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ON THE NOISE
OF THE SUPERCONDUCTING MAGNETS

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INTRODUCTION

It is well known, that the magnetic field acts with considerable mechanical forces upon the electromagnet's coil. In the solenoids a large amount of mechanical energy may be accumulated. For instance, in magnetic field $B = 6T$, the density of the accumulated energy is $B^2/8\pi = 14\ MJ/m^3$. In the case of superconduction solenoids, the magnetic field penetrates as a vortex lines in the volume of the type-II superconducting wires, which the coil is made of. These lines are pinned at the defects — normal metal grains, for example, embedded in the volume of the type-II superconductor of the wire.

In the case of the hopping of the superconducting vortex lines, the magnetic field's energy accumulated in the system, may generate acoustic oscillations. It is observed in experiments, that varying the current in normal or superconducting magnets, sometimes squeaks, cracks and moan alike sounds may be heard. The magnitude and the repeating frequency of these signals increases when current reaches the values, where the transition of the magnet to nirmal state begins [1]. If the value of current is too high, so that the solenoid is near to the transition to normal state it is natural to increase the frequency of vortex line reconnections with the pinning centers. This creeping of the magnetic flux is able to excite elastic waves, propagating in the superconductor's volume. The aim of the present paper is the investigation of the dispersion of the magnetoacoustic waves in homogeneously magnetized type-II superconductor. The approach we use to derive this dispersion is the consideration of the linear magnetoelastic equations. Precisely, we will analyse the hybridization between the magnetohydrodynamic Alfven waves, investigated in a conducting fluid and transversal acoustic waves in elastic medium, non existing in a fluid at all. The possible experiments for observations of these waves are discussed.

THE MODEL

Before we begin our consideration, it should be pointed that for application of the magnetohydrodynamics it is necessary that wavelength and the wave period must be considerably greater than the length and the time of the mean free paths of the current carriers (electrons and ions in normal metal).

Further in this work we will show that the MHD equations can be applied for type-II superconductor with strong pinning force.

In the normal metal the averaged Lorentz force, acting upon the electrons acts also upon the crystal lattice, because of the electron's scattering over the impurities. The Ohm's law $\mathbf{j} = \sigma \mathbf{E}$ ($\sigma = ne^2 \tau_{tr}/m$) supposes that the Hall's angle (of the order of $\omega_c \tau_{tr}$) is extremely small. That is, using the cyclotron frequency $\omega_c = eB/(mc)$, it can be obtained that Hall's angle is small for magnetic field $B << mc/(e\tau_{tr})$.

The scattering is that effective momentum exchange mechanism between electrons and ions, which makes it possible to consider the Lorentz force volume density $\mathbf{j} \times \mathbf{B}$ as an external force, acting upon the ion component as in the magnetohydrodynamics or magnetoelastodynamics. The frequency of collisions between the electrons and the impurities in the normal metal, providing small Hall's angle in the case of small magnetic fields, plays the same role as the frequency of collisions between the electrons and ions with maxwellian distribution of the velocity in gaseous plasma. Here it should be mentioned that the vector $\mathbf{j} \times \mathbf{B}$ is external force density in the MHD considerations, applicable for example in the investigation of Alfven waves in liquid mercury and in plasma. In the superconductors formally $\sigma = \infty$ and $\tau_{tr} = \infty$. But the mechanism of averaged Lorentz force acting upon the crystal lattice by means of superconducting current of the Cooper pairs, absolutely differs from the normal metal case. In the type-II superconductors the magnetic field penetrates by means of vortex lines. Around every line a magnetic flux $\Phi_0 = 2\pi hc/(2e)$ is concentrated at a distance of the order of the penetration depth λ. Let us consider for simplicity the type-II superconductor with large Ginzburg-Landau parameter (such as high T_c superconductors or those, used for technical applications) $\kappa = \lambda/\xi >> 1$, where ξ is the superconducting coherence length. At a distance r from the core of the vortex line ($\xi << r << \lambda$) it is possible to consider the superconduction current as a perfect viscousless incompressible fluid. The curl of the velocity field Γ is quantized

$$\oint \mathbf{v} d\mathbf{r} = \Gamma = 2\pi\hbar/m^*,$$

where $\mathbf{v}(\mathbf{r})$ is the Cooper pairs velocity and m^* is their mass. When this vortex line is flowed by the flux with average drift velocity $\mathbf{v}_{drift}$, according to the Zukovski theorem /2/ a lifting force acting per unit length of the vortex line is $v_{drift}\Gamma$. Let us consider the curl of the Cooper pairs velocity field

$$\boldsymbol{\Omega} = \mathrm{rot}\mathbf{v}.$$

For the Cooper pair superfluid the field Ω has a δ-type singularities at the vortex line cores,

$$\Gamma = \oint \mathbf{v} d\mathbf{r} = \int \mathrm{rot}\mathbf{v} d\mathbf{s} = \int \boldsymbol{\Omega} d\mathbf{s}.$$

The last integral is calculated over an arbitrary surface $\mathbf{S}$ crossing the vortex line with size considerably large than GL coherent length ξ, i.e. the vortex line core size. For a viscousless fluid the Helmholtz theorem is fulfilled:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla\right)\boldsymbol{\Omega} - (\boldsymbol{\Omega}\nabla)\mathbf{v} = 0 \cdot$$

That is, the δ-type singularities at the vortex lines, follow the movement of the superconducting crystal in the same way as the magnetic field, which will be discussed later.

On the other hand, the vortex lines are connected with the crystal lattice by pinning forces. Roughly speaking, the superconductivity (Cooper pairs) is destroyed by the centrifugal forces at a distance ξ from the center of the vortex line. In other words, the creation of the vortex line requires an additional energy $\xi^2 \omega_c$, where ω_c is the superconducting condensation energy. This energy is not necessary if the vortex line core passes through normal metal grain in the composite superconductor. This decrease of energy is exactly the energy of the vortex line pinning to the crystal lattice.

In this way the averaged Lorentz force $(2e)\mathbf{v}_{drift} \times \mathbf{B}$ acts upon the vortex lines and the latter transfer it to the crystal lattice. So, for the type-II superconductors (as in the magnetohydrodynamics) we can regard that $\mathbf{j} \times \mathbf{B}$ is the averaged external magnetic force, which should be added to the elasticity equations

$$\rho\frac{\partial^2 \mathbf{u}}{\partial t^2} = \rho c_t^2 \Delta \mathbf{u} + \rho(c_l^2 - c_t^2)\nabla \mathrm{div}\, \mathbf{u} + \mathbf{j} \times \mathbf{B},$$

$$\mathbf{j} = c\, \mathrm{rot}\mathbf{B}/4\pi \ ,$$

In the linear equation (1) we suppose that all the conponents of the deformation tensor

$$\frac{1}{2}(\partial_i u_k + \partial_k u_i) = u_{ik}$$

are small enough and introduce the following symbols: ρ for the mass density, $\mathbf{u}$ for the displacement vector of the isotropic elastic medium, c_l and c_t are the velocities of the transversal and longitudinal acoustic waves, $\mathbf{B}$ is the magnetic field, averaged over the Abrikosov lattice (in the sense of the electrodynamics of the continuous media). That is, we regard that the magnetic field does not vary considerably at a distances of the order of the Abrikosov lattice cell size $\sim \sqrt{\Phi_0/B}$. When a vortex line hoppes between two pinning centers (for instance under the influence of a strong electric current of heat fluctuations) this will cause a generation of the magnetoelastic waves in the superconductors volume, whose investigation is the aim of our paper.

As we mentioned above, the magnetic field in the type-II superconductors is connected (or "freezed") to the elastic medium (crystal lattice) by means of pinned vortex lines in a direct sense. That is why the magnetohydrodynamic equation is automatically fulfilled:

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v}\nabla)\mathbf{B} = (\mathbf{B}\nabla)\mathbf{v}; \quad \mathbf{v} = \frac{\partial \mathbf{u}}{\partial t}.$$

The equations (1), (2), (3) form the closed set of equations of the magnetoelastodynamics. The eigenmodes of these homogeneous equations represent a magnetoelastic waves. Similar waves, connected with fluid compressibility are investigated in the magnetohydrodynamics ([4], section 69). That is why we will concentrate our interest over a purely transversal (to the wave vector and to the external magnetic field) waves, representing a hybridization between the Alfven waves in the magnetohydrodynamics and the transversal acoustic waves in the theory of the isotropic elastic continuum.

Considering an external magnetic field, directed along the x-axis $\mathbf{B}_0 = (B_0, 0, 0)$, using the linear approximation for the equations (1), (2), (3), we get the following relations for the perturbations $\mathbf{v}$ and $\mathbf{b}$ ($\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$):

$$\frac{\partial v_x}{\partial t} = c_t^2 \Delta u_x + (c_l^2 - c_t^2)\frac{\partial}{\partial x}\mathrm{div}\,\mathbf{u}$$

$$\frac{\partial v_y}{\partial t} = c_t^2 \Delta u_y + (c_l^2 - c_t^2)\frac{\partial}{\partial y}\mathrm{div}\,\mathbf{u} - \frac{1}{4\pi\rho}B_0\left(\frac{\partial b_x}{\partial y} - \frac{\partial b_y}{\partial x}\right)$$

$$\frac{\partial v_z}{\partial t} = c_t^2 \Delta u_z + (c_l^2 - c_t^2)\frac{\partial}{\partial z}\mathrm{div}\,\mathbf{u} - \frac{1}{4\pi\rho}B_0\left(\frac{\partial b_x}{\partial z} - \frac{\partial b_z}{\partial x}\right)$$

$$\frac{\partial \mathbf{b}}{\partial t} = B_0\frac{\partial \mathbf{v}}{\partial x}.$$

After a Fourier transformation:

$$\omega^2 \tilde{v}_x = c_t^2 k^2 \tilde{v}_x (c_l^2 - c_t^2) k_x (\mathbf{k}\tilde{\mathbf{v}})$$

$$\omega^2 \tilde{v}_y = c_t^2 k^2 \tilde{u}_y + (c_l^2 - c_t^2) k_y (\mathbf{k}\tilde{\mathbf{u}}) - \frac{\omega B_0}{4\pi\rho}(k_y \tilde{b}_x - k_x \tilde{b}_y)$$

$$\omega^2 \tilde{v}_z = c_t^2 k^2 \tilde{u}_z + (c_l^2 - c_t^2) k_z (\mathbf{k}\tilde{\mathbf{u}}) - \frac{\omega B_0}{4\pi\rho}(k_z \tilde{b}_x - k_x \tilde{b}_z)$$

$$\omega \tilde{\mathbf{b}} = -(B_0 k_x)\tilde{\mathbf{v}}.$$

Taking into account that we consider incompressible ($\mathrm{div}\mathbf{u} = 0$, i.e. $(\mathbf{k}\tilde{\mathbf{u}}) = 0$) perturbations:

$$(c_t^2 k^2 - \omega^2)\tilde{v}_x = 0$$

$$\frac{B_0^2}{4\pi\rho}k_x k_y \tilde{v}_x + \left(c_t^2 k^2 - \omega^2 + \frac{B_0^2}{4\pi\rho}k_x^2\right)\tilde{v}_y = 0$$

$$\frac{B_0^2}{4\pi\rho}k_x k_z \tilde{v}_x + \left(c_t^2 k^2 - \omega^2 + \frac{B_0^2}{4\pi\rho}k_x^2\right)\tilde{v}_z = 0.$$

Finally we get the following dispersion equation:

$$\omega(\mathbf{k}) = [c_t^2 \mathbf{k}^2 + v_A^2(\mathbf{nk})]^{1/2}; \quad \mathbf{n} = \mathbf{B}/B,$$

where v_A is defined by:

$$\frac{1}{2}\rho v_A^2 = \mathbf{B}^2/8\pi.$$

For the group velocity we obtain:

$$\mathbf{v}_{gr} = \partial\omega/\partial\mathbf{k} = (c_t^2\mathbf{k} + v_A^2(\mathbf{kn})\mathbf{n})/\omega(\mathbf{k}).$$

Formally, when the shear module $\mu = c_t^2\rho$ is equal to zero, we get the well-known result for the Alfven waves group velocity:

$$\partial\omega/\partial\mathbf{k} = [\mathbf{B}/\sqrt{4\pi\rho}]\,\mathrm{sign}(\mathbf{Bk}),$$

$$\mathrm{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

(cf[4], formula (69.9)).

For all the real superconductors, however $c_t >> v_A$ and we obtain only small anisotropic terms to the sound velocity:

$$\omega/k = c_t[1 + (\mathbf{Bk}/k)^2/8\pi\mu].$$

This shift can be observed not only in the investigation of the eigenmodes of the superconducting specimens but also in the case of normal metal crystals in high magnetic fields at low temperatures. In normal metals an attenuation of the investigated waves amplitude $\propto e^{-\gamma t}$, where (see the problem in[4], section 69)

$$\gamma/\omega \sim (k\delta)^2; \quad \delta = c/(2\pi\sigma\omega)^{1/2},$$

where δ is the penetration depth for the normal skin effect ($\delta << l$, l is the electron mean free path). Taking into account the final conductivity of the superconductors leads to only small changes in the oscillation's frequency of the superconducting specimen

$$\Delta\omega/\omega \sim (\lambda k)^2 \ll 1 .$$

DISCUSSION

Unfortunately, there are too few direct experiments /1/, clarifying the role of the mechanical tensions due to the pinning forces in the superconducting magnets. We think that the magnetoacoustic waves, discussed in the present work can be perceived by a gramophone piezocrystal in the regime of the flux creep, attached to the superconductor. The investigation of the changes of the eigenmodes frequency of superconductive specimen in a homogenious magnetic field is another possibility. We obtained that the change of the velocity is highly anisotropic. Moreover, the same anisotropy of the sound velocity should be observed for normal metals as well. External magnetic fields can also excite magnetoacoustic waves in metal. Unfortunately, we do not have proper data to make certain estimations, but principally a magnetic storm may bring into action a coincidence scheme of aluminium detectors for gravity waves. If we want to interpret the coincidence of the registered signals from metal detectors as gravity waves, we have to take into account such weak effects as the one, discussed here.

CONCLUSION

Unfortunately, for the existing magnetic fields there is not such a materials, for which the sound velocity is comparable (of the same order) with the Alfven velocity. This case of strong hybridization can be observed in composite material of superconductive particles (for example high T_c granules), buried in bubble plastics or elastic net with relatively low shear module (comparable with the density of the magnetic field), armouring liquid mercury. In conclusion we think that systematic measurements of the noise associated with the flux creep should be carried out.

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