сообщения
объединенного
института
ядерных
исследований
дубна

E17-91-220

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NONLINEAR SPIN WAVES
AND TWO-DIMENSIONAL CLASSICAL ATTRACTOR

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1991

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Нелинейные спиновые волны и двумерный
классический аттрактор

Проводится исследование легкоосного магнетика Гейзенберга со спином S = 1 посредством обобщенных когерентных состояний группы SU(3). Подтверждено наличие высокочастотной магнонной моды. Получены системы уравнений, описывающие слабонелинейные спиновые волны и в стационарном пределе сводящиеся к НУШ. Аналитические и численные исследования указывают на наличие в 4-мерном спиновом фазовом пространстве двумерного сечения /многообразия/, являющегося аттрактором.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1991

Makhankov V.G. et al. E17-91-220
Nonlinear Spin Waves and
Two-Dimensional Classical Attractor

An easy-axis S = 1 Heisenberg ferromagnet is studied by means of generalized spin coherent states defined on the SU(3) group. An additional high frequency magnon mode is revived. Systems of equations are obtained describing weak nonlinear spin waves which reduce to the familiar NLSE in the stationary limit. Analytical and numerical studies of the system show that in the four dimensional spin phase space there is a two dimensional section (manifold) which is an attractor. This manifold (S^2) coincides with that of the spin phase space of the S = 1/2 (or $S \to \infty$) model which can be described by the SU(2) coherent states.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1991

Investigation of the quasi-one dimensional magnetic systems attracts a great attention because of simplicity of their mathematical description and possibility easy to check theoretical prediction by experimental data. The most investigated models conserve the square of classical spin, but this property of magnets reveals near the Curie temperature only, and at the higher temperatures square of classical spin does not conserve. Our approach in this problem is based on the using of the generalized CS of the SU(3) group, which takes into account this property. Such a CS was constructed in the paper [1], but parametrization used in [1] is not convenient. Therefore we shall use the CS, that up to reparametrization coincides with the abovementioned SU(3) CS.

We construct the coherent states in terms of real functions in the following form (see also [2])

$$| \Psi > = U(\theta,\phi,\gamma) exp(2ig Q_{xy}) | u >, \qquad (1)$$

where

$$Q_{xy} = \frac{i}{2} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix},$$

is quadrupole moment, $| u >$ is a referent state, and

$$U(\theta,\phi,\gamma) = e^{-i\phi \hat{S}^z} e^{-i\theta \hat{S}^y} e^{-i\gamma \hat{S}^z} . \qquad (2)$$

In fact the last function is a unitary operator, being Wigner function, that provide us to proceed into own mobile (trial functions) using the generators of corresponding group, the group SU(3) for model under consideration. However, even in the case of the standard Hamiltonian of the Heisenberg model, the quasiclassical behaviour of the S=1 system can differs radically from one defined by Landau-Lifshitz equation as we shall see below (see also [10]) and its description requires to leave the sphere SU(2)/U(1): $S^2 = 1$. Therefore, description on the basis of SU(3) CS constructed on the CP^2 space is more adequate.

We shall investigate the S=1 easy-axis ($\delta>0$) Heisenberg model with exchange anisotropy

$$\hat{H}_{ea} = -J \sum_j \left(\hat{\vec{S}}_j \hat{\vec{S}}_{j+1} + \delta\ \hat{S}^z_j \hat{S}^z_{j+1} \right) \tag{1}$$

and with single-ion one

$$\hat{H}_{si} = -J \sum_j \left(\hat{\vec{S}}_j \hat{\vec{S}}_{j+1} + \delta\ \hat{S}^z_j \hat{S}^z_j \right) \tag{2}$$

by using the SU(3) and SU(2) CS and then compare results.

Let us remind the GCS constructed in [10], where they have the form

$$|\ \psi\ \rangle = e^{\tilde{1}} |0\rangle$$

here $|0\rangle$ - chosen referent state, $\tilde{1}$ - elements of coset space $CP^{2s} = SU(2S+1)/SU(2S)\otimes U(1)$. Dimension of this space is $2(2S+1)-2=4S$. In the S=1 case the SU(2) CS is

$$|\ \psi\ \rangle = \frac{1}{1+|\psi|^2} \left\{ |0\rangle + \sqrt{2}\psi|1\rangle + \psi^2|2\rangle \right\} \tag{3}$$

and the SU(3) version is

$$|\ \zeta\ \rangle = (1+|\zeta_1|^2+|\zeta_2|^2)^{-1/2} \left\{ |0\rangle + \zeta_1|1\rangle + \zeta_2|2\rangle \right\}. \tag{4}$$

The S=1 quantum systems live in $2(2S+1)-2=4$ - dimensional spin phase space, so the Hamiltonian averaged via the spin GCS (1) governs behaviour of the system in the two- dimensional section

$$\zeta_1 = \sqrt{2}\psi, \qquad \zeta_2 = \frac{1}{2}\,\zeta_1^2 \tag{5}$$

of this space.

It should be stressed, that in the SU(3) representation the conservation law

$$S^2 = 1 \tag{6}$$

breaks down in contrast to the SU(2) one, so we have

$$S^2 + \Big\{ \langle\hat{S}^-\hat{S}^z\rangle\langle\hat{S}^+\hat{S}^z\rangle + \langle\hat{S}^z\hat{S}^-\rangle\langle\hat{S}^z\hat{S}^+\rangle + \langle\hat{S}^-\hat{S}^-\rangle\langle\hat{S}^+\hat{S}^+\rangle + $$

$$+ \Big(1 - \langle\hat{S}^z\hat{S}^z\rangle\Big)^2 \Big\} = 1, \tag{7}$$

i.e. conservation of the squire of classical spin plus some terms of quadrupole nature.

The Hamiltonians (1) and (2), averaged via SU(3) CS are

$$H_{ea} = \int \Big\{ \frac{a_0^2}{2} \Big(\langle\hat{S}^+\rangle_x\langle\hat{S}^-\rangle_x + (\langle\hat{S}^z\rangle_x)^2 \Big) -$$

$$- \langle\hat{S}^+\rangle\langle\hat{S}^-\rangle - (\langle\hat{S}^z\rangle)^2 - \delta(\langle\hat{S}^z\rangle)^2 \Big\} dx \tag{8}$$

and

$$H_{si} = \int \Big\{ \frac{a_0^2}{2} \Big(\langle\hat{S}^+\rangle_x\langle\hat{S}^-\rangle_x + (\langle\hat{S}^z\rangle_x)^2 \Big) -$$

$$- \langle\hat{S}^+\rangle\langle\hat{S}^-\rangle - (\langle\hat{S}^z\rangle)^2 - \delta(\langle\hat{S}^z\hat{S}^z\rangle)^2 \Big\} dx\ . \tag{9}$$

The classical vacuum states in both models (8) and (9) are

$$\zeta_1 = \zeta_2 = 0 \tag{10}$$

hence $\langle\hat{S}^+\rangle = \langle\hat{S}^-\rangle = 0$, $\langle\hat{S}^z\rangle = -1$ and the quadrupole moment in (7) vanishes, i.e. the SU(3) easy-axis vacuum coincides with the SU(2) one. The GCS corresponding to this classical vacuum

$$|\,\psi\,\rangle = \Pi_j |0\rangle_j \tag{11}$$

is just the same as in SU(2) model.

In order to investigate a linear waves propagating in both systems we expand the Hamiltonians (8) and (9) up to $O(\,|\zeta|^2)$ and obtain the dispersion relations

$$\omega_1 = k^2 a_0^2 + 2\delta \tag{12.a}$$

$$\omega_2 = 4(1+\delta) \tag{12.b}$$

for model (1) and

$$\omega_1 = k^2 a_0^2 + \delta \tag{13.a}$$

$$\omega_2 = 4 \tag{13.b}$$

for model (2). The relations (12.a) and (13.a) describe the low frequency (l.f.) waves of field ζ_1 and coincide with those for SU(2) version. However, there appear additional high frequency (h.f.) modes of the field ζ_2 with dispersions (12.b) and (13.b). Note that quadrupole terms in the Hamiltonians do not vanish.

Let us proceed to investigate weak nonlinear excitations in system (1) and (2). Taking into account terms up to $O(|\zeta|^4)$ in classical Hamiltonians (8) and (9) we get

$$H_{ea} \cong a_0^2|\zeta_{1x}|^2 + 2\delta|\zeta_1|^2 - 2(\bar{\zeta}_1^2\zeta_2 + \zeta_1^2\bar{\zeta}_2) + 4(1+\delta)|\zeta_2|^2$$

$$+ (1-3\delta)|\zeta_1|^4 - 8(1+\delta)|\zeta_2|^4 - 2(4+5\delta)|\zeta_1|^2|\zeta_2|^2 \tag{14}$$

for exchange anisotropy and

$$H_{si} \cong a_0^2|\zeta_{1x}|^2 + \delta|\zeta_1|^2 - 2(\bar{\zeta}_1^2\zeta_2 + \zeta_1^2\bar{\zeta}_2) + 4|\zeta_2|^2$$

$$+ (1-\delta)|\zeta_1|^4 - 8|\zeta_2|^4 - (8+\delta)|\zeta_1|^2|\zeta_2|^2 \tag{15}$$

for single-ion anisotropy cases.

By use of the equations of motion in the SU(3)/SU(2)⊗U(1) space

$$i\dot{\zeta}_1 + (1+|\zeta_1|^2+|\zeta_2|^2)\Big\{ (1+|\zeta_1|^2)\frac{\delta H}{\delta\bar{\zeta}_1} + \zeta_1\bar{\zeta}_2\frac{\delta H}{\delta\bar{\zeta}_2}\Big\} = 0,$$

$$i\dot{\zeta}_2 + (1+|\zeta_1|^2+|\zeta_2|^2)\left\{(1+|\zeta_2|^2)\frac{\delta H}{\delta\bar{\zeta}_2} + \bar{\zeta}_1\zeta_2\frac{\delta H}{\delta\bar{\zeta}_1}\right\} = 0 \tag{16}$$

we derive the systems of equations

$$i\dot{\zeta}_1 - \zeta_{1xx} + 2\delta\zeta_1 - 4\bar{\zeta}_1\zeta_2 + 2(1-\delta)|\zeta_1|^2\zeta_2 = 0, \tag{17.a}$$

$$i\dot{\zeta}_2 + 4(1+\delta)\zeta_2 - 2\zeta_1^2 = 0 \tag{17.b}$$

and

$$i\dot{\zeta}_1 - \zeta_{1xx} + 2\delta\zeta_1 - 4\bar{\zeta}_1\zeta_2 + 2|\zeta_1|^2\zeta_2 = 0, \tag{18.a}$$

$$i\dot{\zeta}_2 + 4\zeta_2 - 2\zeta_1^2 = 0, \tag{18.b}$$

where we took into account that $\zeta_2 \simeq \zeta_1^2/2$ in the vicinity of vacuum.

Looking for a solutions of (17) and (18) in the form

$$\zeta_1(x,t) = \eta_1(x)\, e^{i\omega_1 t}$$

$$\zeta_2(x,t) = \eta_2(x)\, e^{i\omega_2 t}$$

we see that for both models

$$\omega_2 = 2\omega_1 \,. \tag{20}$$

From (17.b) and (18.b) (up to $O(\eta_1^3)$) an estimations

$$\eta_2 = \frac{\eta_1^2}{2 - \omega + 2\delta} \tag{21}$$

and

$$\eta_2 = \frac{\eta_1^2}{2 - \omega} \tag{22}$$

follow. So the first two equations (17.a) and (18.a) of systems are reduced to conventional nonlinear Schrödinger equations (NSE) in the above approximation

$$\eta_{1xx} - (2\delta - \omega)\eta_1 + 2\delta\eta_1^3 = 0 \tag{23}$$

and

$$\eta_{1xx} - (\delta - \omega)\eta_1 + \delta\eta_1^3 = 0, \tag{24}$$

which possess well-known solutions.

Expressions (20)-(24) mean, that weak nonlinear spin waves propagating in the easy-axis vacuum create solitons in such a way, that the field ζ_2 is coupled to the ζ_1 one, so the quadrupole terms vanish (naturally, up to $O(\eta^4)$ terms). As a result the system tends "to live" in the two-dimensional SU(2) cross-section of the full spin phase space $\mathbb{CP}^2$.

Consider, whether any relic from the full phase space remains. Comparing equations (23) and (24) with the corresponding ones obtained in the SU(2) model (S=1)

$$i\dot{\psi} = \psi_{xx} + 2\delta\psi - 4\delta|\psi|^2\psi \tag{25}$$

and

$$i\dot{\psi} = \psi_{xx} + \delta\psi - 2\delta|\psi|^2\psi \tag{26}$$

and taking into account (5) we see, that the both corresponding equations coincide, i.e. they describe the same stationary solitons.

Let us consider computational aspect of the problem. We carry out computer simulation of systems (17) and (18) by means of the well-known explicit " leap frog " scheme. This scheme is stable for $\tau \le h^2/4$, so we have chosen $\tau = 0.002$, $h = 0.1$. Computations were carried out in the interval $x \in [-60,60]$ with zero boundary conditions.

We check conservability of scheme by computing the two first integrals of motion:
Hamiltonians in plane metric

$$H_{ea} = -\int\Big\{ |\zeta_{1x}|^2 + 2\delta|\zeta_1|^2 - 2(\bar{\zeta}_1^2\zeta_2 + \zeta_1^2\bar{\zeta}_2) + 4(1+\delta)|\zeta_2|^2 + (1-\delta)|\zeta_1|^4 \Big\}\, dx, \tag{27}$$

$$H_{si} = -\int\Big\{ |\zeta_{1x}|^2 + \delta|\zeta_1|^2 - 2(\bar{\zeta}_1^2\zeta_2 + \zeta_1^2\bar{\zeta}_2) +$$

$$+ 4|\zeta_2|^2 - |\zeta_1|^4 \Big\} dx, \tag{28}$$

integral of particle number

$$N = \int \Big\{ |\zeta_1|^2 + 2|\zeta_2|^2 \Big\} dx. \tag{29}$$

In computer experiments we investigate stationary soliton-like solutions of type (19) where

$$\eta_1 = \frac{b}{\cosh(\sqrt{2\delta-\omega_1}\; x)}, \qquad \eta_2 = \frac{\eta_1^2}{2 - \omega_1 + 2\delta} \tag{30}$$

for the system (17), and

$$\eta_1 = \frac{b}{\cosh(\sqrt{\delta-\omega_1}\; x)}, \qquad \eta_2 = \frac{\eta_1^2}{2 - \omega_1} \tag{31}$$

for the system (18), and b is a free parameter.

Behaviour in time of the value

$$\Delta = \sup_{x \in [-L,L]} |\zeta_2 - \zeta_1^2/2| \tag{32}$$

in computer runs shows us how close to the SU(2) cross-section the system is.

The study of stationary solutions (30) and (31) shows their stability and the both systems lie in the SU(2) section for a long time. For example, experiments with b=0.75 and b=0.4 show that the value of Δ conserves and does not exceed 10^{-2} and 10^{-3}, correspondingly, up to time T=30. Integrals of motion are conserved with accuracy

$$\Delta N/N \propto 10^{-6}, \qquad \Delta E/E \propto 10^{-6} \tag{33}$$

for both systems.

Then we investigate a perturbed solutions. The both systems (17) and (18) display that the fields ζ_1 and ζ_2 alter in such a way, that the value of Δ decreases in time and the systems tend to live in the SU(2) section.

Resume

A vacuum states of easy-axis S=1 magnet lie in the SU(2) section. Linear waves are independent trajectories filling all the fourdimensional space. They consist of two branches: the l.f. one, which lies in the SU(2) section, and the h.f. one, that fills the rest of the space. The weak interaction between brunches ("trajectories") results in that a part of h.f. brunch is captured by the l.f. one and they both come to the SU(2) section. Computer experiments show, that initial states closed to stationary solutions evolve in such a way that value $\sup|\zeta_2 - \zeta_1^2/2|$ conserves or decreases in time. This behaviour of the systems mean, that for both easy-axis models (1) and (2) the SU(2) cross-section is the attractor for classical trajectories in the spin phase space of the initial quantum system.

Classical phenomenological behaviour of the quantum ferromagnet according to the Landau-Lifshitz approach is reached via the asymptotic tend to the SU(2) attractor.

We are grateful to prof.S.S.Moiseev for very illuminative discussion.

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Received by Publishing Department
on May 20, 1991.