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QUANTUM PHASE FLUCTUATIONS IN THE JAYNES-CUMMINGS MODEL: EFFECTS OF CAVITY DAMPING

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1 Introduction

One of the fundamental models in quantum optics is the Jaynes-Cummings model (JCM) of a single two-level atom coupled to a single-mode cavity radiation field [1]. In spite of its apparent mathematical simplicity, the model provides us with a lot of interesting and unexpected results (for a review see [2]–[5]). In particular, in the JCM, the familiar Rabi oscillations are affected by the distribution of photon numbers, which causes a dephasing or "collapse" as the range of possible Rabi frequencies interfere [6]. The discrete nature of the photon number distributions leads to a further purely quantum mechanical effect when the dephased Rabi oscillations rephase or "revive" [7]. This phenomenon has been observed in experiments with Rydberg atoms in superconducting microwave cavities [8].

Recently, it has been shown that the collapses and revivals of Rabi oscillations in the JCM may be treated in terms of interferences of quasiprobabilities in a phase space [9, 10] or phase density probability distribution in a polar diagram [11] (see also [12]). The evolution of the field in the JCM towards a coherent superposition of macroscopically distinct quantum states, that is, a Schrödinger-cat-like-state, has been discussed at length in [13, 14]. The observation of such macroscopic-superposition states remains an outstanding problem in fundamental physics.

In [10], an example of the time evolution of the phase probability distribution in the presence of the cavity damping has been presented graphically. However, little attention has been paid to analyzing effects of photon leakage on phase properties of the field, and since the master equation is solved exactly, the expression for the phase probability distribution is complicated.

Our aim in this Letter is twofold. Firstly, we use the dressed atom approximation technique [15, 16] to derive explicit expressions for the phase variables which prove to have rather simple forms. This (dressed atom) approximation is made under the assumptions of very high-Q and zero temperature cavity. Both these conditions can now be realized in most of the Rydberg maser experiments [8, 15, 17]. Based on the obtained formulae, it is shown that in the presence of the cavity damping, the field goes faster over into a state with uniform phase distribution. Secondly, we compare phase distributions obtained by integrating the Wigner and Q functions over the radial variable with that using the Pegg-Barnett Hermitian phase formalism [18]–[20] and show that they have a quite similar behaviour. This is in agreement with the area-of-overlap-in-phase-space concept [21].

2 Solution for density matrix elements in the high-Q limit

The master equation describing the dynamical model of one two-level atom coupled to a single damped cavity mode at zero temperature reads ($\hbar = 1$), in the interaction picture,

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] - \kappa \left(a^+ a \rho - 2a\rho a^+ + \rho a^+ a\right), \tag{1}$$

where the interaction hamiltonian H in the rotating wave approximation is given by

$$H = g\left(R^+ a + R^- a^+\right). \tag{2}$$

Here $R^{\pm}$ are the transition operators for the atom; a^+ and a are the creation and annihilation operators of the field mode; g is the coupling constant and the exact resonance $\omega_{atom} = \omega_{field} \equiv \omega$ is assumed. The last term in Eq. (1) arises due to finite Q ($\equiv \omega/2\kappa$) of the cavity. Thus, 2κ represents the rate of loss of photons from the cavity.

To solve (1) we work in the dressed-state representation, i.e., the representation consisting of the complete set of eigenstates of H, which are known to be given by

$$\begin{aligned} H|0;g\rangle &= -\frac{\omega}{2}|0;g\rangle, \\ H|\psi_n^{\pm}\rangle &= \lambda_n^{\pm}|\psi_n^{\pm}\rangle, \\ |\psi_n^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|n;e\rangle \pm |n+1;g\rangle\right), \quad n = 0, 1, ..., \infty, \\ \lambda_n^{\pm} &= \pm g\sqrt{n+1}. \end{aligned} \tag{3}$$

Here $|n,e\rangle$ and $|n,g\rangle$ refer to states with n photons in the cavity field mode and the atom in the excited and ground states, respectively. For the annihilation operator a we have

$$\begin{aligned} a &= \sum_{n=0}^{\infty} \sqrt{n}\left(|n-1;e\rangle\langle n;e| + |n-1;g\rangle\langle n;g|\right) \\ &= \frac{|0;g\rangle\langle\psi_0^+| - |0;g\rangle\langle\psi_0^-|}{\sqrt{2}} + \frac{1}{2}\sum_{n=1}^{\infty}\Big[(\sqrt{n+1}+\sqrt{n})(|\psi_{n-1}^+\rangle\langle\psi_n^+| + |\psi_{n-1}^-\rangle\langle\psi_n^-|) \\ &\qquad -(\sqrt{n+1}-\sqrt{n})(|\psi_{n-1}^+\rangle\langle\psi_n^-| + |\psi_{n-1}^-\rangle\langle\psi_n^+|)\Big]. \end{aligned} \tag{4}$$

Similarly, the photon number operator can be written as

$$a^+ a = \frac{1}{2}\sum_{n=0}^{\infty}\Big[(2n+1)\left(|\psi_n^+\rangle\langle\psi_n^+| + |\psi_n^-\rangle\langle\psi_n^-|\right) - \left(|\psi_n^+\rangle\langle\psi_n^-| + |\psi_n^-\rangle\langle\psi_n^+|\right)\Big]. \tag{5}$$

By defining

$$\mathcal{W}(t) = \exp(iHt)\rho(t)\exp(-iHt), \tag{6}$$

and substituting (4) and (5) into (1), we obtain an equation for $\mathcal{W}(t)$ which contains time-dependent and time-independent terms. It can be shown that the contribution of the oscillating terms is of the order κ^2/g^2 and can be neglected within the dressed-atom (secular) approximation [15, 16]. Then the equation for $\mathcal{W}(t)$ is found to be

$$\begin{aligned} \dot{\mathcal{W}}(t) = \kappa\Big[&|0;g\rangle\langle\psi_0^+|\mathcal{W}(t)|\psi_0^+\rangle\langle 0;g| + |0;g\rangle\langle\psi_0^-|\mathcal{W}(t)|\psi_0^-\rangle\langle 0;g|\Big] \\ +\kappa/2\sum_{n=1}^{\infty}\Big\{&(\sqrt{n+1}+\sqrt{n})^2\Big[|\psi_{n-1}^+\rangle\langle\psi_n^+|\mathcal{W}(t)|\psi_n^+\rangle\langle\psi_{n-1}^+| + |\psi_{n-1}^-\rangle\langle\psi_n^-|\mathcal{W}(t)|\psi_n^-\rangle\langle\psi_{n-1}^-|\Big] \\ &+(\sqrt{n+1}-\sqrt{n})^2\Big[|\psi_{n-1}^+\rangle\langle\psi_n^-|\mathcal{W}(t)|\psi_n^-\rangle\langle\psi_{n-1}^+| + |\psi_{n-1}^-\rangle\langle\psi_n^+|\mathcal{W}(t)|\psi_n^+\rangle\langle\psi_{n-1}^-|\Big]\Big\} \\ -\kappa/2\sum_{n=0}^{\infty}&(2n+1)\Big[(|\psi_n^+\rangle\langle\psi_n^+| + |\psi_n^-\rangle\langle\psi_n^-|)\mathcal{W}(t) + \text{H.c.}\Big]. \end{aligned} \tag{7}$$

From Eq. (7), it easily follows that [16]

$$\begin{aligned} \langle\psi_n^{\epsilon}|\mathcal{W}(t)|\psi_k^{\epsilon}\rangle &= \exp[-\kappa t(n+k+1)]\langle\psi_n^{\epsilon}|\mathcal{W}(0)|\psi_k^{\epsilon}\rangle, \qquad (n \neq k,\ \epsilon = +,-), \\ \langle\psi_n^{+}|\mathcal{W}(t)|\psi_k^{-}\rangle &= \exp[-\kappa t(n+k+1)]\langle\psi_n^{+}|\mathcal{W}(0)|\psi_k^{-}\rangle, \qquad (\forall n, k), \\ \langle 0;g|\mathcal{W}(t)|\psi_n^{\pm}\rangle &= \exp[-\kappa t(n+1/2)]\langle 0;g|\mathcal{W}(0)|\psi_n^{\pm}\rangle. \end{aligned} \tag{8}$$

Equation (8) along with its Hermitian conjugates determine all the off-diagonal elements of $\mathcal{W}(t)$. In calculations of the phase variables, the diagonal elements of $\mathcal{W}(t)$ are not needed in the explicit form; so we will not write down them here.

3 Properties of the field phase in a damped cavity

The Pegg-Barnett formalism is based on introducing a finite $(s+1)$-dimensional space Ψ spanned by the number states $|0\rangle$, $|1\rangle, ..., |s\rangle$. A complete orthonormal basis of $(s+1)$ phase states is defined on this finite space as

$$|\theta_m\rangle \equiv \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \exp(in\theta_m)|n\rangle, \tag{9}$$

where

$$\theta_m \equiv \theta_0 + \frac{2\pi m}{s+1}, \qquad (m = 0, 1, ..., s). \tag{10}$$

The value of θ_0 is arbitrary and defines a particular basis set of $(s+1)$ mutually orthogonal phase states. The Hermitian phase operator is defined as

$$\hat{\phi}_\theta \equiv \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m|. \tag{11}$$

The phase states (9) are eigenstates of the phase operator (11) with the eigenvalues θ_m restricted to lie within a phase window between θ_0 and $\theta_0+2\pi$. Note that the Hermitian phase operator in a finite dimensional space has also been constructed in earlier works of Garrison and Wong [22], and Popov and Yarunin [23]. A complete description of the single-mode field involves an infinite set of number states and here this corresponds to the limit as $s \to \infty$. An essential feature of the Pegg-Barnett formalism is that the value of s is allowed to tend to infinity only after all necessary expectation values have been calculated in Ψ.

Now let us assume that the atom is initially in the excited state

$$\rho_a(0) = |e\rangle\langle e|, \tag{12}$$

and the field is in the coherent state

$$\rho_f(0) = \sum_{n,k} b_n b_k \exp[i(n-k)\varphi]|n\rangle\langle k| \tag{13}$$

with

$$b_n = \exp\left(-\frac{\bar{n}}{2}\right)\left(\frac{\bar{n}^n}{n!}\right)^{1/2}. \tag{14}$$

For $\rho_a(0)$ and $\rho_f(0)$ as are given above, the expression for $\mathcal{W}(0)$ in the dressed-state representation becomes

$$\begin{aligned} \mathcal{W}(0) &= \rho(0) = \rho_a(0) \otimes \rho_f(0) \\ &= \frac{1}{2} \sum_{n,k=0}^{\infty} b_n b_k \exp[i(n-k)\varphi]\Big(|\psi_n^+\rangle\langle\psi_k^+| + |\psi_n^-\rangle\langle\psi_k^-| + |\psi_n^+\rangle\langle\psi_k^-| + |\psi_n^-\rangle\langle\psi_k^+|\Big). \end{aligned} \tag{15}$$

Since we are interested in the properties of the light field, we have to perform a trace with respect to the atom. Then, the probability of finding the field in the phase state $|\theta_m\rangle$ is defined as

$$\begin{aligned} \mathrm{Tr}_a[\langle\theta_m|\rho(t)|\theta_m\rangle] &= \langle\theta_m; e|\rho(t)|\theta_m; e\rangle + \langle\theta_m; g|\rho(t)|\theta_m; g\rangle \\ &= \frac{1}{s+1} \sum_{n,k=0}^{s} \exp[-i(n-k)\theta_m]\Big[\langle n; e|\rho(t)|n; e\rangle + \langle n; g|\rho(t)|n; g\rangle\Big]. \end{aligned} \tag{16}$$

By writing $|n,e\rangle$ and $|n,g\rangle$, in terms of $|\psi_n^\pm\rangle$, one obtains

$$\begin{aligned} \frac{1}{s+1} \sum_{n,k=0}^{s} \exp[-i(n-k)\theta_m]\langle n; e|\rho(t)|n; e\rangle &= \frac{1}{2(s+1)} \sum_{n,k=0}^{s} \exp[-i(n-k)\theta_m] \\ &\times\Big\{\exp[-i(\lambda_n - \lambda_k)t]\langle\psi_n^+|\mathcal{W}(t)|\psi_k^+\rangle + \exp[i(\lambda_n - \lambda_k)t]\langle\psi_n^-|\mathcal{W}(t)|\psi_k^-\rangle \\ &+ \exp[-i(\lambda_n + \lambda_k)t]\langle\psi_n^+|\mathcal{W}(t)|\psi_k^-\rangle + \exp[i(\lambda_n + \lambda_k)t]\langle\psi_n^-|\mathcal{W}(t)|\psi_k^+\rangle\Big\}, \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{1}{s+1} \sum_{n,k=0}^{s} \exp[-i(n-k)\theta_m]\langle n; g|\rho(t)|n; g\rangle &= \frac{1}{2(s+1)} \sum_{n,k=0}^{s-1} \exp[-i(n-k)\theta_m] \\ &\times\Big\{\exp[-i(\lambda_n - \lambda_k)t]\langle\psi_n^+|\mathcal{W}(t)|\psi_k^+\rangle + \exp[i(\lambda_n - \lambda_k)t]\langle\psi_n^-|\mathcal{W}(t)|\psi_k^-\rangle \end{aligned}$$

$$+\exp[-i(\lambda_n+\lambda_k)t]\langle\psi_n^+|\mathcal{W}(t)|\psi_k^-\rangle+\exp[i(\lambda_n+\lambda_k)t]\langle\psi_n^-|\mathcal{W}(t)|\psi_k^+\rangle\Big\}$$

$$+\frac{1}{\sqrt{2}(s+1)}\sum_{n=0}^{s-1}\Big\{\exp[i(n+1)\theta_m]\Big[\exp(i\lambda_n t)\langle 0;g|\mathcal{W}(t)|\psi_n^+\rangle$$

$$+\exp(-i\lambda_n t)\langle 0;g|\mathcal{W}(t)|\psi_n^-\rangle\Big]+\text{c.c.}\Big\}+\frac{1}{s+1}\langle 0;g|\mathcal{W}(t)|0;g\rangle. \quad (18)$$

For very large s, we can ignore the difference between s and $(s-1)$ and write

$$\begin{aligned}\mathrm{Tr}_a\langle\theta_m|\rho(t)|\theta_m\rangle &= \frac{1}{s+1}\Big\{\langle 0;g|\mathcal{W}(t)|0;g\rangle+\sum_{n=0}^{s}\Big[\langle\psi_n^+|\mathcal{W}(t)|\psi_n^+\rangle+\langle\psi_n^-|\mathcal{W}(t)|\psi_n^-\rangle\Big]\Big\}\\ &+\frac{1}{\sqrt{2}(s+1)}\sum_{n=0}^{s-1}\Big\{\exp[i(n+1)\theta_m]\Big[\exp(i\lambda_n t)\langle 0;g|\mathcal{W}(t)|\psi_n^+\rangle\\ &\qquad+\exp(-i\lambda_n t)\langle 0;g|\mathcal{W}(t)|\psi_n^-\rangle\Big]+\text{c.c.}\Big\}\\ &+\frac{1}{s+1}\sum_{\substack{n,k=0\\(n\neq k)}}^{s}\exp[-i(n-k)\theta_m]\Big[\exp[-i(\lambda_n-\lambda_k)t]\langle\psi_n^+|\mathcal{W}(t)|\psi_k^+\rangle\\ &\qquad+\exp[i(\lambda_n-\lambda_k)t]\langle\psi_n^-|\mathcal{W}(t)|\psi_k^-\rangle\Big]. \end{aligned} \quad (19)$$

In the case of the coherent state (13), it is convenient to choose the reference phase θ_0 as follows [19]

$$\theta_0=\varphi-\frac{\pi s}{s+1}, \quad (20)$$

that is, we symmetrize the phase window with respect to the initial mean phase φ. The density of phase states is $(s+1)/2\pi$; so in the continuum limit as s tends to infinity, after inserting (8), (15), and (20) into (19), and taking into account the fact that

$$\lim_{s\to\infty}\Big\{\langle 0;g|\mathcal{W}(t)|0;g\rangle+\sum_{n=0}^{s}\Big[\langle\psi_n^+|\mathcal{W}(t)|\psi_n^+\rangle+\langle\psi_n^-|\mathcal{W}(t)|\psi_n^-\rangle\Big]\Big\}=1, \quad (21)$$

we arrive at the continuous phase probability distribution

$$P(\theta,t)=\frac{1}{2\pi}\Big\{1+2\sum_{n>k}^{\infty}b_n b_k\cos[(n-k)\theta]$$

$$\times\cos[(\sqrt{n+1}-\sqrt{k+1})gt]\exp[-\kappa t(n+k+1)]\Big\}, \quad (22)$$

where θ_m has been replaced by the continuous phase variable θ, and the phase window is now from $-\pi$ to π. If $\kappa=0$, the photon leakage is absent and formula (22) reduces to that in the situation of a lossless cavity [11].

The expectation value of the phase operator (11) and its variance are given by

$$\langle\hat{\Phi}_\theta\rangle = \sum_{m=0}^{s}\theta_m\mathrm{Tr}_a\left[\langle\theta_m|\rho(t)|\theta_m\rangle\right], \quad (23)$$

$$\langle(\Delta\hat{\Phi}_\theta)^2\rangle = \sum_{m=0}^{s}\left(\theta_m-\langle\hat{\Phi}_\theta\rangle\right)^2\mathrm{Tr}_a\left[\langle\theta_m|\rho(t)|\theta_m\rangle\right]. \quad (24)$$

After replacing the summations in (23) and (24) by relevant integrals in the limit $s\to\infty$, and using the phase distribution function (22), one finds

$$\langle\hat{\Phi}_\theta\rangle = \varphi, \quad (25)$$

$$\langle(\Delta\hat{\Phi}_\theta)^2\rangle = \frac{\pi^2}{3}+4\sum_{n>k}b_n b_k\frac{(-1)^{n-k}}{(n-k)^2}\cos[(\sqrt{n+1}-\sqrt{k+1})gt]\exp[-\kappa t(n+k+1)]. \quad (26)$$

Formula (25) shows that, differently from the nonzero detuning and the atomic coherence [11], a finite Q does not lead to time changes of the expectation value of the phase operator. The phase distribution evolves, but always remains symmetric with respect to the mean value φ. However, the cavity losses lead to increasing in the phase uncertainty, as can be seen in figures 1 and 2 where we have illustrated the time evolution of $P(\theta,t)$ and $\langle(\Delta\hat{\Phi}_\theta)^2\rangle$ for various values of κ/g. Now besides the noncommensurability between the Rabi frequencies, there appears a second factor – the leakage of photons from the cavity which causes the field phase to be randomized. In the presence of the cavity damping, for long enough times, the field state will eventually become vacuum with the uniform phase distribution $P(\theta,t\to\infty)=1/(2\pi)$ and the variance $\langle(\Delta\hat{\Phi}_\theta)^2\rangle=\pi^2/3$ [20]. The larger the damping rate is, the quicker this regime is established. Without damping no stationary solution can be reached and then for long times, the phase probability distribution, though exhibits apparently chaotic behaviour, still shows some complex structure (see Fig. 1d, solid line) while the phase variance after an interval of complete fading again shows oscillations in a random manner around $\pi^2/3$ (see Fig. 2, solid line). This renewal of oscillations also occurs if the cavity-damping rate is small.

4 Phase distributions

Recently, investigating the Q function and Wigner function for the JCM, Eiselt and Risken [9, 10] have shown that they reveal the collapses and revivals of the Rabi oscillations [7] in a very spectacular way. Starting with a light field in a coherent state and the atom in its upper state, the initial shifted Gaussian quasiprobability distribution splits into peaks counterrotating in the complex α-plane. When the peaks are well separated, the atomic inversion shows no oscillations; when they collide, oscillations of the inversion occur. In [11], it has been found that the same feature holds for the time behaviour of the phase probability distribution and in addition to that, the revivals of the Rabi oscillations can be understood in terms of maxima and minima of the phase variance. This is clearly visible in Figs. 1 and 2, where for convenience, the time has been scaled by the factor equal to the revival period $T_R = 2\pi\sqrt{\bar{n}}/g$. Behind the similarities between the phase distribution and the quasiprobabilities when describing the collapse and revival effect, there exists a more fundamental relation. To show that, we calculate the "classical" phase distribution defined by integrating the Q function over the radial variable. This distribution was referred to as "classical" by Braunstein and Caves [24] since the Q function applies to simultaneous measurement of two noncommuting observables, a process that inevitably introduces an additional noise.

The Q function for the JCM is defined as

$$Q(\alpha, t) = \mathrm{Tr}_a\left[\langle\alpha|\rho(t)|\alpha\rangle\right], \tag{27}$$

where $|\alpha\rangle$ denotes a coherent field state

$$|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{(|\alpha|e^{i\theta})^n}{\sqrt{n!}}|n\rangle. \tag{28}$$

Using formula (6), the solution (8), the initial condition (15), and after performing the integration of $Q(\alpha, t)$ over $|\alpha|$, one gets

$$\begin{aligned} P_Q(\theta, t) &\equiv \int_0^\infty Q(\alpha, t)|\alpha|\frac{d|\alpha|}{\pi} \\ &= \frac{1}{2\pi}\Bigg(1 + 2\sum_{n>k}^{\infty} b_n b_k \cos[(n-k)\theta]\exp\left[-\kappa t\,(n+k+1)\right] \\ &\quad\times\Big\{\frac{1}{2}[F(n,k) + F(n+1,k+1)]\cos[(\sqrt{n+1} - \sqrt{k+1})gt] \\ &\qquad + \frac{1}{2}[F(n,k) - F(n+1,k+1)]\cos[(\sqrt{n+1} + \sqrt{k+1})gt]\Big\}\Bigg), \end{aligned} \tag{29}$$

where

$$F(n,k) = \frac{\Gamma\left(\frac{n+k}{2} + 1\right)}{\sqrt{n!k!}}, \tag{30}$$

and the phase $(\theta - \varphi)$ has been replaced by θ, which is equivalent to the choice of the reference phase (20). Since $Q(\alpha, t)$ is positive definite, $P_Q(\theta, t)$ is also positive definite, and normalized, and it can be treated as a phase distribution. It is evident that these are the extra factors $F(n,k)$ that distinguish the "classical" phase distribution (29) from the Hermitian phase distribution (22). These factors result from the integrating of $Q(\alpha, t)$ over $|\alpha|$, and therefore, are independent of the concrete form of the field state under consideration. In particular, they appear in expressions for the "classical" phase distribution of the anharmonic oscillator states [25], displaced number states [26], and fields generated in multi-photon down converter [27]. One can easily check that the elements $F(n,k)$ are symmetrical $F(n,k) = F(k,n)$, their diagonal elements are unity $F(n,n) = 1$, and farther away we go from the diagonal, the smaller are $F(n,k)$ [27].

The concept of interference in the phase space introduced by Schleich and Wheeler [21] when applied to describe phase properties of the field indicates still another possibility to get the phase distribution [28] by integrating the Wigner distribution over the radial variable. In our case, the Wigner distribution can be defined as [29]

$$W(\alpha,t) = 2\sum_{n=0}^{\infty}(-1)^n \mathrm{Tr}_a[\langle n|\rho(t)D(2\alpha)|n\rangle], \tag{31}$$

where $D(2\alpha)$ is the displacement operator

$$D(2\alpha) = \exp\left(2\alpha a^+ - 2\alpha^* a\right). \tag{32}$$

On inserting (6), (8) into equation (31) and using again the initial condition (15), the integrating of $W(\alpha,t)$ over $|\alpha|$ gives us

$$\begin{aligned} P_W(\theta,t) &\equiv \int_0^\infty W(\alpha,t)|\alpha|\frac{\mathrm{d}|\alpha|}{\pi} \\ &= \frac{1}{2\pi}\Bigg(1 + 2\sum_{n>k}^{\infty} b_n b_k \cos[(n-k)\theta]\exp[-\kappa t(n+k+1)] \\ &\quad \times \Big\{\frac{1}{2}[G(n,k)+G(n+1,k+1)]\cos[(\sqrt{n+1}-\sqrt{k+1})gt] \\ &\quad + \frac{1}{2}[G(n,k)-G(n+1,k+1)]\cos[(\sqrt{n+1}+\sqrt{k+1})gt]\Big\}\Bigg), \end{aligned} \tag{33}$$

where

$$G(n,k) = \sum_{m=0}^{p}(-1)^{p+m}2^{(|n-k|+2m)/2}\sqrt{\binom{p}{m}\binom{q}{p-m}}\,F(m,|n-k|+m), \tag{34}$$

with

$$p = \min(n,k), \qquad q = \max(n,k), \tag{35}$$

and $F(m,|n-k|+m)$ given by Eq. (30). The factors $G(n,k)$ are symmetrical $G(n,k) = G(k,n)$, and $G(n,n) = 1$.

In Fig. 3 we show the plots of the phase distributions calculated according to three formulae (22), (29), and (33), for $\kappa = 0$ that means an ideal lossless cavity. It is clearly seen that they carry the same phase information though $P_Q(\theta,t)$ is broader than $P_W(\theta,t)$ and the Pegg-Barnett phase distribution. This broadening may be explained

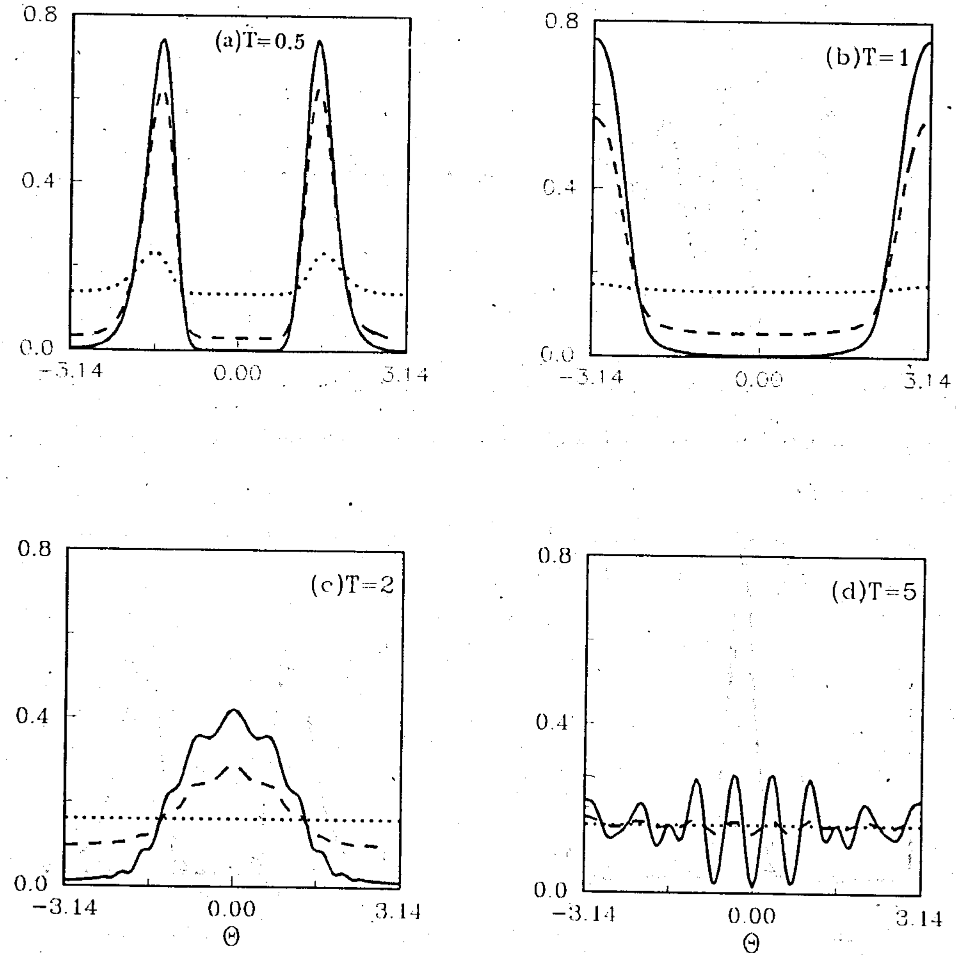

Figure 1

Phase probability distribution $P(\theta,t)$ plotted against θ for various values of $T = gt/(2\pi\sqrt{\bar{n}})$, and for $\kappa/g = 0$ (solid line), $\kappa/g = 0.001$ (dashed line), and $\kappa/g = 0.01$ (dotted line). The average photon number $\bar{n} = 10$.

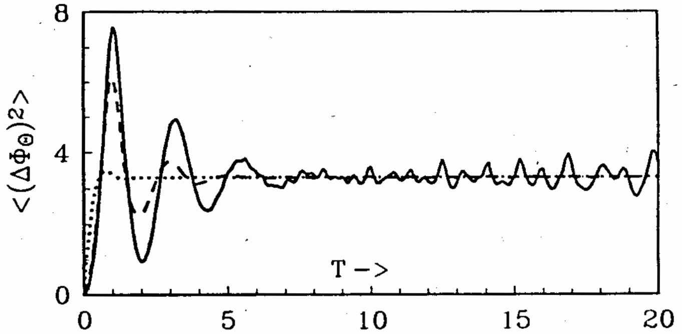

Figure 2
Plot of the variance of the phase operator as a function of T, for $\kappa/g = 0$ (solid line), $\kappa/g = 0.001$ (dashed line), and $\kappa/g = 0.01$ (dotted line). The average photon number $\bar{n} = 10$.

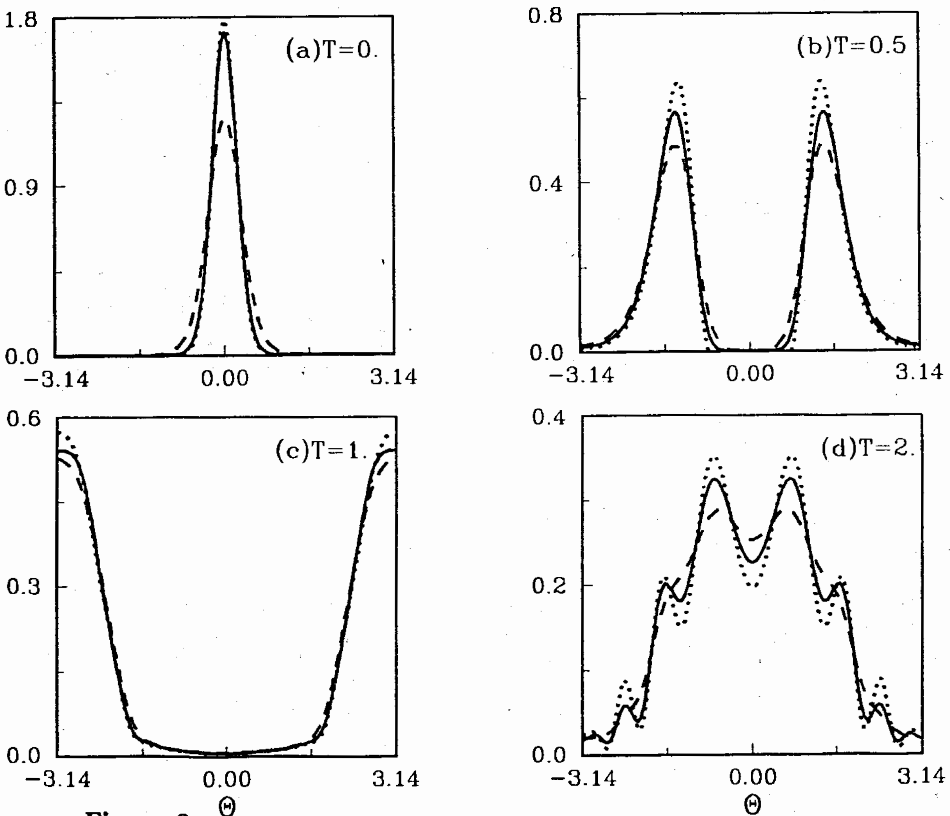

Figure 3
Plots of the phase distributions $P(\theta,t)$ (solid line), $P_Q(\theta,t)$ (dashed line), and $P_W(\theta,t)$ (dotted line) for $\kappa = 0$ and $\bar{n} = 5$

as follows. In Eq. (29), if one neglects the terms proportional to $(1/2)[F(n,k) - F(n+1,k+1)]$, which are small as compared with those proportional to $(1/2)[F(n,k) + F(n+1,k+1)]$, one can treat $P_Q(\theta,t)$ as resulting from averaging the Pegg-Barnett phase distribution with the "probabilities" $(1/2)[F(n,k) + F(n+1,k+1)]$. Since the nondiagonal elements $F(n,k)$, $(n \neq k)$ are smaller than unity, $(1/2)[F(n,k) + F(n+1,k+1)]$ are also smaller than unity and this diminishing of the nondiagonal elements that define the phase structure leads to a phase distribution broader than $P(\theta,t)$. In the case of displaced number states [26], a similar averaging procedure even causes some losses of phase information.

Since the coefficients $G(n,k)$ take on the values that are smaller or larger than unity, their effect on the phase distribution is not as simple as in the case of $P_Q(\theta,t)$. From Fig. 3 we see that the phase peaks of $P_W(\theta,t)$ are slightly narrower than those of $P(\theta,t)$. This similarity is in agreement with the concept of area-of-overlap in phase space, where a quantum state is represented by the Wigner function, a phase state is represented by a diverging beam and the phase probability is associated with the weighted area of overlap between them [28]. The area-of-overlap principle gives a simple visualization and a deeper insight into the phase properties of quantum state. However $P_W(\theta,t)$, in general, can take on negative values while there are no such problems with the Pegg-Barnett phase distribution.

5 Conclusion

We have discussed the effects of cavity damping on properties of the field phase in the JCM. By using the dressed atom approximation, analytical formulae for the phase distribution, the expectation value of the phase operator and its variance have been obtained in rather simple forms. It has been shown that due to the leakage of photons from the cavity, the field phase undergoes a quicker randomization than in the case of an ideal cavity. We have compared the Hermitian phase distribution with those obtained by integrating the Q function and Wigner function over the amplitude and shown that they carry basically the same phase information. This similarity is in agreement with the area-of-overlap-in-phase-space principle.

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Хо Чунг Зунг, Шумовский А.С. E17-92-201
Квантовые флуктуации фазы в модели Джейнса — Каммингса:
влияние конечной добротности резонатора

Исследуются фазовые свойства когерентного поля, взаимодействующего с двухуровневым атомом в резонаторе с очень высокой конечной добротностью. Показывается, что из-за утечки фотонов из резонатора фаза поля хаотизируется быстрее, чем в случае идеального резонатора. Сравнивается распределение эрмитовой фазы с распределениями, связанными с Q-функцией и функцией Вигнера. Подобие между ними имеет ясную интерпретацию через принцип суперпозиции в фазовом пространстве.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Ho Trung Dung, Shumovsky A.S. E17-92-201
Quantum Phase Fluctuations in the Jaynes — Cummings Model:
Effects of Cavity Damping

Phase properties of a coherent field interacting with a two-level atom in a cavity with very high but finite Q are studied. It is shown that due to the cavity damping the field phase is randomized more quickly than in the ideal-lossless-cavity case. The Hermitian phase distribution and the phase distributions associated with the Q function and the Wigner function are compared. The similarities between them have clear interpretation in terms of the area-of-overlap in phase space.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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