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# STIMULATION OF SUPERCONDUCTIVITY IN MULTILAYER STRUCTURES WITH WEAK JOSEPHSON JUNCTION

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## INTRODUCTION

In recent years, there emerged theoretical and experimental papers devoted to the study of simulated superconducting layered structures Nb-Cu, Nb-Ta, V-SI, Nb-NbOx-Nb /1,2/.

Within these structures one can separate three trends of investigations: quasi-two-dimensional superconductivity, pinning and the change of characteristics due to the interaction with other layers.

Simulated layered structures are an ideal model system for studying such fundamental problems of physics as the existence of a nonphonon mechanism of the Copper pairing and the high-temperature superconductivity.

Recently, it has been discovered that multilayer structures from oxide-metal layers of niobium Nb-NbOx-Nb with a predetermined conditions of oxidation provide a discrete increase in the temperature of transformation into a superconducting state depending on the number of layers in the structure /3/.

The niobium layers were evaporated by the magnetometer in the atmosphere of spectroscopically pure argon /2/. The interaction between the layers was controlled by the oxidation conditions. The interaction is specified by the product of resistivity of the oxide layer $\rho_i$ times its thickness $d_i$. It was determined from measurements of tunnel contacts Nb-NbOx-Nb prepared under predetermined oxidation conditions /2,3/. The Auger analysis /2/ ensured the control of oxidation conditions of niobium and indicated the periodic structure, which is especially important for changing the critical temperature of a superconducting transition on an N layer of the structure from a number of identical layers $T_c^{(n)} = f(n)$.

## STATEMENT OF THE PROBLEM

The multilayer structure Nb-NbOx-Nb may thought to be quasi-two-dimensional as the current $(j_\perp \neq 0)$. flows accross the layers; in this case, in an isolated layer of the structure from each thin niobium layer of 100-400 Å with the thickness of the oxide layer 10-20 Å there exists superconductivity ($T_c \sim 4,9$ K). Under given conditions of oxidation the product $(\rho_i d_i)$ changed from

$2 \cdot 10^{-6}$ $OM \cdot CM^2$ up to $2 \cdot 10^{-5}$ $OM \cdot CM^2$ . These values are typical of the Josephson functions.

As is known, in a pure two-dimensional system the phase fluctuations destroy a long-range order in a superconductor. Nevertheless electron transitions between the layers completely suppress those fluctuations, the order parameter in three layers being independent of electron transition probabilities between the layers /4/ . Indeed, as has been shown by Dzyaloshinsky and Kats /5/, even a very small overlapping of the electron wave functions of adjacent layers leads to complete suppression of the phase fluctuations destroying a furthermost order of the two-dimensional system.

## THE RESULTS AND THEIR DISCUSSION

Let us consider a multi-layer structure ("N-sandwich") as a semimacroscopic quantum system in which N tunnelings proceed. The total wave function satisfies the equation

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \cdot \Psi, \tag{I}$$

where $\hat{H}$ is the Hamiltonian operator of the considered system. Following Feynman /6/ the systems of Copper pairs (without taking account of recombination and decay of pairs into electrons with transition into the continuous spectrum) will be treated as a two-level quantum system. Under the assumption of discreteness of states of the system ( $\alpha$ = 1,2,..., $N$ ) the wave function $|\Psi_\alpha\rangle$ is represented as

$$|\Psi(t)\rangle = \sum_\alpha C_\alpha(t) |\Psi_\alpha\rangle. \tag{2}$$

Under the condition of normalisation with the density of the Copper pairs we have $\rho_\alpha = |C_\alpha|^2$ . Then, the Schroedinger equation (I) results in the following system of equations for

$$i\hbar \frac{dC_\alpha}{dt} = \sum_\beta \langle \Psi_\alpha | \hat{H} | \Psi_\beta \rangle C_\beta(t)$$

$$H_{\alpha\beta}^{(n)} = \langle \Psi_\alpha | \hat{H} | \Psi_\beta \rangle = \int \Psi_\alpha^* \cdot \hat{H} \Psi_\beta \cdot dV. \tag{3}$$

Allowance for the interaction only between the nearest layers leads to the following structure of three-diagonal partitioned matrices of the recurrent type:

$$H_{\alpha\beta}^{(n)} = \begin{array}{c} \\ n \\ \\ \end{array} \begin{array}{c} \phantom{0} \quad n \\ \left[ \begin{array}{cccc} 0 & \ddots & & 0 \\ & & \boxed{\begin{array}{cc} U_{n,n} & K_{n,n+1} \\ K_{n+1,n} & U_{n+1,n+1} \end{array}} & \\ & 0 & & \ddots \quad 0 \end{array} \right] \end{array} . \qquad (4)$$

Representing the probability amplitudes as $C_n = \sqrt{\rho_n} \cdot \exp(i\Theta_n)$ taking into account eqs. (3) and (4) we get the system of equations for parameter phase jumps of an order of individual tunnellings in the plane (x,y) model

$$\left( \nabla^2 - c_0^{-2} \cdot \partial^2 t + \lambda_J^{-2} \cdot \sin\left[ \cdot \right] \right) \cdot \begin{bmatrix} \varphi_{1,2} \\ \vdots \\ \varphi_{n,n+1} \\ \varphi_{N-1,N} \end{bmatrix} = 0, \qquad (5)$$

where $\lambda_J^{-2} = \hbar / e \mu_0 \cdot d \cdot j_J(T)$ is the Josephson parameter, $\varphi_{n,n+1} = \Theta_{n+1} - \Theta_n$ is the phase jump from a layer $n$ to a layer (n+1). To derive (5) the "block" form (4) of the matrix $H_{\alpha\beta}^{(n)}$ and the Maxwell local equations a describing superconductive strip resonator for individual layers /4/ were used /6,10/ . Since we are interested in the change of the phase jump in the transverse direction of transition along the $y$ axis (the transition is the plane ( $x-y$ ) (fig.1)), in addition to the Maxwell equation we have the basic equation for the Josephson current in the form /7/ $J_x(y,t) = \lambda_J^{-2} \cdot \sin \varphi_{n-1,n}$ . The projection of the Maxwell equation for the rotor $\vec{B}$ onto the $X$ axis gives

$$\left[ \nabla \times B \right]_x = \mu_0 \cdot J_x + \mu \cdot C_s \cdot \frac{\partial U_{n,n+1}}{\partial t} , \qquad (6)$$

where

$$\frac{\partial \varphi_{n,n+1}}{\partial y} = \frac{e \cdot d}{\hbar} \cdot B_x \quad ,$$

$$\frac{\partial \varphi_{n,n+1}}{\partial t} = \frac{2 \cdot e}{\hbar} U_{n,n+1} ,$$

$$J_x = \frac{\partial S_n}{\partial t} \cdot \lambda_J^{-2} \cdot \sin \varphi_{n,n+1} .$$

$C_0 = (\mu_0 \cdot C_s \cdot d_n)^{-1/2}$ is the velocity of wave propagation in the Josephson distributing junction, $C_s$ is the capacitance per unit of the junction area and $d_n$ is the thickness of the oxide film. Transition from (6) to (5) is obvious. The full change of the phase jump in passing from the first layer to an N layer is defined by

$$(\nabla^2 - c_0^{-2} \cdot \partial^2 t) \cdot \varphi_{1,N} = - J_\Sigma , \qquad (7)$$

$$J_\Sigma = \lambda_J^{-2} \cdot \sum_{i=2}^{N} \sin \varphi_{i-1,i} ,$$

where $J_\Sigma$ is the effective Josephson current for the N layered SDS ..., SDS structure. Note that using (7) one can easily derive the expression found in ref. /8/ for supercurrent in the system of N layers

$$J(\varphi) = \lambda_J^{-2} \cdot \sum_{i=2}^{N} \sin(\varphi_{i-1,i} \cdot (n-1)) \cdot j_{n-1}^{(1)}$$

which has been presented in ref. /8,9/ from phenomenological considerations. The free energy of the system consisting of N layers is represented as

$$F = \frac{\hbar}{e} j_J \cdot \int_{-d/2}^{d/2} dy \int dt \left\{ \frac{1}{2} \lambda_J^2 (T) \cdot \sum_n \left[ (\nabla \varphi_{n,n+1})^2 - \right.\right.$$
$$\left.\left. - c_0^{-2} \cdot (\partial_t \varphi_{n,n+1})^2 - (1 - \cos \varphi_{n,n+1}) \right] + C \sum \cdot \varphi_{n,n+1}^2 \right\} . \qquad (8)$$

As will be shown below, the last term is to be taken into account. Varying (8) with respect to $\varphi_{n,n+1}$ we get

$$\delta F \sim \int_V \Big[ j_J \cdot \Big( \sum_n \sin\varphi_{n,n+1} - \lambda_J^2 \cdot \sum \big( \nabla^2 \varphi_{n,n+1} - c_0^{-2} \cdot \partial^2 t\, \varphi_{n,n+1} \big)\Big)\Big]\, dt \cdot dy \cdot d\varphi_{n,n+1} + \int_S \sum \big( (\nabla_\mu \cdot \varphi_{n,n+1}) \cdot S_\mu + C \cdot \varphi_{n,n+1} \big) \cdot \delta\varphi_{n,n+1} \cdot dS, \quad (9)$$

where $\nabla_\mu = (-\partial t, \partial y)$, $S_\mu$ is the unit vector orthogonal to the surface $\delta\varphi_{n,n+1}$ .

Using the coordinate representation as a "plane" moving wave /10/ $\bar{\tau} = K_\mu \cdot X_\mu$ , $K_\mu = \lambda_J^{-1} \cdot \left(1 - \frac{V^2}{c_0^2}\right)^{-1/2} \cdot \left(\frac{V}{c_0}, \alpha, \beta\right)$, $X_\mu = (t, x, y)$, where $\alpha, \beta$ are the direction cosines to the coordinate axes. Now we choose axes (fig.1)

$$\tau = \bar{\tau} - \tau^0_{n,n+1} = K_\mu \cdot X_\mu = \lambda_J^{-1} \cdot \left(1 - \left(\frac{V}{c_0}\right)^2\right)^{-1/2} \cdot \left[\frac{V}{c_0} t - \alpha (x - x_0) - \beta (y - y_0)\right]$$

at $V = 0$ , $X = X_0$ and $y_0 = 0$ ; thus, the phase jump for the order parameter may be thought to be a function $\varphi_{n,n+1} = \varphi_{n,n+1}(\tau)$, where $\tau = -\lambda_J^{-1} \cdot \beta \cdot y$ . Taking into account $\delta F = 0$ in (9) and the dependence of the function $\varphi_{n,n+1} = \varphi_{n,n+1}(\tau)$ , we get the

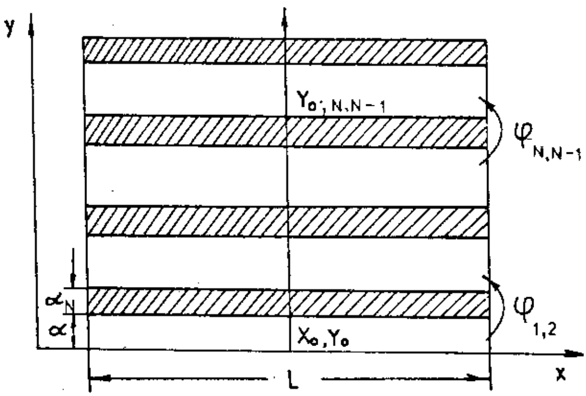


Fig.1. Scheme of the N layer structure (S_D_S).

system of difference equations for the N layered structure with the corresponding boundary conditions on the surface of separation of layers

$$\partial^2 z \cdot \varphi_{n,n+1}(z) = -\sin[\varphi_{n,n+1}(z)]$$
$$\partial z \cdot \varphi_{n,n+1}\Big|_{y=y^{\circ}_{n,n+1}} = c \cdot \varphi_{n,n+1} . \qquad (10)$$

Allowing for the fact that in (10) the jump of the order parameter phase changes slightly at the layer boundary at $y^{\circ}_{n,n+1} = n \cdot \bar{d}$, where $\bar{d} = d + d_n$ is the thickness of the film consisting of the layer of a superconductor of thickness $d$ and oxide layers of thickness $d_n$, $\varphi_{n,n+1} \sim O(\varphi_{n,n+1})$, we have $\sin \varphi_{n,n+1} \simeq \varphi_{n,n+1}$. Then, the system of the Sin-Gordon equations (5) transforms into the system of linearised equations with the boundary conditions defined by the constant $c$

$$\partial^2 z \cdot \varphi_{n,n+1} = -\varphi_{n,n+1}$$
$$\partial z \cdot \varphi_{n,n+1}\Big|_{y=y^{\circ}_{n,n+1}} = c \cdot \varphi(y^{\circ}_{n,n+1}) . \qquad (11)$$

The solution (11) has the form

$$\varphi_{n,n+1} = \cos(\beta \cdot y / \lambda_J) . \qquad (12)$$

Taking account of (12) we can write the boundary conditions in (11) as

$$\frac{\beta}{\lambda_J} \cdot tg\left(\frac{\beta \cdot y^{\circ}_{n,n+1}}{\lambda_J}\right) = c . \qquad (13)$$

The transcendental equation (13) is solved with respect to $\lambda_J$ by the iteration method. As a result, we get the characteristics of $T_c(n)$. Without loss of generality we consider (13) in the case of $\lambda_J^{-1} \cdot \beta \cdot y^{\circ}_{n,n+1} \ll 1$. Then,

$$\frac{n\,(d+d_n)}{c} = \lambda_J^{-2} \qquad (14)$$

at $\beta = 1$ the current is perpendicular to the layer surface.

Using the expression for the temperature dependence $\lambda_J(T)$ from the BCS theory /11/ we get $\lambda_J(T)$

$$\lambda_J(T) = \left(\frac{\hbar}{e\mu_0 \cdot d}\right) \frac{q \cdot R_{nn}}{\pi} \cdot \frac{1}{\Delta(T) \cdot th\,(\Delta(T)/2K \cdot T)} \tag{15}$$

$$\Delta(T)\Big|_{T=T_c} \simeq 3{,}15 \cdot K \cdot T_c \cdot (1 - T/T_c)^{1/2},$$

where $R_{nn}$ is the tunnelling resistance per unit of the transition area provided that both the metals are in the normal state /11/ and $\Delta(T)$ is the energy gap width at the temperature $T$ $(o<T<T_c)$.

Considering (14),(15) near $T_c$ we get

$$\frac{n(d+d_n)}{d} = \lambda_J^2(0) \cdot A^2 \cdot \frac{T}{T_c^2} \cdot \left(1 - \frac{T}{T_c}\right)^{-1}, \tag{16}$$

$$\lambda_J^2(0) = \frac{\hbar \cdot R_{nn}}{\mu_0 \cdot d \cdot \pi}, \quad A^2 = 2/(3{,}15^2 \cdot K).$$

From (16) we get the temperature dependence $T_c(n)$ on the number of layers $n$ and thickness of the oxidemetal layer

$$T_c^{(n)} = T_c^{(max)} \cdot \left[1 + \lambda_J^2(0) \cdot \frac{d \cdot A^2}{T_c^{(max)} \cdot (d+d_n) \cdot n}\right]^{-1}, \tag{17}$$

where N is the maximal number of layers at which in fact $T_c^{max} \simeq T_c^{(n)}\Big/_{n-N}$ $(N=8\div10)$. Now we rewrite (17) introducing the film thickness $\tilde{d}_n = d_n + d$ :

$$T_c^{(n)} = T_c^{(max)} \cdot \left[1 + \lambda_J^2(0) \frac{const}{T_c^{(max)} \cdot \tilde{d}_n \cdot n}\right]^{-1}. \tag{18}$$

In eq. (18) $const$ is a constant depending on the type of the oxidemetal structure with dimension $(L^{-1} \cdot T)$. Thus, for the system $Nb - NbO_x - Nb$, const $\sim \lambda^{-1}(0) \cdot T_c^{(max)}$ see ref. /12/. The solution (18) can be compared with that of the total transcendental equation (13)

$$tg\left[\frac{n(d+d_n)}{A \cdot (T_c - T)^{1/2}} \cdot \lambda_J^{-1}(0) \cdot \left(\frac{T}{T_c}\right)^{1/2}\right] = \lambda_J(0) \cdot c \cdot A \left[\frac{T}{T_c(T_c - T)}\right]^{1/2}. \tag{19}$$

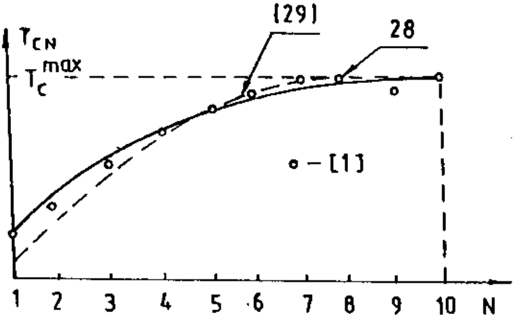


Fig.2. Dependence of $T_c(N)$ on $N$ given by (18),(20). Experiment [1] is denoted by the points – o –[1].

The diagrams of solutions (18) and (19) are given in fig.2. Thus, in our case the quasi-two-dimensional superconductivity is the current between the layers not violating superconductivity inside the layers. The current density $j_{n,n+1}$ between the layers $n$ and $n+1$ can be calculated by perturbation theory under the assumption that the order parameters of the layers $\Delta_n$ and $\Delta_{n+1}$ are constant inside the layer $n$. As in the case of usual Josephson junctions, the current is determined as $j_{n,n+1} = J_c(\Delta) \cdot \sin(\varphi_{n+1} - \varphi_n)$, where $\varphi_n$ is the phase of the order parameter in the layer $n$.

Recently, in the case of metal superlattices within the model of inhomogeneous electron-electron interaction Shapiro and Efimova have found the dependences of the transition temperature into a superconducting state for the quasi-two-dimensional structure depending on the number of layers (N) and their width $d$ /12/.

The free parameter of the theory $C$ (see the boundary condition at the boundary of separation of layers) allows one to take into account the specific properties of the N layered structure, especially, a type of a good superconducting material and layer.

In conclusion, we should like to note that the calculation of $T_c(n)$ in the N layered SDS structure is not associated with any mechanism providing pairing, for instance, with an electron interaction.

Apparently, this fact will be useful when investigating the physical properties of N-layer structures in high-$T_c$ superconducters.

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