by increasing the average photon number of the thermal input field. It is worth to note the following interesting feature of the squeezed thermal states: the minimum value of the quantum phase fluctuations can be approached by increasing the thermal fluctuations.

Finally, we consider the phase distribution obtained by integration of the Glauber-Sudarshan $\mathcal{P}$ function over the radial variable. The Glauber-Sudarshan $\mathcal{P}$ function is well-defined for a classical state, but it is either negative or does not exist for states exhibiting nonclassical behaviour. As it has been shown by Kim et. al. [19], if the quadrature variances are larger than the minimum uncertainty limit, i. e. $(2\bar{n}+1)e^{-2r} > 1$, it is possible to describe the squeezed thermal state in terms of a well-behaved $\mathcal{P}$ function which is positive everywhere and can be written as

$$\mathcal{P}_{\rm ST}(\alpha) = \frac{1}{\pi[(\bar{n}e^{r}+\sinh r)(\bar{n}e^{-r}-\sinh r)]^{1/2}} \times \exp\left[\frac{(\alpha-\alpha^*)^2 e^{-r}}{4(\bar{n}e^{r}+\sinh r)} - \frac{(\alpha+\alpha^*)^2 e^{r}}{4(\bar{n}e^{-r}-\sinh r)}\right]. \quad (25)$$

The phase distribution $P_{\rm ST}^{(\mathcal{P})}(\theta)$ associated with the $\mathcal{P}$ function is then given by

$$P_{\rm ST}^{(\mathcal{P})}(\theta) = \int_0^\infty \mathcal{P}_{\rm ST}(\alpha)|\alpha|d|\alpha| = \frac{1}{2\pi}\frac{[(\bar{n}e^{r}+\sinh r)(\bar{n}e^{-r}-\sinh r)]^{1/2}}{(\bar{n}+1/2)(\cosh 2r+\sinh 2r\cos 2\theta)-1/2}. \quad (26)$$

In the large $\bar{n}$ limit the distribution $P_{\rm ST}^{(\mathcal{P})}(\theta)$ (26) goes over into $P_{\rm ST}^{(W)}(\theta)$ given by Eq. (21). In Fig. 3, we display $P_{\rm ST}^{(\mathcal{P})}(\theta)$ for $r = 0.5$ and $\bar{n} = 1, 5$. For comparison we

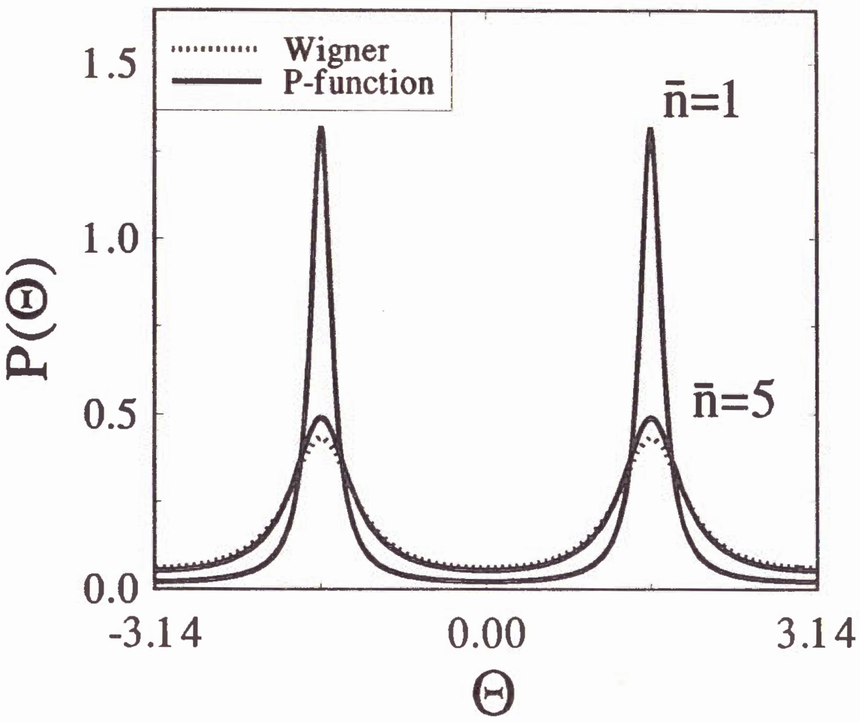

Figure 3: Plots of the phase distributions $P^{(\mathcal{P})}(\theta)$ (full curves) and $P^{(W)}(\theta)$ (dotted curve) for the squeezed thermal states with $r = 0.5$ and $\bar{n} = 1, 5$

have also plotted the phase distribution $P_{\rm ST}^{(W)}(\theta)$ calculated according to formula (21) with r=0.5. It is clearly seen that the larger $\bar{n}$ the broader $P_{\rm ST}^{(\mathcal{P})}(\theta)$.

3 Conclusions

In this paper we have studied the phase properties of the squeezed number states and the squeezed thermal states applying the Pegg-Barnett Hermitian phase formalism. We have compared the Pegg-Barnett phase distribution with the phase distributions $P^{(W)}(\theta)$, $P^{(Q)}(\theta)$ and $P^{(\mathcal{P})}(\theta)$ obtained by integrating the Wigner function, the Q function and the Glauber-Sudarshan $\mathcal{P}$ function, respectively, over the radial variable. We

have shown that the phase distribution associated with the Wigner function does not depend on the photon number and has the same form for both kinds of squeezed states considered in the paper while all other phase distributions approach $P^{(W)}(\theta)$ in the limit of highly excited states. We have also shown that in the case of squeezed thermal states a rise in the thermal fluctuations leads to a decrease in the quantum phase fluctuations, and the minimum value of the phase fluctuations, that can be approached in this way, is defined by the phase distribution $P^{(W)}(\theta)$ obtained by integrating the Wigner function.

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Чижов А.В., Ганцог Ц., Мурзахметов Б.К. E17-92-271
Фазовые распределения для сжатого
фоковского и сжатого хаотического состояний

Исследованы фазовые свойства сжатого фоковского и сжатого хаотического состояний. Получены точные аналитические формулы для фазовых распределений, основанные на различных подходах к описанию фазы, и проиллюстрированы графически. Показано, что фазовое распределение $P^{(W)}(\theta)$, связанное с функцией Вигнера, не зависит от числа фотонов и имеет одинаковую форму для обоих рассматриваемых состояний, тогда как другие распределения, такие как фазовое распределение Пегга-Барнетта и фазовые распределения, связанные с Q-функцией и P-функцией Глаубера-Сударшана, совпадают с $P^{(W)}(\theta)$ в пределе высоковозбужденных состояний.

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Phase Distributions of Squeezed Number
States and Squeezed Thermal States

Phase properties of squeezed number states and squeezed thermal states are studied. Exact analytical formulae for phase distributions based on different phase approaches are derived and illustrated graphically. It is shown that the phase distribution $P^{(W)}(\theta)$ associated with the Wigner function does not depend on the photon number and has the same form for both kinds of squeezed states under consideration while all other phase distributions, such as the Pegg-Barnett phase distribution and the phase distributions associated with the Q function and the Glauber-Sudarshan P function, approach $P^{(W)}(\theta)$ in the limit of highly excited states.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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