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# CONDITIONS FOR ARISING THE SUB-POISSON EQUILIBRIUM DISTRIBUTION OF PHONONS IN POLARITON-LIKE SYSTEMS

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## 1. Introduction.

Recently, in quantum optics new collective states of the electromagnetic field have been predicted and experimentally detected, which essentially differ from the traditional (chaotic and coherent) states in quantum fluctuation and correlation properties [1-3]. In particular, states have been found with a sub-Poisson distribution of the number of field quanta and a "sub-Poisson laser" has been constructed [4].

Recall that the coherent light possesses the Poisson distribution for the number of photons whereas the chaotic light has the Gaussian distribution [5,6]. It is conventional to call the sub-Poisson distributions those distributions which are more narrow than that for the coherent light. For example, for the Fock state of the electromagnetic field the distribution of the number of quanta is a δ-function.

Such "new" states of the electromagnetic field are nonequilibrium, and their generation is due to nonlinear interaction of light with matter in the processes of generation or scattering.

It is undoubtedly interesting to study the problem of existence of thermodynamically equilibrium states of Bose fields with nonstandard statistical properties. The main reason is that the mechanism of interaction of bosons of a different physical nature in condensed matter possesses the same nonlinearity as the processes used for generation of sub-Poisson states in optics.

As has recently been shown, for the simplest model of a degenerate parametric process in the state of thermodynamic equilibrium below a certain temperature, there may occur squeezing of quantum fluctuations of the Bose-field amplitudes [7]; and the distribution of the number of quanta is sub-Poisson.

In this paper, we will consider some simple models used in the solid-state physics to demonstrate possible changes in the statistical properties of the Bose field with lowering the temperature and to establish the condition under which the sub-Poisson distribution develops.

## 2. Polariton-like systems.

The theory of polariton considers model problems with Hamiltonians that are bilinear forms of Bose operators of two types, photons and phonons [9]:

$$H = \sum_{k,l} \left\{ A_{kl} a_k^+ a_l^+ + A_{kl}^* a_k a_l + B_{kl} a_k^+ a_l \right\},$$

$$B_{kl} = B_{lk}^*, \; A_{kl} = A_{lk}. \qquad (1)$$

This form is rather general and can be used for the description of a wide class of phenomena in solid states, including the exiton-phonon interaction in molecular crystals, scattering of light on phonons, and some problems of the theory of magnetism.

By the known linear transformation (see, e.g. [8])

$$a_k = \sum_n \left( u_{kn} \alpha_n + v_{kn}^* \alpha_n^+ \right), \qquad [\alpha_m, \alpha_n^*] = \delta_{mn}.$$

a Hamiltonian of that type is reduced to the diagonal form

$$H = E_o + \sum_n E_n \alpha_n^+ \alpha_n,$$

and than various thermodynamic characteristics of a system of free quasiparticles described by operators $\alpha_n^+$, $\alpha_n$ with the spectrum $E_n$ are calculated. These quasiparticles are of a complicated structure; polaritons, for instance, consist of an optical phonon interacting with photons [9]. In this case, experimental methods allow the study of one of the components of the quasiparticle, for instance, with the use of the Raman scattering [10] that allows us to determine the spectral characteristics of phonons. In what follows, we will be interested in the quantum statistical properties of phonon subsystem, specifically, in the character of distribution of the number of phonons in polaritons in an equilibrium state with a temperature T. In this case we should first calculate variance of the number of quanta in different modes:



$$V_k = \langle (a_k^+ a_k)^2 \rangle - \langle a_k^+ a_k \rangle^2,$$

where averaging runs over the equilibrium state of system (1) with a certain temperature T:

$$\langle \ldots \rangle = \mathrm{Tr} \, \langle \ldots \rho \rangle, \qquad \rho = \prod_n \rho_n(m) \, |m\rangle_{nn} \langle m|,$$

$$\rho_n(m) = \frac{\langle \alpha_n^+ \alpha_n \rangle^m}{\left(1 + \langle \alpha_n^+ \alpha_n \rangle\right)^{1+m}}, \qquad \langle \alpha_n^+ \alpha_n \rangle = \left[\exp(E_n / k_b T) - 1\right]^{-1}.$$

The condition for the sub-Poisson distribution of the number of quanta in the k-th mode looks as follows:

$$V_k < \langle a_k^+ a_k \rangle \qquad (2)$$

This inequality connects the temperature of the system with microscopic characteristics entering into the Hamiltonian (1) (interaction parameters and characteristic frequencies $A_{kl}$, $B_{kl}$; $E_n = E_n(A, B)$). From a physical point of view, it is natural to expect that at sufficiently high temperatures the distribution of the number of quanta should be chaotic (Gaussian); consequently, for the condition (2) to be valid at a fixed **A** and **B** in (1) the temperature should be lowering. In this case the equality

$$V_k = \langle a_k^+ a_k \rangle \qquad (3)$$

may be treated as an equation for determining the threshold temperature

$$T^{th} = T^{th}(A, B),$$

below which there occur true quantum fluctuations not distorted by thermal noice.

## 3. A two-mode system.

For simplicity, we will consider one mode of the photon field interacting with a quasiresonance mode of optical phonons. The Hamiltonian of this system is of the form

$$H = \omega_a a^+ a + \omega_b b^+ b + \kappa (a^+ b^+ + ba). \tag{4}$$

To diagonalize the Hamiltonian (4), we will apply the canonical transformation:

$$a = u\alpha + v\beta^+, \quad u^2 - v^2 = 1,$$
$$b = \mu\alpha^+ + \nu\beta, \quad \nu^2 - \mu^2 = 1, \tag{5}$$

where the operators $\alpha$ и $\beta$ obey the commutation relations:

$$[\alpha, \alpha^+] = [\beta, \beta^+] = 1, \quad [\alpha^\#, \beta^\#] = 0,$$

and the transformation parameters are given by:

$$u = -\nu = -\left[\frac{1+\sqrt{1-k^2}}{2\sqrt{1-k^2}}\right]^{\frac{1}{2}}, \quad v = -\mu = \left[\frac{1-\sqrt{1-k^2}}{2\sqrt{1-k^2}}\right]^{\frac{1}{2}}, \tag{6}$$

where $k = \dfrac{2\kappa}{\omega_a + \omega_b}$. (7)

and $k < 1$ due to the stability condition for Hamiltonian (1) [11].

As a result, we arrive at the diagonalized Hamiltonian

$$H_d = E_\alpha \alpha^+ \alpha + E_\beta \beta^+ \beta + E_o \tag{8}$$

with dimensionless eigenvalues



$$\frac{E_\alpha}{\theta} = \frac{1}{S}\left(\frac{1+k^2}{\sqrt{1-k^2}} + \frac{\omega}{2}\right), \quad \frac{E_\beta}{\theta} = \frac{1}{S}\left(\frac{1+k^2}{\sqrt{1-k^2}} - \frac{\omega}{2}\right); \quad \frac{E_0}{\theta} = \frac{1}{S}\left(\frac{1+k^2}{\sqrt{1-k^2}} - 1\right).$$

and the following notation we have here introduced:

$$\omega = 2\,\frac{\omega_a - \omega_b}{\omega_a + \omega_b}, \quad S = \frac{2\theta}{\omega_a + \omega_b}, \quad \text{where } \theta = k_b T. \tag{9}$$

Then we compute the averages:

$$\langle a^+ a\rangle = v^2\left[n_\alpha + n_\beta + 1\right] + n_\alpha, \quad \langle b^+ b\rangle = v^2\left[n_\alpha + n_\beta + 1\right] + n_\beta,$$
$$V_a = V_b = v^2(v^2+1)\left[2n_\alpha n_\beta + n_\alpha + n_\beta + 1\right],$$

where $n_\alpha = \left(\exp(E_\alpha/kT) - 1\right)^{-1}, \quad n_\beta = \left(\exp(E_\beta/kT) - 1\right)^{-1}$ (10)

Here averaging runs over the eigenvalues of Hamiltonian (8).

The distribution for the mode a will be sub-Poisson provided that the inequality (2) holds in the following form:

$$v^4\left[2n_\alpha n_\beta + n_\alpha + n_\beta + 1\right] + 2n_\alpha n_\beta v^2 - n_\alpha < 0. \tag{11}$$

Solutions of the biquadratic inequality (11) are in the region limited by the roots $v^2_{\pm a}$:

$$v^2_{-a} < v^2 < v^2_{+a},$$

where $v^2_{\pm a} = \dfrac{-n_\alpha n_\beta \pm \sqrt{d_a}}{2n_\alpha n_\beta + n_\alpha + n_\beta + 1}, \quad d_a = n_\alpha^2 n_\beta^2 + n_\alpha\left[2n_\alpha n_\beta + n_\alpha + n_\beta + 1\right].$

Since $v^2 > 0$, and $v^2_{-a} < 0$, in any case, then the actual constraint on $v^2$ will be of the form:

$$0 < v^2 < v_{+a}^2. \qquad (12)$$

For the mode **b** to have the sub-Poisson distribution the similar inequality:

$$0 < v^2 < v_{+b}^2, \qquad (13)$$

where $v_{+b}^2 = \dfrac{-n_\alpha n_\beta + \sqrt{d_b}}{2n_\alpha n_\beta + n_\alpha + n_\beta + 1}$, $\quad d_b = n_\alpha^2 n_\beta^2 + n_\beta\left[2n_\alpha n_\beta + n_\alpha + n_\beta + 1\right]$

should hold valid.

The threshold temperature $T^{th}$ for the mode **a** as a function of parameters $\omega_a$, $\omega_b$ and $\kappa$ can be determined from the equation

$$v^2(v^2+1)(2n_\alpha^{th} n_\beta^{th} + n_\alpha^{th} + n_\beta^{th} + 1) = v^2(n_\alpha^{th} + n_\beta^{th} + 1) + n_\alpha^{th}; \qquad (14)$$

whereas for mode **b**, from the equation

$$v^2(v^2+1)(2n_\alpha^{th} n_\beta^{th} + n_\alpha^{th} + n_\beta^{th} + 1) = v^2(n_\alpha^{th} + n_\beta^{th} + 1) + n_\beta^{th}, \qquad (15)$$

where $n_\alpha^{th}$ и $n_\beta^{th}$ are given by expression (10) at $T = T^{th}$.

Numerical solutions to equation (14) are drown in Fig.1 for various values of the dimensionless parameter $\omega$.

## 4. Discussion of results.

The simplest polariton model we have considered shows the possibility of appearing the sub-Poisson distribution for the number of phonons at temperatures below a certain threshold temperature $T^{th}$ that is function of Hamiltonian parameters. Since the condition

$$V = \langle a^+ a \rangle$$

implies the Poisson distribution realized for the coherent



state of the Bose-field [5] which is closest to the classical state as it possesses minimal symmetric quantum fluctuations, then $T^{th}$ can be considered as the threshold temperature of transition from a state with the true quantum behavior ($T<T^{th}$) into the region of classical behavior ($T>T^{th}$). There is no phase transition at the point $T^{th}$ in the conventional sense because the latter should be related with spontaneous symmetry breaking of the collective state of the system (see, for instance, [12]).

The question now arises of how to observe experimentally the above-described "nonclassical" behavior of phonons in thermal equilibrium. It seems that for this purpose the methods of Raman scattering can be used. However, since the information on statistical properties of phonons is contained in the correlation function of the second order **V**, the measurement of spectral characteristics of scattered light is not sufficient. It is necessary to perform measurement of the

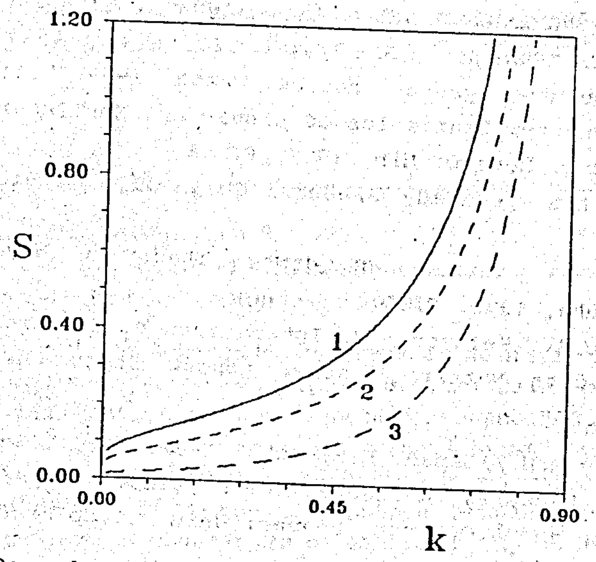


Fig.1. Dependence of the threshold temperature $S^n$ on the coupling constant **k** at the following values of the detuning parameter $\omega$:

1) $\omega = 1$; 2) $\omega = 0$; 3) $\omega = -1.5$.

Normalized variables $S^n$ and $\omega$ are given via original parameters of system by (9); and **k**, by (7).

correlation functions of scattered light and to reconstruct the correlation function of phonons from their relationship. This relationship will be thoroughly considered in a subsequent paper.

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**References**

1. Yamomoto Y., Haus H.A.//Rew. of Mod. Phys. 1986. V.58. P.1001.
2. Loudon R., Knight P.//J. of Mod. Opt. 1987.V.34.P.709.
3. Kilin S.Ya. Quantum Optics. Fields and their detecting. Minsk: Nauka & Tachnika, 1990. P.176.
4. Machida S., Yamomoto Y.//Opt. Comm. 1986. V.57.P.290.
5. Malkin I.A., Man'ko V.I. Dynamic Systems and Coherent States of Quantum Systems. Moscow: Nauka, 1979. P.319.
6. Perina Y. Quantum Statistics of Linear and Nonlinear Optical Phenomena. Moscow: Mir, 1987. P.368.
7. Shumovsky A.S.//Doklady Akademii Nauk USSR. 1991.V316.-P.894.
8. Tyablikov S.V. Methods of Quantum Theory of Magnetism. Moscow: Nauka, 1975. P.527.
9. Il'insky Yu.A., Keldysh A.V. Interaction of Electromagnetic Radiation with Matter. Moscow: Moscow State University Pub.1989.P.299.
10. Suschinsky M.M.//Uspehi Fiz. Nauk. 1988.V.154.P.353.
11. Haag R.// Lectures in Theoretical Physics. V.20/ Ed. by N.E.Brittin and A.O.Barut. Denver: Univ. of Colorado, 1960. P.353.
12. Yukalov V.I.,Shumovsky A.S. Lectures on Phase Transitions. Singapore-New Jersey-London-Hong Kong: World Scientific, 1990. P.238.

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