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ON SEMICLASSICAL BEHAVIOUR OF THE S = 1 UNIAXIAL HEISENBERG MAGNET

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Абдуллоев Х.О. и др. E17-91-219
Полуклассическое поведение одноосного магнетика Гейзенберга со спином

Исследуется одноосный магнетик Гейзенберга со спином S = 1 посредством техники обобщенных когерентных состояний группы SU(3) в действительной параметризации. Получены уравнения, описывающие спиновую и квадрупольную динамики ферромагнетика на полуклассическом уровне, которые могут быть сведены к уравнениям Ландау-Лифшица и sin - Гордон в пределе исчезающего квадрупольного момента.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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On Semiclassical Behaviour of the S = 1 Uniaxial Heisenberg Magnet

The S = 1 Heisenberg magnet with uniaxis anisotropy is investigated by use of the SU(3) coherent states technique in the real function parametrization. The equations are derived, which describe spin and quadrupole moment dynamics of the ferromagnet on the semiclassical level and may be reduced to the well-known sine-Gordon and Landau-Lifshitz equations in the limit of vanishing quadrupole moment.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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The aim of this paper is to investigate quasiclassical behaviour of Heisenberg magnet by use of generalized coherent states (in Perelomov sense, [1]). Papers [2] and [3] devoted to applications of Heisenberg-Weyl and SU(2) coherent states (CS) for studying spin systems revealed that such states well work for magnetics with spin $S \to \infty$. Both approaches give rise in fact to the Landau-lifshitz equation. But in real systems spin magnitude, which is defined by an assemblage of valency electrons does not usually exceed several unities [4]. The question naturally arises of, to what extent, the classical description in the framework of Landau-Lifshitz model is adequate to study systems with S=1, such as $CsNiF_3$, $[(CH_3)_4N]$ $[NiCl_3]$ and so on. Moreover experimental data on the quasi-one-dimensional magnetic $CsNiF_3$ can be qualitatively explained in the scope of present theory based on the classical Landau-Lifshitz approach (see review [5] and papers cited). Quite a vast literature is now devoted to the theoretical studies of S=1 magnetics using alternative grounds [6-10]. In [11] an approach proposed which, in a sense, was a generalization of the well-known mean field method, we mean a trial function technique.

If quantum Hamiltonian of the S=1 magnetic system contains single-ion anisotropy or higher spin operator moments, then the SU(2) CS did not adapted to study such systems. It is necessary to construct more adequate states coordinate system for every site. Two Eiler angles θ and ϕ define the orientation of the classical spin vector, the angle γ - rotation of the quadrupole moment around the spin vector, and parameter g characterizes the length change of the vector of classical spin and the quadrupole moment. Then the CS takes the form

$$|\,\Psi > = C_1 |\, u > + C_0 |\, m > + C_{-1} |\, d >, \tag{3}$$

where

$$C_1 = e^{-i\phi} \left(\cos^2\frac{\theta}{2} \cos g \, e^{-i\gamma} + \sin^2\frac{\theta}{2} \sin g \, e^{i\gamma} \right),$$

$$C_0 = \frac{\sin\theta}{\sqrt{2}} \left(\cos g \, e^{-i\gamma} + \sin g \, e^{i\gamma} \right),$$

$$C_{-1} = e^{i\phi} \left(\sin^2\frac{\theta}{2} \cos g \, e^{-i\gamma} + \cos^2\frac{\theta}{2} \sin g \, e^{i\gamma} \right)$$

and satisfies the normalization condition.

Now the spin operators averaged by CS (3) have the physically illustrative form

$$\begin{aligned} < \hat{S}^{+} > &= e^{i\phi} \cos 2g \sin\theta, \\ < \hat{S}^{-} > &= e^{-i\phi} \cos 2g \sin\theta, \\ < \hat{S}^{z} > &= \cos 2g \cos\theta. \end{aligned} \tag{4}$$

By vector of the classical spin we shall mean

$$< \hat{S} > = \cos 2g \ (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta).$$

So the square of classical spin is

$$s^2 = \cos^2 2g.$$

As it was shown in paper [1] for the SU(3) CS the following equality takes place

$$s^2 + q^2 = 1, \tag{5}$$

where

$$q^2 = <\hat{S}^{+}\hat{S}^{z}><\hat{S}^{z}\hat{S}^{-}> + <\hat{S}^{-}\hat{S}^{z}><\hat{S}^{z}\hat{S}^{+}> + <\hat{S}^{+}\hat{S}^{+}><\hat{S}^{-}\hat{S}^{-}> + \\ + (1 - <\hat{S}^{z}\hat{S}^{z}>)^2 .$$

The simple calculations show us that in our case

$q^2 = \sin^2 2g.$

Now we shall investigate the s=1 Heisenberg magnet described by the Hamiltonian

$$H = -J \sum_j \left(\hat{S}_j \hat{S}_{j+1} + \delta\, \hat{S}^z_j \hat{S}^z_{j+1} \right) - h \sum_j \hat{S}^x_j, \qquad (6)$$

where h is magnetic field and δ is constant of anisotropy. If $\delta>0$ then Hamiltonian (6) describes "easy-axis" and when $\delta<0$ then (6) corresponds to the magnet with "easy plane" anisotropy.

Averaging Hamiltonian (6) by the CS (3) and transiting from summation to integration we find the classical Hamiltonian

$$H = -J \int \Big(\cos^2 2g\, (1 + \delta \cos^2\theta) - \frac{a_0^2}{2} \left(4 \sin^2 2g g_x + \cos^2 2g\, \theta_x^2 + \cos^2 2g \sin^2\theta\, \phi_x \right) \Big) \frac{dx}{a_0}. \qquad (7)$$

To obtain the dynamical equations we use the method expounded in the paper [3] and construct Lagrangian for corresponding CS (3) acting in SU(3)/SU(2)⊗U(1) space

$$L = \cos 2g\, \dot{\gamma} + \cos 2g \cos\theta\, \dot{\phi} - H. \qquad (8)$$

Varying this Lagrangian we get equations of motion in Hamilton form

$$\begin{aligned}
\dot{\phi} &= - \frac{1}{\cos 2g \sin\theta} \frac{\delta H}{\delta\theta}, \\
\dot{\theta} &= \frac{1}{\cos 2g \sin\theta} \left(\frac{\delta H}{\delta\phi} - \cos\theta \frac{\delta H}{\delta\gamma} \right), \\
\dot{g} &= \frac{1}{2 \sin 2g} \frac{\delta H}{\delta\gamma}, \\
\dot{\gamma} &= \frac{\cos\theta}{\cos 2g \sin\theta} \frac{\delta H}{\delta\theta} - \frac{1}{2 \sin 2g} \frac{\delta H}{\delta\gamma}.
\end{aligned} \qquad (9)$$

Hamiltonian (7) with equations (9) generates the system of equations

$$\begin{aligned}
\dot{\phi} &= -2\delta \cos 2g \cos\theta + a_0^2 \Big(\frac{\cos 2g}{\sin\theta} \theta_{xx} - \\
&\quad - 4 \frac{\sin 2g}{\sin\theta} g_x \theta_x - 2 \cos 2g \cos\theta\, \phi_x^2 \Big) - h \,\mathrm{ctg}\theta \cos\phi, \\
\dot{\theta} &= a_0^2 (\cos 2g \sin\theta\, \phi_{xx} + 2 \cos 2g \cos\theta\, \theta_x \phi_x - \\
&\quad - 4 \sin 2g \sin\theta\, \phi_x g_x) + h \sin\phi, \\
\dot{g} &= 0, \\
\dot{\gamma} &= -2\cos 2g - h\frac{\cos\phi}{\sin\theta} + a_0^2 \Big(2 \sin 2g g_{xx} - 4 \sin 2g \,\mathrm{ctg}\theta\, g_x \theta_x + \\
&\quad + \cos 2g \,(\phi_x^2 + \theta_x^2 + 4 g_x^2 + \mathrm{ctg}\theta\, \theta_{xx}) \Big).
\end{aligned} \qquad (10)$$

In the long-wave limit and weak magnetic field $a_0^2 < \delta$, $h \ll \delta$ and near the "easy plane", i.e. when $\theta = \pi/2 - \bar{\theta}$ and magnitude $g \ll 1$, $\bar{\theta} \ll 1$ we derive the sin-Gordon equation

$$\phi_{\tau\tau} - \phi_{\xi\xi} + \sin\phi = 0. \qquad (11)$$

When we put $g=0$ first two equations of the system (10) take the form of Landau-Lifshitz equation (see [4]). Classical vacuum state of the "easy axis" ferromagnet is $\phi = 0$, $g = 0$ and $\theta = 0$, $\theta = \pi$ i.e. it is two time degenerated. Ground state of the "easy plane " ferromagnet is $g = 0$, $\theta = \pi/2$, i.e. multiple degenerated. Dispersion of linear waves propagating in the "easy axis" ground state has two branches

$$\begin{aligned}
\omega_1 &= 2(\delta + k^2), \\
\omega_2 &= 2(\delta + 1),
\end{aligned} \qquad (12)$$

i.e. there are high frequency mode ω_2 in addition to the low frequency one ω_1. In the case of "easy plane " we have also two modes: the first one coincides with the well- known Bogolyubov dispersion

$$\omega_1^2 = \frac{1}{2} k^2 (k^2 + 2\delta) \qquad (13.a)$$

and the second one is the high frequency oscillations

$$\omega_2^2 = 4 . \qquad (13.b)$$

If we take into account the single-ion anisotropy in the Hamiltonian (6), then g_t is not equal to zero in the system (10), whereby we have ferromagnets with nonconserving square of classical spin.

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