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DYNAMICAL EQUATIONS FOR SPIN CHAIN IN THE SU(2S+1)/SU(2S)⊗U(1) SPACE AND S = 3/2 EASY AXIS MAGNET

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Динамические уравнения для спиновой цепочки в пространстве SU(2S+1)/SU(2S)⊗U(1) и легкоосный магнетик со спином

Построены гамильтоновы уравнения в пространстве SU(2S+1)/SU(2S)⊗U(1) для полуклассических моделей магнетиков с произвольным спином. Исследован магнетик Гейзенберга со спином S = 3/2 при помощи обобщенных когерентных состояний группы SU(4). Показано, что классические вакуум-системы лежат в SU(2) сечении 6-мерного спинового фазового пространства CP^3. Найдены две дополнительные высокочастотные моды в магнонном спектре легкоосного магнетика.

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Dynamical Equations for Spin Chain in the SU(2S+1)/SU(2S)⊗U(1) Space and S = 3/2 Easy Axis Magnet

The Hamiltonian equations of motion in the SU(2S+1)/SU(2S)⊗U(1) space for semiclassical models of magnets with arbitrary spins are constructed. The S = 3/2 Heisenberg magnet is investigated by use of the generalized coherent states of SU(4) group. We show that classical vacuum of system lie in the SU(2) section of total 6-dimensional spin phase space CP^3. Two additional high frequency modes are found in magnon spectrum of the easy-axis magnet.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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A great attention was recently paid to theoretical study and experimental investigations of the S=1 magnets. There only three spin states exist in every site of cristall lattice and all possible transitions between these states are described by eight independent operators of the SU(3) group. The main problem in general case (arbitrary spins) is to construct correctly Hamiltonian equations of motion in the corresponding $SU(2S+1)/SU(2S)\otimes U(1)$ space of the SU(2S+1) group. Such equations in the $SU(3)/SU(2)\otimes U(1)$ space were derived in paper [4].

Here we come to more general problem i.e. to derive Hamiltonian equations of motion in the $SU(3)/SU(2)\otimes U(1)$ space. As a concrete example the spin S=3/2 Heisenberg magnet is considered.

Note, that in the S=3/2 case all possible transition between four spin states in every site are described by fifteen undependent operators of the SU(4) group. This operators are related by linear guadratic and cubic combinations of spin operators.

Dimension of spin state spase for spin S=3/2,is

$$\dim H=6. \tag{1}$$

The symmetrical space

$$\mathbb{C}P^2=SU(4)/SU(3)\otimes U(1) \tag{2}$$

has the same dimension.

In the paper [3] the generalized spin coherent states are constructed to give for the SU(4) group

$$|\Psi> = exp\left\{ \sum_{i=1}^{3} \left(\xi_i \hat{T}_i^{+} - \overset{''}{\xi}_i \hat{T}_i^{+} \right) \right\} \qquad (3)$$

where generators $\hat{T}_i^{\pm}$ are defined by the formulae

$$\hat{T}_1^{+} = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0 \end{pmatrix}, \qquad \hat{T}_1^{-} = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0 \end{pmatrix},$$

$$\hat{T}_2^{+} = \begin{pmatrix} 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix}, \qquad \hat{T}_2^{-} = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&1&0&0 \end{pmatrix},$$

$$\hat{T}_3^{+} = \begin{pmatrix} 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix}, \qquad \hat{T}_3^{-} = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix}.$$

Corresponding coherent state for spin S=3/2 in the SU(4)/SU(3)⊗U(1) space assumes the form

$$|\Psi>=\left(1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2\right)^{-1/2}\left\{|0>+ \Psi_1|1>+ \Psi_2|2>+ \Psi_3|3>\right\} \qquad (4)$$

where

$$\Psi_i = \frac{\xi_i}{|\xi|}\tan|\xi|, \qquad |\xi|^2=|\xi_1|^2+|\xi_2|^2+|\xi_3|^2.$$

As spin operators for S=3/2 we choose the following operators

$$\hat{S}^{+} = \begin{pmatrix} 0&\sqrt{3}&0&0\\ 0&0&2&0\\ 0&0&0&\sqrt{3}\\ 0&0&0&0 \end{pmatrix}, \quad \hat{S}^{-} = \begin{pmatrix} 0&0&0&0\\ \sqrt{3}&0&0&0\\ 0&2&0&0\\ 0&0&\sqrt{3}&0 \end{pmatrix}, \quad \hat{S}^{z} = \frac{1}{2}\begin{pmatrix} 3&0&0&0\\ 0&1&0&0\\ 0&0&-1&0\\ 0&0&0&-3 \end{pmatrix}, \qquad (5)$$

where $\hat{S}^{x}= \frac{1}{2}\left(\hat{S}^{+}+ \hat{S}^{-}\right)$, $\hat{S}^{y}= \frac{1}{2i}\left(\hat{S}^{+}- \hat{S}^{-}\right)$,

that satisfy the commutation relations of the SU(2) algebra

$$\left[\hat{S}^{+}, \hat{S}^{-}\right] = 2\hat{S}^{z}, \quad \left[\hat{S}^{z}, \hat{S}^{+}\right] = \hat{S}^{+}, \quad \left[\hat{S}^{z}, \hat{S}^{-}\right] = -\hat{S}^{-}.$$

Remaining generators of the SU(4) group can be taken as guadratic and cubic combinations of spin operators.

Averaging the spin operators via generalized coherent state (GCS) (4) we have

$$< \hat{S}^{+}> = \frac{\sqrt{3}\Psi_2\bar{\Psi}_3+2\Psi_1\bar{\Psi}_2+\sqrt{3}\bar{\Psi}_1}{1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2},$$

$$< \hat{S}^{-}> = \frac{\sqrt{3}\bar{\Psi}_2\Psi_3+2\bar{\Psi}_1\Psi_2+\sqrt{3}\Psi_1}{1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2}, \qquad (7)$$

$$< \hat{S}^{z}> = \frac{1}{2}\,\frac{3|\Psi_3|^2+|\Psi_2|^2-|\Psi_1|^2-3}{1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2}.$$

We also give double correlators that will be of use in what follow

$$< \hat{S}^{z}\hat{S}^{z}> = \frac{N}{4}\left(9|\Psi_3|^2 + |\Psi_2|^2 + |\Psi_1|^2+9\right),$$

$$< \hat{S}^{+}\hat{S}^{+}> = 2\sqrt{3}(\Psi_1\bar{\Psi}_3 + \bar{\Psi}_2),$$

$$< \hat{S}^{-}\hat{S}^{-}> = 2\sqrt{3}(\bar{\Psi}\ \Psi\ + \Psi\ ), \qquad (8)$$

$$< \hat{S}^{+}\hat{S}^{-}> = N(3|\Psi_3|^2+ 4|\Psi_2|^2 + 3|\Psi_1|^2),$$

$$< \hat{S}^{+}\hat{S}^{z}> = \frac{N}{2}\sqrt{3}(\Psi_2\bar{\Psi}_3 - 2\Psi_1\bar{\Psi}_2 - 3\sqrt{3}\bar{\Psi}_1),$$

$$< \hat{S}^{z}\hat{S}^{-}> = \overline{< \hat{S}^{+}\hat{S}^{z}>},$$

where

$$N=1/(1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2).$$

Hence, the Casimir operator is

$$< \hat{C} > = \frac{1}{2}\left(< \hat{S}^{+}\hat{S}^{-}> + < \hat{S}^{-}\hat{S}^{+}> \right) + < \hat{S}^{z}\hat{S}^{z}> = \frac{15}{4}. \qquad (9)$$

Contrary to the SU(3) GCS the conservation law s^2=const and even s^2+q^2=const are now broken, and the identity

$$s^2+q^2+f^2=1 \qquad (10)$$

is valid, where q^2 and f^2 are the corresponding sums of double and triple correlators.

Whence in order to obtain dynamical equations we construct the Lagrangian following to paper [6]

$$L= i\hbar\,\frac{\dot{\Psi}_1\bar{\Psi}_1-\Psi_1\dot{\bar{\Psi}}_1+\dot{\Psi}_2\bar{\Psi}_2-\Psi_2\dot{\bar{\Psi}}_2+\dot{\Psi}_3\bar{\Psi}_3-\Psi_3\dot{\bar{\Psi}}_3}{1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2} - H(\Psi_1,\Psi_2,\Psi_3). \qquad (11)$$

Varying Lagrangian (11) we get the following equations in the SU(4)/SU(3)⊗U(1) space

$$i\dot{\Psi}_1=\left(1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2\right)\left[(1+|\Psi_1|^2)\frac{\delta H}{\delta\bar{\Psi}_1} + \Psi_1\bar{\Psi}_2\frac{\delta H}{\delta\bar{\Psi}_2} + \Psi_1\bar{\Psi}_3\frac{\delta H}{\delta\bar{\Psi}_3}\right]$$

$$i\dot{\Psi}_2=\left(1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2\right)\left[(1+|\Psi_2|^2)\frac{\delta H}{\delta\bar{\Psi}_2} + \Psi_2\bar{\Psi}_1\frac{\delta H}{\delta\bar{\Psi}_1} + \Psi_2\bar{\Psi}_3\frac{\delta H}{\delta\bar{\Psi}_3}\right]$$

$$i\dot{\Psi}_3=\left(1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2\right)\left[(1+|\Psi_3|^2)\frac{\delta H}{\delta\bar{\Psi}_3} + \Psi_3\bar{\Psi}_1\frac{\delta H}{\delta\bar{\Psi}_1} + \Psi_3\bar{\Psi}_2\frac{\delta H}{\delta\bar{\Psi}_2}\right].$$

Analogous equations one can get for the conjugate functions $\bar{\Psi}_1,\bar{\Psi}_2,\bar{\Psi}_3$. One can now extend result (12) to the case of an arbitrary spins in the space SU(2S+1)/SU(2S)⊗U(1) to obtain

$$i\dot{\Psi}_l=\left(1+\sum_{k=1}^{n}|\Psi_k|^2\right)\left[(1+|\Psi_l|^2)\frac{\delta H}{\delta\bar{\Psi}_l} + \sum_{k=1}^{n}\Psi_l\bar{\Psi}_k\frac{\delta H}{\delta\bar{\Psi}_k}\alpha_{lk}\right] \qquad (13)$$

where

$$\alpha_{lk}=\begin{cases} 0, & l=k \\ 1, & l\neq k. \end{cases}$$

Let us now compare coherent states for spin S=3/2 in the SU(4) version with that in the SU(3) one. GCS for S=3/2 in the SU(2) version (see [7]) has the form

$$|\eta>=\left(1+|\eta|^2\right)^{-1/2}\left\{|0>+ \sqrt{3}\eta|1>+ \sqrt{3}\eta^2|2>+ \eta^3|3>\right\} \qquad (14)$$

and in the SU(4) version

$$|\Psi>=\left(1+|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2\right)^{-1/2}\left\{|0>+ \Psi_1|1>+ \Psi_2|2>+ \Psi_3|3>\right\}.$$

The states $|\eta>$ are defined by the points of sphere S^2 and the state $|\Psi>$ by points of $\mathbb{CP}^3$. Quantum system with the spin S=3/2 lives in the 6-dimensional state space and the Hamiltonian averaged over the SU(2)/U(1) describes its behaviour in a two-dimensional section of this space. It is easy to check that this section is

$$\Psi_3= \frac{1}{3\sqrt{3}}\Psi_1^3,\quad \Psi_2= \frac{1}{\sqrt{3}}\Psi_1^2,\quad \Psi_3= \sqrt{3}\,\Psi_1^3. \qquad (15)$$

Hereafter we investigate S=3/2 spin chain that describes the ferromagnet with uni-axis exchange anisotropy. As usual, Hamiltonian is given by

$$\hat{H} = -J\sum \hat{\vec{S}}_j\hat{\vec{S}}_{j+1} - \delta\sum \hat{S}^z_j\hat{S}^z_{j+1}. \qquad (16)$$

In the continuum limit, we get the classical Hamiltonian

$$H=-J\int\left\{<\hat{S}^{+}><\hat{S}^{-}>+(1+\delta)(<\hat{S}^{z}>)^{2}-\frac{a_0^2}{2}\left(<\hat{S}^{+}>_x<\hat{S}^{-}>_x+(1+\delta)(<\hat{S}^{z}>_x)^{2}\right)\right\}dx .$$

Let us calculate classical vacuum states of this Hamiltonian. In the case $\delta>0$, that corresponds to the "easy axis" model of ferromagnet, the ground state is

$$\Psi_1=\Psi_2=\Psi_3=0$$

that is terms of GCS $|\Psi_0>=|0>$, which coincides with the ground states of SU(2) and SU(3) versions (see [3]). When $\delta<0$, the "easy plane" model, Hamiltonian acquires its minimum at $\Psi_1=\Psi_2=\sqrt{3}$ and $\Psi_3=1$ and GCS takes the form $|\Psi_0>=\sqrt{8}(|0>+\sqrt{3}|1>+\sqrt{3}|2>+|3>)$, that is also the same as in the SU(2) version for spin S=3/2.

Now we derive dispersions for linear waves propagating in the system. In order to do this we expand Hamiltonian (17) near the easy-axis ground state and keeping the second order terms we get

$$H=-\frac{3}{2}J\int\left\{3|\Psi_1|^2-|\Psi_2|^2-3|\Psi_3|^2+\delta\left(|\Psi_1|^2+|\Psi_2|^2+|\Psi_3|^2\right)-a_0^2|\Psi_{1x}|^2\right\}\frac{dx}{a_0} .$$

Linearized eguations have the form

$$i\dot{\Psi}_1=-(3+\delta)\Psi_1-\Psi_{1xx},$$

$$i\dot{\Psi}_2=(\delta+1)\Psi_2,$$

$$i\dot{\Psi}_3=3(\delta+1)\Psi_3.$$

Hence the dispersion relations are

$$\omega_1=k^2-(3+\delta), \qquad (19)$$

$$\omega_2=\delta+1, \qquad (20)$$

$$\omega_3=3(\delta+1). \qquad (21)$$

Thus , in the linear approximation there is a low frequency mode in the S=3/2 easy-axis mgnet, with dispersion formula (19) being the same to that of the SU(2) model [3], and unlike S=1 magnet there are two additional high frequency modes (20) and (21). In the spin state space l. f. branch lie in the SU(2) section of the total 6-dimensional spin phase space, and h. f. branches span all the rest phase space.

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