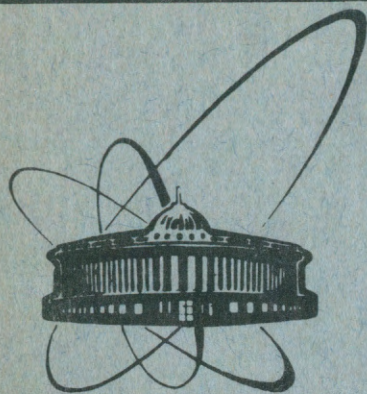


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SUPERFLUORESCENCE FROM AN INITIAL
SQUEEZED VACUUM STATE

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Сверхизлучение из начального состояния
сжатого вакуума

Рассматривается процесс сверхизлучения в системе N двухуровневых атомов, взаимодействующих с полем, для которого начальным состоянием выбран вакуум. Сжатие начального поля приводит к появлению остаточной инверсии и к увеличению времени релаксации поляризации. Процесс излучения происходит за более короткое время при увеличении степени сжатия начального поля. Наблюдается поглощение энергии поля атомной системой после процесса излучения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Superfluorescence from an Initial Squeezed
Vacuum State

Superfluorescence in the point-like two-level N -atom system interacting with an initially squeezed vacuum field state is considered. Squeezing of the initial field leads to the appearance of the residual inversion and to the increase of the polarization relaxation time. The radiation process occurs in a shorter time as the squeezing degree of the initial field increases. The field energy absorption by the atomic system after the radiation process is observed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

In the previous paper [1] exact expressions for the mean values of the field and atomic operator products were obtained. Using these expressions one can eliminate the field operators from the hierarchic equation describing the time evolution of a system with atom-field interaction. In particular, these expressions can be applied to examine the superfluorescence problem [2-5].

Usually the superfluorescence process is considered for the case of a vacuum initial state of the field (spontaneous generation process). In the recent experiment [6] the chaotic (thermal equilibrium) state of the field was used to initiate the superfluorescence generation by the inverted nuclear spin-1/2 system (see also [7]). In principle, any state of the field can be used to initiate the superfluorescence of the two-level macroscopic system. So, in the present paper an important case of the initial squeezed vacuum state of the field will be considered.

The standard approach to the theoretical description of the superfluorescence by the point-like two-level N-atom system is based on the Dicke hamiltonian [3]

$$H = \sum_k \hbar \omega_k a_k^+ a_k + \hbar \Omega R_3 + \sum_k \hbar g_k (R^+ a_k + a_k^+ R^-). \quad (1)$$

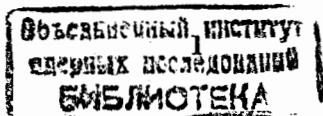
Here the operators a_k^+ , a_k correspond to the k-th mode photons, and the operators R describe the two-level N-atom system

$$[a_k, a_q^+] = \delta_{kq},$$

$$[R_3, R^\pm] = \pm R^\pm, [R^+, R^-] = 2R_3, \|R_3\| \leq N/2. \quad (2)$$

To describe the superfluorescence process in the initially inverted atomic system, it is necessary to calculate the time dependence of the mean photon number $\langle n_k \rangle \equiv \langle a_k^+ a_k \rangle$ and of its derivative $d\langle n_k \rangle / dt$.

In the Heisenberg representation the equations of motion



for the field operators

$$i da_k/dt = \omega_k a_k + g_k R^-, \quad i da_k^*/dt = -\omega_k a_k^* - g_k R^+$$

have formal solutions of the form

$$a_k(t) = \tilde{a}_k(t) - iA_k, \quad a_k^*(t) = \tilde{a}_k^*(t) + iA_k^* \quad (3)$$

where

$$\tilde{a}_k(t) \equiv a_k(t_0) \exp\{-i\omega_k(t-t_0)\},$$

$$A_k \equiv g_k \int_{t_0}^t d\tau \exp\{-i\omega_k(t-\tau)\} R^-(\tau).$$

Therefore for the mean photon number in the k -th mode we obtain

$$\langle n_k \rangle_t = \langle n_k \rangle_{t_0} + \langle A_k^+ A_k \rangle_t - i(\langle \tilde{a}_k^+ A_k \rangle_t - \langle A_k^+ \tilde{a}_k \rangle_t). \quad (4)$$

Here

$$\langle n_k \rangle_t \equiv \text{Tr } n_k(t) \rho_t,$$

where ρ_{t_0} is the statistical operator of the system under consideration at the initial time t_0 . If we suppose that the interaction between the field and the atoms is absent for $t < t_0$, we can choose ρ_{t_0} in the following form

$$\rho_{t_0} = \rho_{\text{field}} \circ \rho_{\text{atom}}.$$

In our case ρ_{field} has the form

$$\rho_{\text{field}} = S(\xi) |0\rangle \langle 0| S^+(\xi), \quad (5)$$

where $|0\rangle$ is the vacuum state of the field and $S(\xi)$ is a

squeezing operator by Stoler [8]

$$S(\xi) = \prod_k \exp\left(\frac{1}{2} \xi_k^* a_k^2 - \frac{1}{2} \xi_k a_k^{+2}\right).$$

Therefore

$$\langle n_k \rangle_{t_0} = |\nu_k|^2, \quad (6)$$

where

$$\nu_k = e^{i\theta_k} \sinh r_k, \quad \xi_k = r_k e^{i\theta_k}.$$

To calculate the mean values

$$\langle \tilde{a}_k^+ A_k \rangle_t, \quad \langle A_k^+ \tilde{a}_k \rangle_t$$

let us use the expressions obtained in [1]:

$$\langle \tilde{a}_k^+ A_k \rangle_t = \langle \tilde{a}_k^+ \rangle_t \langle A_k \rangle_t + \langle \tilde{a}_k^+, \tilde{a}_k \rangle_t \langle [A_k, \tilde{a}_k^+] \rangle_t -$$

$$\langle \tilde{a}_k^+, \tilde{a}_k^+ \rangle_t \langle [A_k, \tilde{a}_k] \rangle_t,$$

$$\langle A_k^+ \tilde{a}_k \rangle_t = \langle \tilde{a}_k \rangle_t \langle A_k^+ \rangle_t + \langle \tilde{a}_k^+, \tilde{a}_k \rangle_t \langle [\tilde{a}_k, A_k^+] \rangle_t -$$

$$\langle \tilde{a}_k, \tilde{a}_k \rangle_t \langle [\tilde{a}_k^+, A_k^+] \rangle_t.$$

Here $\langle B, C \rangle \equiv \langle BC \rangle - \langle B \rangle \langle C \rangle$.

For the initial state of the field described by the statistical operator (5) we have

$$\langle \tilde{a}_k^+, \tilde{a}_k \rangle_{t_0} = |\nu_k|^2,$$

$$\langle \tilde{a}_k, \tilde{a}_k \rangle_{t_0} = \langle \tilde{a}_k^+, \tilde{a}_k^+ \rangle_{t_0}^* = -\mu_k \nu_k e^{-2i\omega_k(t-t_0)},$$

where

$$\mu_k = \cosh r_k.$$

Therefore instead of (7) we get

$$\langle \tilde{a}_k^+ A_k \rangle_t = |\nu_k|^2 \langle [A_k, \tilde{a}_k^+] \rangle_t + \mu_k^* \nu_k^* e^{2i\omega_k(t-t_0)} \langle [A_k, \tilde{a}_k] \rangle_t \quad (8)$$

$$\langle A_k^+ \tilde{a}_k \rangle_t = |\nu_k|^2 \langle [\tilde{a}_k, A_k^+] \rangle_t + \mu_k \nu_k e^{-2i\omega_k(t-t_0)} \langle [\tilde{a}_k^+, A_k^+] \rangle_t.$$

Using now expressions (3) to calculate the commutator in the right-hand side of (8) we finally obtain

$$i(\langle \tilde{a}_k^+ A_k \rangle_t + \langle A_k^+ \tilde{a}_k \rangle_t) = |\nu_k|^2 \langle [A_k, A_k^+] \rangle_t. \quad (9)$$

So, for the mean number of photons (4) we can now write

$$\langle n_k \rangle_t = |\nu_k|^2 + \langle A_k^+ A_k \rangle_t + |\nu_k|^2 \langle [A_k^+, A_k] \rangle_t. \quad (10)$$

Then the radiation intensity is defined by

$$I(t) = \sum_k \hbar \omega_k \frac{d}{dt} \langle n_k \rangle_t. \quad (11)$$

One can see easily that

$$dA_k/dt = g_k R^-(t), \quad dA_k^+/dt = g_k R^+(t).$$

Therefore from the expressions (10) and (11) we have

$$I(t) = \sum_k \hbar \omega_k g_k \{ \langle R^+ A_k \rangle_t + \langle A_k^+ R^- \rangle_t + |\nu_k|^2 (\langle [R^+, A_k] \rangle_t + \langle [A_k^+, R^-] \rangle_t) \}. \quad (12)$$

Now to obtain the time dependence of the radiation intensity we should calculate the correlation functions

$$\langle R^+ A_k \rangle_t = g_k \int_{t_0}^t d\tau e^{-i\omega_k(t-\tau)} \langle R^+(t) R^-(\tau) \rangle, \quad (13)$$

$$\langle A_k^+ R^- \rangle_t = g_k \int_{t_0}^t d\tau e^{-i\omega_k(t-\tau)} \langle R^+(\tau) R^-(t) \rangle.$$

For this aim let us use the so-called Markoff approximation [3-5]

$$R^\pm(\tau) \cong R^\pm(t) e^{\mp i\Omega(t-\tau)} e^{-|t-\tau|/2T}, \quad \tau \leq t. \quad (14)$$

Substituting (14) into the expressions (13) we obtain

$$\langle R^+ A_k \rangle_t \cong \Gamma_k \langle R^+ R^- \rangle_t \quad (15)$$

$$\langle A_k^+ R^- \rangle_t \cong \Gamma_k^* \langle R^+ R^- \rangle_t$$

where

$$\Gamma_k \equiv g_k \int_{t_0}^t d\tau e^{i(\Omega-\omega_k)(t-\tau)} e^{-|t-\tau|/2T}.$$

It should be noted that usually the case of the adiabatic switching on of the interaction is considered [4, 5] and so one should put $t_0 \rightarrow -\infty$.

In the one-mode resonance approximation

$$\Gamma = 2gT$$

and we get the time evolution of the radiation intensity in the case of the initial squeezed vacuum state of the field:

$$I(t) = \hbar \omega \{ 2g\Gamma \langle R^+ R^- \rangle_t + 4g\Gamma |\nu|^2 \langle R_3 \rangle_t \}.$$

Now one ought to calculate the one-time correlation functions $\langle R^+ R^- \rangle_t$ and $\langle R_3 \rangle_t$. For this aim let us use the hierarchic equation method [4, 5, 1]. Let O be an atomic operator (e. g. $O = R^+ R^-$). Then the corresponding equation of motion for its mean value is

$$i\hbar \frac{d}{dt} \langle O \rangle_t = \langle [O, H] \rangle_t$$

Taking into account the explicit form of our hamiltonian and using again the exact expressions of paper [1] to eliminate the field variables one can obtain the following exact hierarchic equation for the atomic system in the case of the initial squeezed vacuum state of the field:

$$\begin{aligned} \frac{d}{dt} \langle O \rangle_t + i\Omega \langle [O, R_3] \rangle_t &= g \{ \langle A^\dagger [O, R^-] \rangle_t + \langle [R^\dagger, O] A \rangle_t + \\ &|\nu|^2 (\langle [A, [O, R^+]] \rangle_t + \langle [A^\dagger, [O, R^-]] \rangle_t) + \\ &\mu\nu e^{-2i\omega(t-t_0)} \langle [[O, R^+], A^\dagger] \rangle_t + \\ &\mu^* \nu^* e^{2i\omega(t-t_0)} \langle [[O, R^-], A] \rangle_t \}. \end{aligned} \quad (16)$$

Then for $O=R^+R^-$ we get

$$\begin{aligned} \frac{d}{dt} \langle R^+R^- \rangle_t &= 2g \{ \langle R^+R_3R^- \rangle_t + \langle R^+R_3A \rangle_t + |\nu|^2 (\langle [R^+R_3, A] \rangle_t + \\ &\langle [A^\dagger, R_3R^-] \rangle_t) + \mu\nu e^{-2i\omega(t-t_0)} \langle [A^\dagger, R^+R_3] \rangle_t + \\ &\mu^* \nu^* e^{2i\omega(t-t_0)} \langle [R_3R^-, A] \rangle_t \}. \end{aligned} \quad (17)$$

If we use here the approximation (14) together with the decoupling

$$\langle R^+R^-R_3 \rangle_t \cong \langle R^+R^- \rangle_t \langle R_3 \rangle_t \quad (18)$$

instead of the exact equation (17) we obtain the following approximate equation

$$dy/dp = 4|\nu|^2 x - 2(1 + 3|\nu|^2)y - z + 2xy + 4|\nu|^2 C. \quad (19)$$

Here

$$x \equiv \langle R_3 \rangle_t, \quad y \equiv \langle R^+R^- \rangle_t,$$

$$z \equiv \mu\nu e^{-2i\omega(t-t_0)} \langle R^{+2} \rangle_t + \mu^* \nu^* e^{2i\omega(t-t_0)} \langle R^{-2} \rangle_t$$

$$C \equiv \langle R^+R^- + R_3^2 - R_3 \rangle_t = \frac{N}{2} \left(\frac{N}{2} + 1 \right)$$

and we have used the time scale $p=2g\Gamma t$. Analogously, for $O=R_3$ we obtain

$$dx/dp = -2|\nu|^2 x - y. \quad (20)$$

It should be emphasized that $C=\text{const}$ because the operator $R^+R^- + R_3^2 - R_3$ is an integral of motion for the hamiltonian (1).

To solve the equations (19), (20) we ought now to write also an equation for the variable z . Using the hierarchic equation (16) together with the approximations (14), (18) and with decoupling

$$\langle R^{+2}R_3 \rangle_t \cong \langle R^{+2} \rangle_t \langle R_3 \rangle_t - \langle R^{+2} \rangle_t$$

$$\langle R_3R^{-2} \rangle_t \cong \langle R_3 \rangle_t \langle R^{-2} \rangle_t - \langle R^{-2} \rangle_t$$

we get

$$\begin{aligned} dz/dp &= 12|\nu|^2(1 + |\nu|^2)x - 12|\nu|^2(1 + |\nu|^2)y - \\ &(1 + 2|\nu|^2)z + 2xz + 8|\nu|^2(1 + |\nu|^2)C. \end{aligned} \quad (21)$$

Thus, we obtain the closed system of the ordinary differential equations (19)-(21). To solve this system, we ought to choose the initial values of the variables x , y , z . For this aim let us consider at the initial time the atomic system independently of the field and introduce the free energy

$$F = -\theta \ln \text{Tr} \exp\{-\hbar\Omega R_3 / \theta\} \quad (22)$$

where θ is the atomic temperature. The calculation of the Trace in (22) leads to

$$F = -N \theta \ln 2 \cosh(\hbar\Omega/2\theta). \quad (23)$$

Using (22), (23) and the constant of motion C one can easily get

$$x(t_0) = \partial F / \partial(\hbar\Omega) = - (N/2) \tanh(\hbar\Omega/2\theta) \quad (24)$$

$$y(t_0) = N^2/4 + N/2 - (N/2) \tanh(\hbar\Omega/2\theta) - (N/2) \operatorname{sech}^2(\hbar\Omega/2\theta) - (N^2/4) \tanh^2(\hbar\Omega/2\theta). \quad (25)$$

Now, setting the initial inversion value $x(t_0)$, it is possible to define the initial polarization value $y(t_0)$ using (24) and (25). Note, that for the Dicke states the operators R^+ and R^- are the raising and lowering operators respectively. Hence,

$$z(t_0) = \mu \nu \langle R^+ \rangle_{t_0}^2 + \mu^* \nu^* \langle R^- \rangle_{t_0}^2 = 0$$

due to an orthogonality of Dicke states with different j .

Let us consider now a process in which the field is initially in a chaotic state:

$$\rho_a(t_0) = e^{-\beta n} / \operatorname{Tr}(e^{-\beta n}), \quad n = a^+ a, \quad \beta = \hbar\omega/k_B T.$$

For the inversion $x(t)$ and polarization $y(t)$ one can obtain the closed system of ordinary differential equations:

$$\partial x / \partial p = -2 \alpha x - y,$$

$$\partial y / \partial p = 4 \alpha x - 2(1 + 3 \alpha) y + 2xy + 4 \alpha C,$$

where $p = 2g\Gamma t$, $\alpha = (e^\beta - 1)^{-1}$. For the radiation intensity we have

the expression:

$$I(t) = \hbar\omega (2g\Gamma y + 4g\Gamma \alpha x). \quad (26)$$

It is interesting to compare the time behaviour of the inversion $x(t)$, the polarization $y(t)$ and the radiation intensity $I(t)$ in the both cases of choosing the field initial state.

Let us start consideration of particular cases of preparation of the atomic system initial state:

(i) the total inversion of the atomic system is given by initial conditions

$$x(t_0) = N/2, \quad y(t_0) = N, \quad z(t_0) = 0.$$

The atomic system inversion evolution $x(t)$ for different values of the parameter $|\nu|^2$, which characterizes the squeezing degree of the field at an initial moment $t=t_0$, is given in Fig.1. We have chosen $gT=10^{-5}$. At $|\nu|^2=0$ the initial state of the field is nonsqueezed vacuum. In this case the inversion $x(t)$ falls down from $N/2$ to $-N/2$. At a nonzero value of $|\nu|^2$ one can observe the residual inversion, increasing as the squeezing degree of the initial field increases. This can be easily understood if we take into consideration the fact that the mean value of the photon number in squeezed vacuum is equal to $|\nu|^2$. Hence, the mean field energy in this state is proportional to $|\nu|^2$. There is an energy exchange between the field and the atomic system in the superfluorescence process. The residual inversion is explained through a partial absorption of the field energy by atoms. Note that the increase of the residual inversion is limited by value $x=0$. At this value of the inversion $N/2$ atoms are in the excited state. In Fig.2 the time behaviour of the polarization $y(t)$ is shown. The polarization pick falls down and shifts to the right as the squeezing degree of the initial field increases. On the contrary, in the superfluorescence process with a chaotic state of the field at the initial moment the shift of

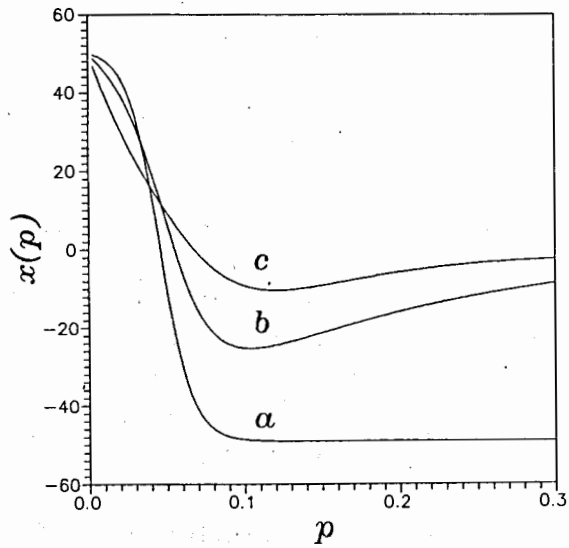


Fig. 1. Time dependence of atomic inversion for an initially excited atomic system ($N=100$, $x(t_0)=N/2$) interacting with an initially squeezed vacuum field state with : (a) $|\nu|^2=0$; (b) $|\nu|^2=3$; (c) $|\nu|^2=10$. The time scale $p=2g\Gamma t$ is used.

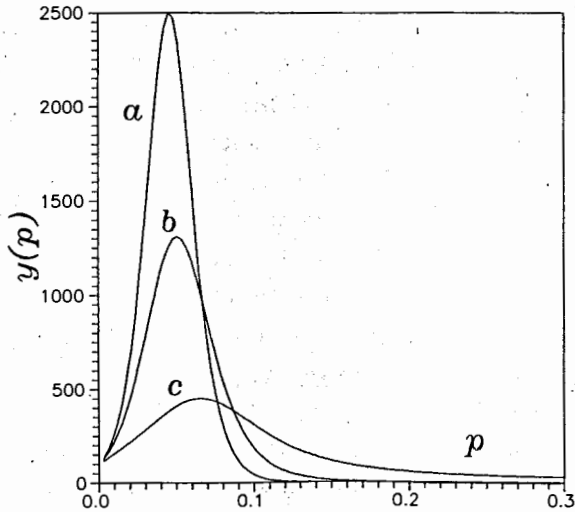


Fig. 2. Time dependence of atomic polarization for an initially excited atomic system ($N=100$, $x(t_0)=N/2$) interacting with an initially squeezed vacuum field state with : (a) $|\nu|^2=0$; (b) $|\nu|^2=3$; (c) $|\nu|^2=10$. The time scale $p=2g\Gamma t$ is used.

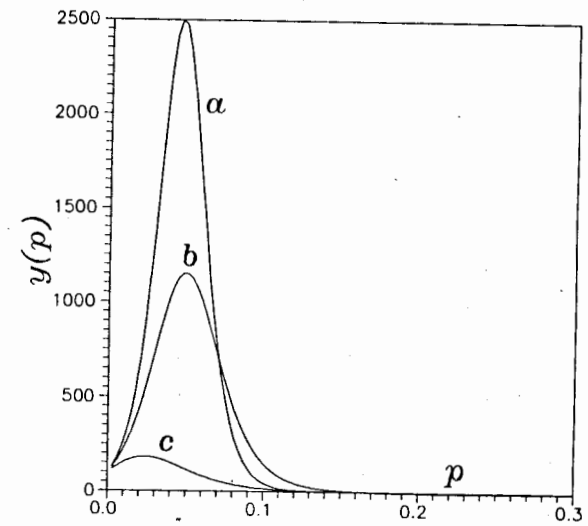


Fig. 3. Time dependence of atomic polarization for an initially excited atomic system ($N=100$, $x(t_0)=N/2$) interacting with an initially chaotic field state with : (a) $\alpha=0$; (b) $\alpha=3$; (c) $\alpha=10$. The time scale $p=2g\Gamma t$ is used.

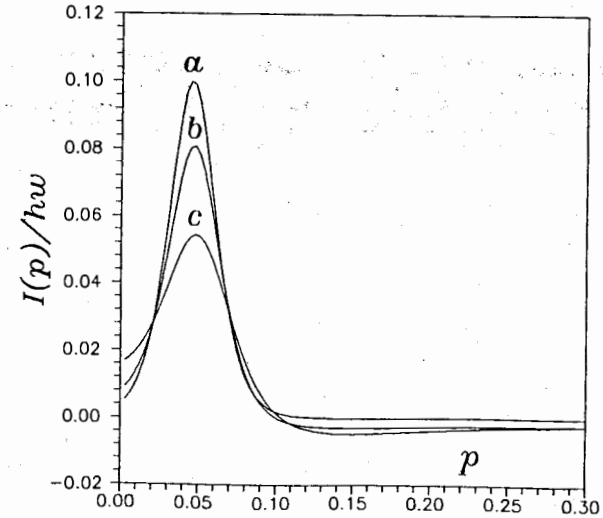


Fig. 4. Time dependence of normalized radiation intensity for an initially excited atomic system ($N=100$, $x(t_0)=N/2$) interacting with an initially squeezed vacuum field state with (a) $|\nu|^2=0$; (b) $|\nu|^2=1$; (c) $|\nu|^2=3$. The time scale $p=2g\Gamma t$ is used.

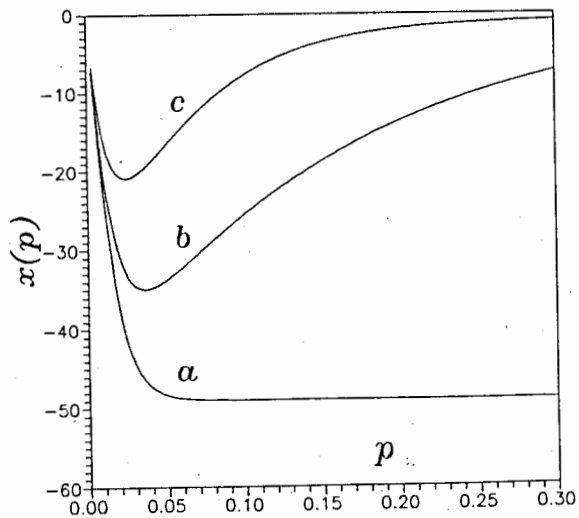


Fig. 5. Time dependence of atomic inversion for an initially excited atomic system ($N=100, x(t_0)=0$) interacting with an initially squeezed vacuum field state with : (a) $|\nu|^2=0$; (b) $|\nu|^2=3$; (c) $|\nu|^2=10$. The time scale $p=2g\Gamma t$ is used.

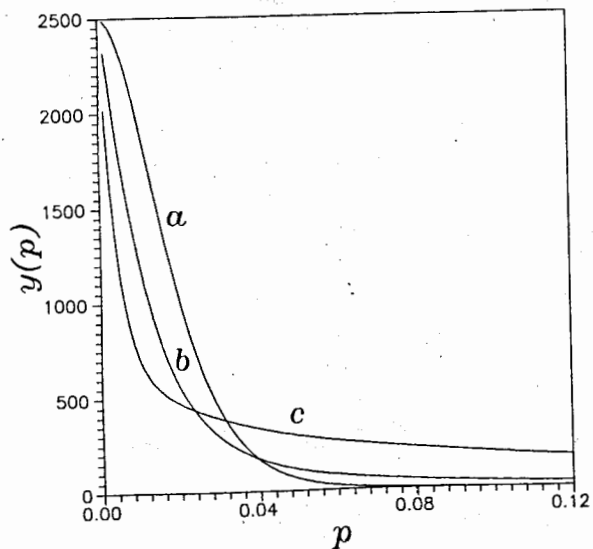


Fig. 6. Time dependence of atomic polarization for an initially excited atomic system ($N=100, x(t_0)=0$) interacting with an initially squeezed vacuum field state with : (a) $|\nu|^2=0$; (b) $|\nu|^2=10$; (c) $|\nu|^2=30$. The time scale $p=2g\Gamma t$ is used.

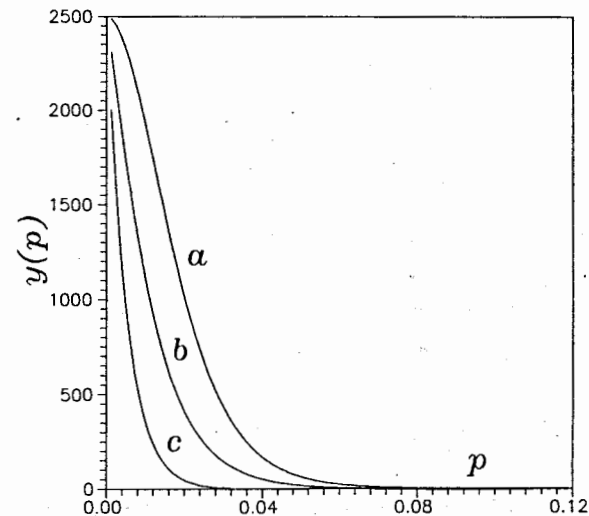


Fig. 7. Time dependence of atomic polarization for an initially excited atomic system ($N=100, x(t_0)=0$) interacting with an initially chaotic field state with : (a) $\alpha=0$; (b) $\alpha=10$; (c) $\alpha=30$. The time scale $p=2g\Gamma t$ is used.

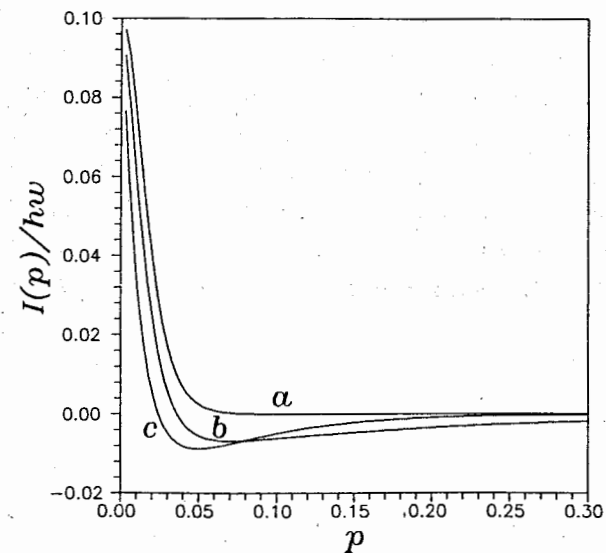


Fig. 8. Time dependence of normalized radiation intensity for an initially excited atomic system ($N=100, x(t_0)=0$) interacting with an initially squeezed vacuum field state with : (a) $|\nu|^2=0$; (b) $|\nu|^2=3$; (c) $|\nu|^2=10$. The time scale $p=2g\Gamma t$ is used.

the polarization pick to the right is absent (Fig.3). It seems like the increase of the polarization relaxation time is an effect of the initial field squeezing. In Fig.4 the time behaviour of the radiation intensity is given. As $|\nu|^2$ increases, the intensity maximum decreases and the radiation absorption by the atomic system increases, which is consistent with the presence of the residual inversion (Fig.1).

(ii) The partial inversion of the atomic system is given by the initial conditions

$$x(t_0) = 0, y(t_0) = N^2/4, z(t_0) = 0.$$

As in the case (i), the residual inversion increases as the squeezing parameter $|\nu|^2$ increases (Fig.5). The greater the squeezing of the initial field, the sooner a saturation of the residual inversion is reached. The behaviour of the polarization $y(t)$ is given in Fig.6. The squeezing of the initial field increases the polarization relaxation time. On the contrary, in the case of the chaotic initial state of the field the increase of the field energy, corresponding to the increase of the parameter $\alpha = (e^\beta - 1)^{-1}$, leads to the decrease of the polarization relaxation time (Fig.7). In Fig.8 the radiation intensity evolution is shown. The radiation process occurs in a shorter time as $|\nu|^2$ increases. The field energy absorption by the atomic system is observed after the radiation.

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