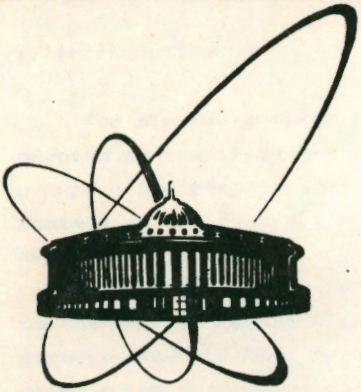

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E17-90-389

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PERMUTATIONAL MAGNETIC POINT GROUPS AND THEIR APPLICATION IN THE LANDAU THEORY OF PHASE TRANSITIONS

Transitive Permutational Representations

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1990

1. Introduction

The present article is the last of a series of articles[1,2] devoted to the study of a new type of colour crystallographic groups - the permutational magnetic groups. The results and the content of the tables should be used together with the tables from Ref.1,2.

The construction of the transitive permutational representations is a basic stage in the construction of the colour permutational groups. Here we have implemented mainly the method proposed by Van-der-Waerden , which is more convenient for computer calculations. As an alternative method we used the algorithm, based on the theory of induced representations[4,5].This is possible, because every transitive permutational representation $D_G^{H'}$,generated by the subgroup $H' \subset G$, is equivalent to that representation, induced by the trivial one - $D_{H'}^{1}$, of the subgroup $H' \subset G$. The purpose of our work was to obtain colour groups and representations, useful for symmetry analysis of magnetic phase transitions (basing on our previous works[6-10]).

2. Construction of the transitive representations

A specific program has been designed in Turbo Pascal 5.0 for microcomputer IBM-PC in order to construct the transitive representations of all 122 magnetic point groups . Each of the groups G is decomposed to left cosets with respect to fixed group H' - a representative of a class of conjugated subgroups Permutation of the left co-classes under the action from the left side by $g \in G$ models the required permutations $D_G^{H'}(g)$.We obtained 1997 nonequivalent permutational representations for the 122 magnetic point groups , and we classified them into 70 images $\mathrm{Im}\ D_G^{H'} = (A,A')_n$ (Ref.1). The representations are decomposed to irreducible components using their characters :

$$D_G^{H'} = \sum_j (D_G^{H'} \mid D_G^{j}) D_G^{j} \qquad (1)$$

As the 122 magnetic point groups are isomorphic to 25 abstract groups, it follows that there exist 25 sets of irreducible representations D_G^j and the same number of $D_G^{H'}$. In tables C.1-25 all permutational representations are identified through the number of the table and a specific row number (for example C.14-28 means $D^{D_6}_{D_{6h}}$). The ordered by isomorphism groups and subgroups have the same number and constitute the symbol of the permutational colour group in Ref.2 (for example A.14-28 means not only D_{6h}/D_6, but also through an isomorphism (see Tab. A.14 in Ref.2) $D_{6h}(D_{3d})/D_6(D_3)$; $D_6 \times \Theta/D_6$, $D_{3d} \times \Theta/D_{3d}$, $D_{6h}(C_{6v})/D_6(C_6)$, etc. For the sake of briefness there are presented just the subgroups of only one representative of a class of isomorphic groups in the first column, and the notation of the irreps is related to the leading group representative. In tables C.19-25 are shown the representations of the 7 centrosymmetrical grey groups, which have no more than one representative. The number of the chromomorphic class, in which they fall according to Table 1 in Ref.1., is placed for those groups in the column 'No cl'.

In tables C.1-25 the zeros are replaced with points, and the numbers present the nonzero values - the multiplicities of a fixed irrep D_G^j in the decomposition of $D_G^{H'}$ (Eq.(1)).

3. Application to the Landau's Theory

As it was shown for the first time from Birman[11,12], in a phase transition with a symmetry breaking $G \longrightarrow H' \subset G$ the active symmetry breaking irrep D_G^j must satisfy the subduction criterion $(D_G^j | D_{H'}^1) \neq 0$. From the Frobenius Reciprocity Theorem follows, that D_G^j is one of the irreps, contained in a representation of G, induced from the trivial one $D_{H'}^1$ of H' (the same result is used in Ref.7-10,13). As $D_{H'}^1 \uparrow G \cong D_G^{H'}$, the role of the tabulated representations for analysis of magnetic phase transitions is obvious. More profound results can be obtained with the help of colour groups[7-10].

As a specific test we shall discuss a classical example - magnetic phase transitions in a crystal αFe_2O_3.

Tab. C.1

C_1/	Γ_1
C_1	1

Tab. C.2

C_2/	No	Γ. 12
C_1	1	11
C_2	2	1.

Tab. C.3

C_3/	No	Γ. 123
C_1	1	111
C_3	2	1..

Tab. C.4

C_4/	No	Γ. 1234
C_1	1	1111
C_2	2	11..
C_4	3	1...

Tab. C.5

C_6/	No	Γ. 123456
C_1	1	111111
C_2	2	111...
C_3	3	1..1..
C_6	4	1.....

Tab. C.6

C_{4h}/	No	Γ.g 1234	Γ.u 1234
C_1	1	1111	1111
C_2	2	11..	11..
C_s	3	11..	..11
C_i	4	1111	....
S_4	5	1...	.1..
C_4	6	1...	1...
C_{2h}	7	11..	....
C_{4h}	8	1...	....

Tab. C.8

D_2/	No	Γ. 1234
C_1	1	1111
C_2'	2	1..1
C_2	3	1.1.
C_2''	4	11..
D_2	5	1...
$C_2=C_2^z$		
$C_2'=C_2^x$ $C_2''=C_2^y$		

Tab. C.7

C_{6h}/	No	Γ.g 123456	Γ.u 123456
C_1	1	111111	111111
C_s	2	1.11..	.1..11
C_2	3	1.11..	1.11..
C_i	4	111111	......
C_3	5	11....	11....
C_{2h}	6	1.11..	......
C_{3h}	7	1.....	.1....
C_6	8	1.....	1.....
S_6	9	11....	......
C_{6h}	10	1.....	......

Tab. C.9

D_{2h}/	No	Γ.g 1234	Γ.u 1234
C_1	1	1111	1111
C_i	2	1111	....
C_2''	3	11..	11..
C_2	4	1.1.	1.1.
C_2'	5	1..1	1..1
C_s''	6	11..	..11
C_s	7	1.1.	.1.1
C_s'	8	1..1	1..1
C_{2v}'	9	1...	...1
C_{2v}	10	1...	..1.
C_{2v}''	11	1...	.1..
D_2	12	1...	1...
C_{2h}'	13	1..1	....
C_{2h}	14	1.1.	....
C_{2h}''	15	11..	....
D_{2h}	16	1...	....
$C_2=C_2^z$ $C_2'=C_2^x$ $C_2''=C_2^y$			

Tab. C.11

D_4/	No	Γ. 12345
C_1	1	11112
C_2'/C_1	2	1.1.1
C_2''/C_1	3	1..11
C_2	4	1111.
D_2'	5	1..1.
D_2	6	1.1..
C_4	7	11...
D_4	8	1....

Tab. C.12

D_{4h}/	No	Γ.g 12345	Γ.u 12345
C_1	1	11112	11112
C_2'/C_1	2	1.1.1	1.1.1
C_2''/C_1	3	1..11	1..11
C_s'/C_1	4	1.1.1	.1.11
C_s''/C_1	5	1..11	.11.1
C_s	6	1111.	2
C_{2v}''/C_s	7	1.1..	1
C_{2v}'/C_s	8	1..1.	1
C_i	9	11112	.....
C_{2h}'/C_i	10	1.1.1	.....
C_{2h}''/C_i	11	1..11	.....
C_2	12	1111.	1111.
C_4	13	11...	11...
S_4	14	11...	..11.
D_2	15	1.1..	1.1..
C_{2v}	16	1.1..	.1.1.
D_2'	17	1..1.	1..1.
C_{2v}'	18	1..1.	.11..
C_{2h}	19	1111.	.....
D_{2d}'	20	1....	...1.
D_{2d}	21	1....	..1..
C_{4v}	22	1....	.1...
D_4	23	1....	1....
D_{2h}'	24	1..1.	.....
D_{2h}	25	1.1..	.....
C_{4h}	26	11...	.....
D_{4h}	27	1....	.....

Tab. C.10

D_3/	No	Γ. 123
C_1	1	112
C_2/C_1	2	1.1
C_3	3	11.
D_3	4	1..

Tab. C.13

D_6/	No	Γ. 123456
C_1	1	111122
C_2'/C_1	2	1.1.11
C_2''/C_1	3	1..111
C_2	4	11..2.
D_2/C_2	5	1...1.
C_3	6	1111..
D_3'	7	1..1..
D_3	8	1.1...
C_6	9	11....
D_6	10	1.....

$C_2'=C_2^y$ $C_2''=C_2^x$

Tab. C.14

D_{6h} /	No	Γ.g 123456	Γ.u 123456
C_1	1	111122	111122
C_2'/C_1	2	1.1.11	1.1.11
C_s'/C_1	3	1.1.11	.1.111
C_2''/C_1	4	1..111	1..111
C_s''/C_1	5	1..111	.11.11
C_2	6	11...2	11...2
D_2/C_2	7	1....1	1....1
C_{2v}/C_2	8	1....1	.1...1
C_s	9	11...2	..112.
C_{2v}'/C_s	10	1....1	..1.1.
C_{2v}''/C_s	11	1....1	...11.
C_i	12	111122	.....
C_{2h}'/C_i	13	1.1.11	.....
C_{2h}''/C_i	14	1..111	.....
C_{2h}	15	11...2	.....
D_{2h}/C_{2h}	16	1....1	.....
C_3	17	1111..	1111..
C_6	18	11....	11....
C_{3h}	19	11....	..11..
D_3	20	1.1...	1.1...
C_{3v}	21	1.1...	.1.1..
D_3'	22	1..1..	1..1..
C_{3v}'	23	1..1..	.11...
S_6	24	1111..	.....
D_{3d}'	25	1.....	...1..
D_{3d}	26	1.....	..1...
C_{6v}	27	1.....	.1....
D_6	28	1.....	1.....
D_{3h}'	29	1..1..	.....
D_{3h}	30	1.1...	.....
C_{6h}	31	11....	.....
D_{6h}	32	1.....	.....

Tab. C.16

T_h/	No	Γ.g 1234	Γ.u 1234
C_1	1	1113	1113
C_2/C_1	2	1111	1111
C_s/C_1	3	1111	...2
C_3/C_1	4	1..1	1..1
C_{2v}/C_1	5	111.	...1
C_i	6	1113	....
C_{2h}/C_i	7	1111	....
S_6/C_i	8	1..1	....
D_2	9	111.	111.
T	10	1...	....
D_{2h}	11	111.	....
T_h	12	1...	....

Tab. C.15

T/	No	Γ. 1234
C_1	1	1113
C_2/C_1	2	1111
C_3/C_1	3	1..1
D_2	4	111.
T	5	1...

Tab. C.18

O_h/	No	Γ.g 12345	Γ.u 12345
C_1	1	11233	11233
C_2/C_1	2	11211	11211
C_s/C_1	3	11211	...22
C_2'/C_1	4	1.112	1.112
C_s'/C_1	5	1.112	.1121
C_3/C_1	6	11.11	11.11
C_{2v}/C_1	7	112..	...11
C_4/C_1	8	1.11.	1.11.
S_4/C_1	9	1.11.	.11.1
D_2'/C_1	10	1.1.1	1.1.1
C_{2v}'/C_1	11	1.1.1	.111.
C_{2v}''/C_1	12	1.1.1	...11
D_3/C_1	13	1...1	1...1
C_{3v}/C_1	14	1...1	.1.1.
C_{4v}/C_1	15	1.1..	...1.
D_{2d}'/C_1	16	1.1..	1
C_i	17	11233	.....
C_{2h}/C_i	18	11211	.....
C_{2h}'/C_i	19	1.112	.....
S_6/C_i	20	11.11	.....
C_{4h}/C_i	21	1.11.	.....
D_{2h}'/C_i	22	1.1.1	.....
D_{3d}/C_i	23	1...1	.....
D_2	24	112..	112..
D_4/D_2	25	1.1..	1.1..
D_{2d}/D_2	26	1.1..	.11..
D_{2h}	27	112..	.....
D_{4h}/D_{2h}	28	1.1..	.....
T	29	11...	11...
T_d	30	1....	.1...
O	31	1....	1....
T_h	32	11...	.....
O_h	33	1....	.....

Tab. C.19

No	C_{4h}^* /	No cl	Γ^+.g 1234	Γ^+.u 1234	Γ^-.g 1234	Γ^-.u 1234
1	C_1	16	1111	1111	1111	1111
2	C_2	9	11..	11..	11..	11..
3	$C_2(C_1)$	6	11..	11..	..11	..11
4	C_s	6	11..	..11	11..	..11
5	$C_s(C_1)$	6	11..	..11	..11	11..
6	$C_i(C_1)$	6	1111	....	....	1111
7	C_i	6	1111	....	1111	....
8	C_1^*	6	1111	1111	....	....
9	C_4	8	1...	1...	1...	1...
10	$C_4(C_2)$	8	1...	1...	.1..	.1..
11	S_4	8	1...	.1..	1...	.1..
12	$S_4(C_2)$	8	1...	.1..	.1..	1...
13	$C_{2h}(C_s)$	4	11..	....	....	..11
14	$C_{2h}(C_2)$	8	11..	....	....	11..
15	$C_{2h}(C_i)$	4	11..	....	..11	....
16	C_{2h}	8	11..	....	11..	....
17	C_s^*	4	11..	..11	....	....
18	C_2^*	8	11..	11..	....	....
19	C_i^*	4	1111	....	....	....
20	$C_{4h}(S_4)$	2	1...	....	....	.1..
21	$C_{4h}(C_4)$	2	1...	....	....	1...
22	$C_{4h}(C_{2h})$	2	1...	....	.1..	....
23	C_{4h}	2	1...	....	1...	....
24	S_4^*	2	1...	.1..	....	....
25	C_4^*	2	1...	1...	....	....
26	C_{2h}^*	2	11..	....	....	....
27	C_{4h}^*	1	1...	....	....	....

Tab. C.20

No	C_{6h}^* /	No cl	Γ^+.g 123456	Γ^+.u 123456	Γ^-.g 123456	Γ^-.u 123456
1	C_1	47	111111	111111	111111	111111
2	C_2	27	1.11..	1.11..	1.11..	1.11..
3	$C_2(C_1)$	27	1.11..	1.11..	.1..11	.1..11
4	C_s	27	1.11..	.1..11	1.11..	.1..11
5	$C_s(C_1)$	27	1.11..	.1..11	.1..11	1.11..
6	$C_i(C_1)$	27	111111	.....	.....	111111
7	C_i	27	111111	.....	111111	.....
8	C_1^*	27	111111	111111	.....	.....
9	C_3	9	11....	11....	11....	11....
10	$C_{2h}(C_s)$	5	1.11..	.....	.....	.1..11
11	$C_{2h}(C_2)$	5	1.11..	.....	.....	1.11..
12	$C_{2h}(C_i)$	5	1.11..	.....	.1..11	.....
13	C_{2h}	5	1.11..	.....	1.11.	.....
14	C_s^*	5	1.11..	.1..11	.....	.....
15	C_2^*	5	1.11..	1.11..	.....	.....
16	C_i^*	5	111111	.....	.....	.....
17	C_6	8	1.....	1.....	1.....	1.....
18	$C_6(C_3)$	8	1.....	1.....	.1....	.1....
19	C_{3h}	8	1.....	.1....	1.....	.1....
20	$C_{3h}(C_3)$	8	1.....	.1....	.1....	1.....
21	$S_6(C_3)$	8	11....	.....	.....	11....
22	S_6	8	11....	.....	11....	.....
23	C_3^*	8	11....	11....	.....	.....
24	C_{2h}^*	3	1.11..	.....	.....	1.....
25	$C_{6h}(C_{3h})$	2	1.....	.....	.....	.1....
26	$C_{6h}(C_6)$	2	1.....	.....	.....	1.....
27	$C_{6h}(S_6)$	2	1.....	.....	.1....	.....
28	C_{6h}	2	1.....	.....	1.....	.....
29	C_{3h}^*	2	1.....	.1....	.....	.....
30	C_6^*	2	1.....	1.....	.....	.....
31	S_6^*	2	11....	.....	.....	.....
32	C_{6h}^*	1	1.....	.....	.....	.....

Tab. C.21

No	D_{2h}^{*} /	No cl	Γ^{+}_{g} 1234	Γ^{+}_{u} 1234	Γ^{-}_{g} 1234	Γ^{-}_{u} 1234
1	C_1	48	1111	1111	1111	1111
2	C_2'	9	11..	11..	11..	11..
3	C_2	9	1.1.	1.1.	1.1.	1.1.
4	C_2''	9	1..1	1..1	1..1	1..1
5	C_s'	9	11..	..11	11..	..11
6	C_s	9	1.1.	.1.1	1.1.	.1.1
7	C_s''	9	1..1	.11.	1..1	.11.
8	$C_2'(C_1)$	9	11..	11..	..11	..11
9	$C_2(C_1)$	9	1.1.	1.1.	.1.1	.1.1
10	$C_2''(C_1)$	9	1..1	1..1	.11.	.11.
11	$C_s'(C_1)$	9	11..	..11	..11	11..
12	$C_s(C_1)$	9	1.1.	.1.1	.1.1	1.1.
13	$C_s''(C_1)$	9	1..1	.11.	.11.	1..1
14	$C_i(C_1)$	9	1111	....	....	1111
15	C_i	9	1111	....	1111	....
16	C_1^{*}	9	1111	1111	....	....
17	D_2	8	1...	1...	1...	1...
18	$D_2(C_2')$	8	1...	1...	.1..	.1..
19	$D_2(C_2)$	8	1...	1...	..1.	..1.
20	$D_2(C_2'')$	8	1...	1...	...1	...1
21	C_{2v}'	8	1...	.1..	1...	.1..
22	$C_{2v}'(C_2')$	8	1...	.1..	.1..	1...
23	$C_{2v}'(C_s)$	8	1...	.1..	..1.	...1
24	$C_{2v}'(C_s'')$	8	1...	.1..	...1	..1.
25	C_{2v}	8	1...	..1.	1...	..1.
26	$C_{2v}(C_s')$	8	1...	..1.	.1..	...1
27	$C_{2v}(C_2)$	8	1...	..1.	..1.	1...
28	$C_{2v}(C_s'')$	8	1...	..1.	...1	.1..
29	C_{2v}''	8	1...	...1	1...	...1
30	$C_{2v}''(C_s')$	8	1...	...1	.1..	..1.
31	$C_{2v}''(C_s)$	8	1...	...1	..1.	.1..
32	$C_{2v}''(C_2'')$	8	1...	...1	...1	1...
33	$C_2''^{*}$	8	11..	11..	....	....
34	$C_s''^{*}$	8	11..	..11	....	....

Tab. C.21 (continued)

No	D_{2h}^{*} /	No cl	Γ^{+}_{g} 1234	Γ^{+}_{u} 1234	Γ^{-}_{g} 1234	Γ^{-}_{u} 1234
35	C_{2h}'	8	11..	....	11..	....
36	$C_{2h}'(C_i)$	8	11..	....	..11	....
37	$C_{2h}'(C_2')$	8	11..	....	....	11..
38	$C_{2h}'(C_s')$	8	11..	....	....	..11
39	$C_2''^{*}$	8	1.1.	1.1.	....	....
40	$C_s''^{*}$	8	1.1.	.1.1	....	....
41	C_{2h}	8	1.1.	....	1.1.	....
42	$C_{2h}(C_i)$	8	1.1.	....	.1.1	....
43	$C_{2h}(C_2)$	8	1.1.	....	....	1.1.
44	$C_{2h}(C_s)$	8	1.1.	....	....	.1.1
45	$C_2''^{*}$	8	1..1	1..1	....	....
46	$C_s''^{*}$	8	1..1	.11.	....	....
47	C_{2h}''	8	1..1	....	1..1	....
48	$C_{2h}''(C_i)$	8	1..1	....	.11.	....
49	$C_{2h}''(C_2'')$	8	1..1	....	....	1..1
50	$C_{2h}''(C_s'')$	8	1..1	....	....	.11.
51	C_i^{*}	8	1111	....	....	....
52	$D_{2h}(C_{2v}'')$	2	1...	....	....	...1
53	$D_{2h}(C_{2v}')$	2	1...	....	....	..1.
54	$D_{2h}(C_{2v}')$	2	1...	....	....	.1..
55	$D_{2h}(D_2)$	2	1...	....	....	1...
56	$D_{2h}(C_{2h}'')$	2	1...	....	...1	....
57	$D_{2h}(C_{2h})$	2	1...	....	..1.	....
58	$D_{2h}(C_{2h}')$	2	1...	....	.1..	....
59	D_{2h}	2	1...	....	1...	....
60	$C_{2v}''^{*}$	2	1...	...1	....	....
61	$C_{2v}''^{*}$	2	1...	..1.	....	....
62	$C_{2v}''^{*}$	2	1...	.1..	....	....
63	$D_2''^{*}$	2	1...	1...	....	....
64	$C_{2h}''^{*}$	2	1..1	....	....	....
65	$C_{2h}''^{*}$	2	1.1.	....	....	....
66	$C_{2h}''^{*}$	2	11..	....	....	....
67	$D_{2h}''^{*}$	1	1...	....	....	....

NOTE: $C'=C^{X}$ $C''=C^{Y}$ $C=C^{Z}$

Tab. C.22

No	D_{4h}^{*} /	No cl	Γ^{+}_{g} 12345	Γ^{+}_{u} 12345	Γ^{-}_{g} 12345	Γ^{-}_{u} 12345
1	C_1	49	11112	11112	11112	11112
2	C_2'/C_1	50	1.1.1	1.1.1	1.1.1	1.1.1
3	$C_2''(C_1)/C_1$	50	1.1.1	1.1.1	.1.11	.1.11
4	C_s'/C_1	50	1.1.1	.1.11	1.1.1	.1.11
5	$C_s''(C_1)/C_1$	50	1.1.1	.1.11	.1.11	1.1.1
6	C_2''/C_1	50	1..11	1..11	1..11	1..11
7	$C_2''(C_1)/C_1$	50	1..11	1..11	.11.1	.11.1
8	C_s''/C_1	50	1..11	.11.1	1..11	.11.1
9	$C_s''(C_1)/C_1$	50	1..11	.11.1	.11.1	1..11
10	$C_2''(C_1)$	14	1111.	1111.	2	2
11	$D_2(C_2')/C_2(C_1)$	15	1.1..	1.1..	1	1
12	$C_{2v}'(C_s')/C_2(C_1)$	15	1.1..	.1.1.	1	1
13	$D_2'(C_2'')/C_2(C_1)$	15	1..1.	1..1.	1	1
14	$C_{2v}''(C_s'')/C_2(C_1)$	15	1..1.	.11..	1	1
15	C_s	14	1111.	2	1111.	2
16	C_{2v}''/C_s	15	1.1..	1	1.1..	1
17	$C_{2v}''(C_s')/C_s$	15	1.1..	1	.1.1.	1
18	C_{2v}''/C_s	15	1..1.	1	1..1.	1
19	$C_{2v}''(C_s'')/C_s$	15	1..1.	1	.11..	1
20	$C_s(C_1)$	14	1111.	2	2	1111.

Tab. C.22 (continued)

No	D_{4h}^{*} /	No cl	Γ^{+}_{g} 12345	Γ^{+}_{u} 12345	Γ^{-}_{g} 12345	Γ^{-}_{u} 12345
21	$C_{2v}''(C_2')/C_s(C_1)$	15	1.1..	1	1	1.1..
22	$C_{2v}''(C_s')/C_s(C_1)$	15	1.1..	1	1	.1.1.
23	$C_{2v}''(C_2'')/C_s(C_1)$	15	1..1.	1	1	1..1.
24	$C_{2v}''(C_s'')/C_s(C_1)$	15	1..1.	1	1	.11..
25	$C_1(C_1)$	14	11112	.....	.....	11112
26	$C_{2h}'(C_2')/C_1(C_1)$	15	1.1.1	.....	.....	1.1.1
27	$C_{2h}'(C_s')/C_1(C_1)$	15	1.1.1	.....	.....	.1.11
28	$C_{2h}''(C_2'')/C_1(C_1)$	15	1..11	.....	.....	1..11
29	$C_{2h}''(C_s'')/C_1(C_1)$	15	1..11	.....	.....	.11.1
30	C_i	14	11112	.....	11112	.....
31	C_{2h}'/C_1	15	1.1.1	.....	1.1.1	.....
32	$C_{2h}(C_1)/C_1$	15	1.1.1	.....	.1.11	.....
33	C_{2h}''/C_1	15	1..11	.....	1..11	.....
34	$C_{2h}''(C_1)/C_1$	15	1..11	.....	.11.1	.....
35	C_1^{*}	14	11112	11112	.....	.....
36	$C_2''^{*}/C_1$	15	1.1.1	1.1.1	.....	.....
37	$C_s''^{*}/C_1$	15	1.1.1	.1.11	.....	.....
38	$C_2''^{*}/C_1$	15	1..11	1..11	.....	.....
39	$C_s''^{*}/C_1$	15	1..11	.11.1	.....	.....
40	$C_{2h}(C_s)$	12	1111.	.....	.....	2

Tab. C.22 (continued)

No	D_{4h}^* /	No cl	Γ^+_g 12345	Γ^+_u 12345	Γ^-_g 12345	Γ^-_u 12345
41	$D_{2h}(C_{2v}'')/C_{2h}(C_s)$	13	1.1..	.....	.....	1
42	$D_{2h}'(C_{2v}\hat{})/C_{2h}(C_s)$	13	1..1.	.....	.....	1
43	$C_{2h}(C_i)$	12	1111.	.....	2	.....
44	$D_{2h}(C_{2h}')/C_{2h}(C_i)$	13	1.1..	.....	1	.....
45	$D_{2h}'(C_{2h}'')/C_{2h}(C_i)$	13	1..1.	.....	1	.....
46	C_s^*	12	1111.	2	.....	.....
47	$C_{2v}''^*/C_s^*$	13	1.1..	1	.....	.....
48	$C_{2v}\hat{}^*/C_s^*$	13	1..1.	1	.....	.....
49	C_i^*	12	1111 2	.....	.....	.....
50	$C_{2h}'^*/C_i^*$	13	1.1.1	.....	.....	.....
51	$C_{2h}''^*/C_i^*$	13	1..1 1	.....	.....	.....
52	C_2	48	1111.	1111.	1111.	1111.
53	C_4	9	11...	11...	11...	11...
54	$C_4(C_2)$	9	11...	11...	..11.	..11.
55	S_4	9	11...	..11.	11...	..11.
56	$S_4(C_2)$	9	11...	..11.	..11.	11...
57	D_2	9	1.1..	1.1..	1.1..	1.1..
58	$D_2(C_2)$	9	1.1..	1.1..	.1.1.	.1.1.
59	C_{2v}	9	1.1..	.1.1.	1.1..	.1.1.
60	$C_{2v}(C_2)$	9	1.1..	.1.1.	.1.1.	1.1..

Tab. C.22 (continued)

No	D_{4h}^* /	No cl	Γ^+_g 12345	Γ^+_u 12345	Γ^-_g 12345	Γ^-_u 12345
61	D_2'	9	1..1.	1..1.	1..1.	1..1.
62	$D_2'(C_2)$	9	1..1.	1..1.	.11..	.11..
63	C_{2v}'	9	1..1.	.11..	1..1.	.11..
64	$C_{2v}'(C_2)$	9	1..1.	.11..	.11..	1..1.
65	$C_{2h}(C_2)$	9	1111.	.....	.....	1111.
66	C_{2h}	9	1111.	.....	1111.	.....
67	C_2^*	9	1111.	1111.	.....	.....
68	D_4	8	1....	1....	1....	1....
69	$D_4(C_4)$	8	1....	1....	.1...	.1...
70	$D_4(D_2)$	8	1....	1....	..1..	..1..
71	$D_4(D_2')$	8	1....	1....	...1.	...1.
72	C_{4v}	8	1....	.1...	1....	.1...
73	$C_{4v}(C_4)$	8	1....	.1...	.1...	1....
74	$C_{4v}(C_{2v})$	8	1....	.1...	..1..	...1.
75	$C_{4v}(C_{2v}')$	8	1....	.1...	...1.	..1..
76	D_{2d}	8	1....	..1..	1....	..1..
77	$D_{2d}(S_4)$	8	1....	..1..	.1...	...1.
78	$D_{2d}(D_2)$	8	1....	..1..	..1..	1....
79	$D_{2d}(C_{2v}')$	8	1....	..1..	...1.	.1...
80	D_{2d}'	8	1....	...1.	1....	...1.

Tab. C.22 (continued)

No	D_{4h}^* /	No cl	Γ^+_g 12345	Γ^+_u 12345	Γ^-_g 12345	Γ^-_u 12345
81	$D_{2d}'(S_4)$	8	1....	...1.	.1...	..1..
82	$D_{2d}'(C_{2v})$	8	1....	...1.	..1..	.1...
83	$D_{2d}'(D_2')$	8	1....	...1.	...1.	1....
84	$C_{4h}(S_4)$	8	11...	.....	.....	..11.
85	$C_{4h}(C_4)$	8	11...	.....	.....	11...
86	$C_{4h}(C_{2h})$	8	11...	.....	..11.	.....
87	C_{4h}	8	11...	.....	11...	.....
88	S_4^*	8	11...	..11.	.....	.....
89	C_4^*	8	11...	11...	.....	.....
90	$D_{2h}(C_{2v})$	8	1.1..	.....	.....	.1.1.
91	$D_{2h}(D_2)$	8	1.1..	.....	.....	1.1..
92	$D_{2h}(C_{2h})$	8	1.1..	.....	.1.1.	.....
93	D_{2h}	8	1.1..	.....	1.1..	.....
94	C_{2v}^*	8	1.1..	.1.1.	.....	.....
95	D_2^*	8	1.1..	1.1..	.....	.....
96	$D_{2h}'(D_2')$	8	1..1.	.....	.....	1..1.
97	$D_{2h}'(C_{2v}')$	8	1..1.	.....	.....	.11..
98	D_{2h}'	8	1..1.	.....	1..1.	.....
99	$D_{2h}'(C_{2h})$	8	1..1.	.....	.11..	.....
100	$D_2'^*$	8	1..1.	1..1.	.....	.....

Tab. C.22 (continued)

No	D_{4h}^* /	No cl	Γ^+_g 12345	Γ^+_u 12345	Γ^-_g 12345	Γ^-_u 12345
101	$C_{2v}'^*$	8	1..1.	.11..	.....	.....
102	C_{2h}^*	8	1111.	.....	.....	.....
103	$D_{4h}(D_{2d}')$	2	1....	.....	.....	...1.
104	$D_{4h}(D_{2d})$	2	1....	.....	.....	..1..
105	$D_{4h}(C_{4v})$	2	1....	.....	.....	.1...
106	$D_{4h}(D_4)$	2	1....	.....	.....	1....
107	$D_{4h}(D_{2h}')$	2	1....	.....	...1.	.....
108	$D_{4h}(D_{2h})$	2	1....	.....	..1..	.....
109	$D_{4h}(C_{4h})$	2	1....	.....	.1...	.....
110	D_{4h}	2	1....	.....	1....	.....
111	$D_{2d}'^*$	2	1....	...1.	.....	.....
112	D_{2d}^*	2	1....	..1..	.....	.....
113	C_{4v}^*	2	1....	.1...	.....	.....
114	D_4^*	2	1....	1....	.....	.....
115	$D_{2h}'^*$	2	1..1.	.....	.....	.....
116	D_{2h}^*	2	1.1..	.....	.....	.....
117	C_{4h}^*	2	11...	.....	.....	.....
118	D_{4h}^*	1	1....	.....	.....	.....

$C_2=C_2^z; C_2'=C_2^x; C_2''=C_2^y; C_{2h}=2^x/m^x; C_{2h}''=2^{xy}/m^{xy}$

$C_{2v}=m^x m^y 2^z; C_{2v}'=m^{xy} m^{\bar{x}y} 2^z; C_{2v}''=2^x m^y m^z; C_{2v}\hat{}=2^{xy} m^{\bar{x}y} m^z; D_2=2^x 2^y 2^z; D_2'=2^{xy} 2^{\bar{x}y} 2^z.$

Tab. C.17

O/	No	Γ. 12345
C_1	1	11233
C_2/C_1	2	11211
C_2'/C_1	3	1.112
C_3/C_1	4	11.11
D_2'/C_1	5	1.1.1
C_4/C_1	6	1.11.
D_3/C_1	7	1...1
D_2	8	112..
D_4/D_2	9	1.1..
T	10	11...
O	11	1....

Tab. C.23

No	D_{6h}^{*} /	No cl	Γ^{+}_{q} 123456	Γ^{+}_{u} 123456	Γ^{-}_{q} 123456	Γ^{-}_{u} 123456
1	C_1	51	111122	111122	111122	111122
2	C_2'/C_1	52	1.1.11	1.1.11	1.1.11	1.1.11
3	$C_2'(C_1)/C_1$	52	1.1.11	1.1.11	.1.111	.1.111
4	C_s'/C_1	52	1.1.11	.1.111	1.1.11	.1.111
5	$C_s'(C_1)/C_1$	52	1.1.11	.1.111	.1.111	1.1.11
6	C_2''/C_1	52	1..111	1..111	1..111	1..111
7	$C_2''(C_1)/C_1$	52	1..111	1..111	.11.11	.11.11
8	C_s''/C_1	52	1..111	.11.11	1..111	.11.11
9	$C_s''(C_1)/C_1$	52	1..111	.11.11	.11.11	1..111
10	C_2	18	11...2	11...2	11..2	11...2
11	D_2/C_2	19	1....1	1....1	1....1	1....1
12	$D_2(C_2)/C_2$	19	1....1	1....1	.1...1	.1...1
13	C_{2v}/C_2	19	1....1	.1...1	1....1	.1...1
14	$C_{2v}(C_2)/C_2$	19	1....1	.1...1	.1...1	1....1
15	$C_2(C_1)$	18	11...2	11...2	..112.	..112.
16	$D_2(C_2')/C_2(C_1)$	19	1....1	1....1	..1.1.	..1.1.
17	$D_2(C_2'')/C_2(C_1)$	19	1....1	1....1	...11.	...11.
18	$C_{2v}(C_s')/C_2(C_1)$	19	1....1	.1...1	..1.1.	...11.
19	$C_{2v}(C_s'')/C_2(C_1)$	19	1....1	.1...1	...11.	..1.1.
20	C_s	18	11...2	..112.	11...2	..112.
21	C_{2v}'/C_s	19	1....1	..1.1.	1....1	..1.1.
22	$C_{2v}'(C_s)/C_s$	19	1....1	..1.1.	.1...1	...11.
23	C_{2v}''/C_s	19	1....1	...11.	1....1	...11.
24	$C_{2v}''(C_s)/C_s$	19	1....1	...11.	.1...1	..1.1.
25	$C_s(C_1)$	18	11...2	..112.	..112.	11...2
26	$C_{2v}'(C_2')/C_s(C_1)$	19	1....1	..1.1.	..1.1.	1....1
27	$C_{2v}'(C_s'')/C_s(C_1)$	19	1....1	..1.1.	...11.	.1...1
28	$C_{2v}''(C_s')/C_s(C_1)$	19	1....1	...11.	..1.1.	.1...1
29	$C_{2v}''(C_2'')/C_s(C_1)$	19	1....1	...11.	...11.	1....1
30	$C_i(C_1)$	18	111122	.....	.....	111122
31	$C_{2h}'(C_2')/C_i(C_1)$	19	1.1.11	.....	.....	1.1.11
32	$C_{2h}'(C_s')/C_i(C_1)$	19	1.1.11	.....	.....	.1.111
33	$C_{2h}''(C_2'')/C_i(C_1)$	19	1..111	.....	.....	1..111
34	$C_{2h}''(C_s'')/C_i(C_1)$	19	1..111	.....	.....	.11.11
35	C_i	18	111122	.....	111122	.....
36	C_{2h}'/C_i	19	1.1.11	.....	1.1.11	.....
37	$C_{2h}'(C_i)/C_i$	19	1.1.11	.....	.1.111	.....
38	C_{2h}''/C_i	19	1..111	.....	1..111	.....
39	$C_{2h}''(C_i)/C_i$	19	1..111	.....	.11.11	.....
40	C_1^{*}	18	111122	111122	.....	.....
41	C_2^{*}/C_1^{*}	19	1.1.11	1.1.11	.....	.....
42	$C_s'^{*}/C_1^{*}$	19	1.1.11	.1.111	.....	.....
43	$C_2''^{*}/C_1^{*}$	19	1..111	1..111	.....	.....
44	$C_s''^{*}/C_1^{*}$	19	1..111	.11.11	.....	.....
45	$C_{2h}(C_2)$	16	11...2	.....	.....	11...2
46	$D_{2h}(D_2)/C_{2h}(C_2)$	17	1....1	.....	.....	1....1

Tab. C.23 (continued)

No	D_{6h}^{*} /	No cl	Γ^{+}_{q} 123456	Γ^{+}_{u} 123456	Γ^{-}_{q} 123456	Γ^{-}_{u} 123456
47	$D_{2h}(C_{2v})/C_{2h}(C_2)$	17	1....1	.....	.....	.1...1
48	$C_{2h}(C_s)$	16	11...2	.....	.....	..112.
49	$D_{2h}(C_{2v}')/C_{2h}(C_2)$	17	1....1	.....	.....	..1.1.
50	$D_{2h}(C_{2v}'')/C_{2h}(C_2)$	17	1....1	.....	.....	...11.
51	C_{2h}	16	11...2	.....	11...2	.....
52	D_{2h}/C_{2h}	17	1....1	.....	1....1	.....
53	$D_{2h}(C_{2h})/C_{2h}$	17	1....1	.....	.1...1	.....
54	$C_{2h}(C_i)$	16	11...2	.....	..112.	.....
55	$D_{2h}(C_{2h}')/C_{2h}(C_i)$	17	1....1	.....	..1.1.	.....
56	$D_{2h}(C_{2h}'')/C_{2h}(C_i)$	17	1....1	.....	...11.	.....
57	C_2^{*}	16	11...2	11...2	.....	.....
58	D_2^{*}/C_2^{*}	17	1....1	1....1	.....	.....
59	C_{2v}^{*}/C_2^{*}	17	1....1	.1...1	.....	.....
60	C_s^{*}	16	11...2	..112.	.....	.....
61	$C_{2v}'^{*}/C_s^{*}$	17	1....1	..1.1.	.....	.....
62	$C_{2v}''^{*}/C_s^{*}$	17	1....1	...11.	.....	.....
63	C_i^{*}	16	111122	.....	.....	.....
64	$C_{2h}'^{*}/C_i^{*}$	17	1.1.11	.....	.....	.....
65	$C_{2h}''^{*}/C_i^{*}$	17	1..111	.....	.....	.....
66	C_{2h}^{*}	10	11...2	.....	.....	.....
67	D_{2h}^{*}/C_{2h}^{*}	11	1....1	.....	.....	.....
68	C_3	48	1111..	1111..	1111..	1111..
69	C_6	9	11....	11....	11....	11....
70	$C_6(C_3)$	9	11....	11....	..11..	..11..
71	C_{3h}	9	11....	..11..	11....	..11..
72	$C_{3h}(C_3)$	9	11....	..11..	..11..	11....
73	D_3	9	1.1...	1.1...	1.1...	1.1...
74	$D_3(C_3)$	9	1.1...	1.1...	.1.1..	.1.1..
75	C_{3v}	9	1.1...	.1.1..	1.1...	.1.1..
76	$C_{3v}(C_3)$	9	1.1...	.1.1..	.1.1..	1.1...
77	D_3'	9	1..1..	1..1..	1..1..	1..1..
78	$D_3'(C_3)$	9	1..1..	1..1..	.11...	.11...
79	C_{3v}'	9	1..1..	.11..	1..1.	.11...
80	$C_{3v}'(C_3)$	9	1..1.	.11..	.11..	1..1..
81	$C_{3i}(C_3)$	9	1111.	.....	.....	1111..
82	C_{3i}	9	1111.	.....	1111.	.....
83	C_3^{*}	9	1111.	1111.	.....	.....
84	D_6	8	1....	1....	1....	1....
85	$D_6(C_6)$	8	1....	1....	.1...	.1...
86	$D_6(D_3)$	8	1....	1....	..1..	..1..
87	$D_6(D_3')$	8	1....	1....	...1.	...1.
88	C_{6v}	8	1....	.1...	1....	.1...
89	$C_{6v}(C_6)$	8	1....	.1...	.1...	1....
90	$C_{6v}(C_{3v})$	8	1....	.1...	..1..	...1.
91	$C_{6v}(C_{3v}')$	8	1....	.1...	...1.	..1...
92	D_{3d}	8	1....	..1..	1....	..1...

Tab. C.23 (continued)

No	D^*_{6h} /	No cl	Γ^+_{ag} 123456	Γ^+_{au} 123456	Γ^-_{ag} 123456	Γ^-_{au} 123456
93	$D_{3d}(C_{3h})$	8	1.....	..1...	.1....	...1..
94	$D_{3d}(D_3)$	8	1.....	..1...	.1....	.1....
95	$D_{3d}(C'_{3v})$	8	1.....	..1...	...1..	.1....
96	D'_{3d}	8	1.....	....1.	1.....	....1.
97	$D_{3d}(C_{3h})$	8	1.....	...1..	.1....	...1..
98	$D_{3d}(C_{3v})$	8	1.....	...1..	.1....	.1....
99	$D_{3d}(D'_3)$	8	1.....	...1..	...1..	1.....
100	$C_{6h}(C_{3h})$	8	11....	..11..	..11..	..11..
101	$C_{6h}(C_6)$	8	11....	..11..	11....	..11..
102	$C_{6h}(C_{3i})$	8	11....	..11..	.11...	..11..
103	C_{6h}	8	11....	..11..	11....	..11..
104	C^*_{3h}	8	11....	..11..	..11..	..11..
105	C^*_6	8	11....	11....	..11..	..11..
106	$D_{3h}(C_{3v})$	8	1.1...	..1.1.	..1.1.	..1.1.
107	$D_{3h}(D_3)$	8	1.1...	..11..	..11..	..11..
108	$D_{3h}(C_{3i})$	8	1.1...	..11..	.1.1..	..11..
109	D_{3h}	8	1.1...	..11..	1.1...	..11..
110	C^*_{3v}	8	1.1...	1.1...	..11..	..11..
111	D^*_3	8	1.1...	1.1...	..11..	..11..
112	$D'_{3h}(D'_3)$	8	1.1...	..1.1.	1.1...	..1.1.
113	$D'_{3h}(C'_{3v})$	8	1.1...	..11..	..11..	.11...
114	D'_{3h}	8	1.1...	..11..	1.1...	..1.1.
115	$D'_{3h}(C_{3i})$	8	1.1...	..11..	.11...	..11..

Tab. C.23 (continued)

No	D^*_{6h} /	No cl	Γ^+_{ag} 123456	Γ^+_{au} 123456	Γ^-_{ag} 123456	Γ^-_{au} 123456
116	D'^*_3	8	1..1..	1..1..	.....	.....
117	C'^*_{3v}	8	1..1..	.11...	.....	.....
118	C^*_{3i}	8	1111..	.....	.....	....1.
119	$D_{6h}(D'^*_{3d})$	2	1.....	.....	.....	....1.
120	$D_{6h}(D_{3d})$	2	1.....	.....	.....	...1..
121	$D_{6h}(C_{6v})$	2	1.....	.....	.....	1.....
122	$D_{6h}(D_6)$	2	1.....	.....	.....	1.....
123	$D_{6h}(D'_{3h})$	2	1.....	.....	...1..	.....
124	$D_{6h}(D_{3h})$	2	1.....	.....	..1...	.....
125	$D_{6h}(C_{6h})$	2	1.....	.....	.1....	.....
126	D_{6h}	2	1.....	.....	1.....	.....
127	D'^*_{3d}	2	1.....	...1..	.....	.....
128	D^*_{3d}	2	1.....	..1...	.....	.....
129	C'^*_{6v}	2	1.....	.1....	.....	.....
130	D'^*_6	2	1.....	1.....	.....	.....
131	D'^*_{3h}	2	1..1..	.1.1..	.....	.....
132	D^*_{3h}	2	1.1...	.....	.....	.....
133	C'^*_{6h}	2	11....	.....	.....	.....
134	D'^*_{6h}	1	1.....	.....	.....	.....

Tab. C.24

No	T^*_h /	No cl	Γ^+_{ag} 1234	Γ^+_{au} 1234	Γ^-_{au} 1234	Γ^-_{au} 1234
1	C_1	53	1113	1113	1113	1113
2	$C_2(C_1)/C_1$	55	1111	1111	...2	...2
3	C_s/C_1	55	1111	...2	1111	...2
4	$C_s(C_1)/C_1$	55	1111	...2	...2	1111
5	C_2/C_1	54	1111	1111	1111	1111
6	C_3/C_1	56	1..1	1..1	1..1	1..1
7	$D_2(C_2)/C_1$	57	111.	111.	...1	...1
8	C_{2v}/C_1	57	111.	...1	111.	...1
9	$C_{2v}(C_2)/C_1$	57	111.	...1	...1	111.
10	$C_{2v}(C_s)/C_1$ $(m_x2'_ym'_z)$	58	111.	...1	...1	...1
11	$C_{2v}(C_s)/C_1$ $(m_xm'_y2'_z)$	58	111.	...1	...1	...1
12	$C_i(C_1)$	23	1113	....	....	1113
13	$C_{2h}(C_s)/C_i(C_1)$	25	1111	....	....	...2
14	$C_{2h}(C_2)/C_i(C_1)$	24	1111	....	....	1111
15	$C_{3i}(C_3)/C_i(C_1)$	26	1..1	....	....	1..1
16	$D_{2h}(C_{2v})/C_i(C_1)$	27	111.	....	....	...1
17	C_i	23	1113	....	1113	....
18	$C_{2h}(C_i)/C_i$	25	1111	....	...2	....
19	C_{2h}/C_i	24	1111	....	1111	....

Tab. C.24 (continued)

No	T^*_h /	No cl	Γ^+_{ag} 1234	Γ^+_{au} 1234	Γ^-_{au} 1234	Γ^-_{au} 1234
20	C_{3i}/C_i	26	1..1	....	1..1	....
21	$D_{2h}(C_{2h})/C_i$	27	111.	....	...1	....
22	C^*_1	23	1113	1113	....	....
23	C^*_s/C^*_1	25	1111	...2	....	....
24	C^*_2/C^*_1	24	1111	1111	....	....
25	C^*_3/C^*_1	26	1..1	1..1	....	....
26	C^*_{2v}/C^*_1	27	111.	...1	....	....
27	C^*_i	20	1113	....	....	....
28	C^*_{2h}/C^*_i	21	1111	....	....	....
29	C^*_{3i}/C^*_i	22	1..1	....	....	....
30	D_2	7	111.	111.	111.	111.
31	$D_{2h}(D_2)$	5	111.	....	....	111.
32	D_{2h}	5	111.	....	111.	....
33	D^*_2	5	111.	111.	....	....
34	D^*_{2h}	3	111.	....	....	....
35	T	8	1...	1...	1...	1...
36	$T_h(T)$	2	1...	....	....	1...
37	T_h	2	1...	....	1...	....
38	T^*	2	1...	1...	....	....
39	T^*_h	1	1...	....	....	....

The crystallographic symmetry is D^6_{3d}, the transitions are equitranslational and it is sufficient to consider only the point groups. Paramagnetic phase has a $D_{3d} \times \Theta = D^*_{3d}$ symmetry. The ordered phases are characterized with the following groups and representations taken from Tab. A.14 and C. 14 :

No28: D^*_{3d}/D_{3d} $(C_2,C_1)_2$ $D^{H'}_G = \Gamma^+_{1g}+\Gamma^-_{1g}$ * : A_z

No 7: $D^*_{3d}/C_{2h}/C_i$ $(D_6,C'_2)_6$ $D^{H'}_G = \Gamma^+_{1g}+\Gamma^+_{3g}+\Gamma^-_{1g}+\Gamma^-_{3g}$ * : $F_xA_yA_z$

No 8: $D^*_{3d}/C_{2h}(C_i)/C_i$ $(D_6,C'_2)_6$ $D^{H'}_G = \Gamma^+_{1g}+\Gamma^+_{3g}+\Gamma^-_{2g}+\Gamma^-_{3g}$ * : $A_xF_yF_z$

No 6: D^*_{3d}/C_i $(D_6,C_1)_2$ $D^{H'}_G = \Gamma^+_{1g}+\Gamma^+_{2g}+2\Gamma^+_{3g}+\Gamma^-_{1g}+\Gamma^-_{2g}+\Gamma^-_{3g}$ * : $\vec{F}$, $\vec{A}$

Tab. C.25

No	O_h^* /	No cl	Γ^+_q 12345	Γ^+_u 12345	Γ^-_q 12345	Γ^-_u 12345
1	C_1	59	11233	11233	11233	11233
2	C_2/C_1	60	11211	11211	11211	11211
3	$C_2(C_1)/C_1$	61	11211	11211	...22	...22
4	C_s/C_1	61	11211	...22	11211	...22
5	$C_s(C_1)/C_1$	61	11211	...22	...22	11211
6	C'_2/C_1	62	1.112	1.112	1.112	1.112
7	$C'_2(C_1)/C_1$	62	1.112	1.112	.1121	.1121
8	C'_s/C_1	62	1.112	.1121	1.112	.1121
9	$C'_s(C_1)/C_1$	62	1.112	.1121	.1121	1.112
10	C_3/C_1	63	11.11	11.11	11.11	11.11
11	$D_2(C_2)/C_1$	65	112..	112..	...11	...11
12	C_{2v}/C_1	65	112..	...11	112..	...11
13	$C_{2v}(C_2)/C_1$	65	112..	...11	...11	112..
14	$C_{2v}(C_s)/C_1$	68	112..	...11	...11	...11
15	C_4/C_1	64	1.11.	1.11.	1.11.	1.11.
16	$C_4(C_2)/C_1$	64	1.11.	1.11.	.11.1	.11.1
17	S_4/C_1	64	1.11.	.11.1	1.11.	.11.1
18	$S_4(C_2)/C_1$	64	1.11.	.11.1	.11.1	1.11.
19	D'_2/C_1	66	1.1.1	1.1.1	1.1.1	1.1.1
20	$D'_2(C_2)/C_1$	66	1.1.1	1.1.1	.111.	.111.
21	$D'_2(C'_2)/C_1$	67	1.1.1	1.1.1	...11	...11
22	C'_{2v}/C_1	66	1.1.1	.111.	1.1.1	.111.
23	$C'_{2v}(C_2)/C_1$	66	1.1.1	.111.	.111.	1.1.1

Tab. C.25 (continued)

No	O_h^* /	No cl	Γ^+_q 12345	Γ^+_u 12345	Γ^-_q 12345	Γ^-_u 12345
24	$C'_{2v}(C'_s)/C_1$	67	1.1.1	.111.	...11	...11
25	C''_{2v}/C_1	67	1.1.1	...11	1.1.1	...11
26	$C''_{2v}(C_s)/C_1$	67	1.1.1	...11	.111.	...11
27	$C''_{2v}(C'_2)/C_1$	67	1.1.1	...11	...11	1.1.1
28	$C''_{2v}(C'_s)/C_1$	67	1.1.1	...11	...11	.111.
29	D_3/C_1	69	1...1	1...1	1...1	1...1
30	$D_3(C_3)/C_1$	69	1...1	1...1	.1.1.	.1.1.
31	C_{3v}/C_1	69	1...1	.1.1.	1...1	.1.1.
32	$C_{3v}(C_3)/C_1$	69	1...1	.1.1.	.1.1.	1...1
33	$D_4(C_4)/C_1$	70	1.1..	1.1..	...1.	...1.
34	$D_4(D'_2)/C_1$	70	1.1..	1.1..	1	1
35	$D_{2d}(S_4)/C_1$	70	1.1..	.11..	...1.	1
36	$D_{2d}(C'_{2v})/C_1$	70	1.1..	.11..	1	...1.
37	C_{4v}/C_1	70	1.1..	...1.	1.1..	...1.
38	$C_{4v}(C_{2v})/C_1$	70	1.1..	...1.	.11..	1
39	$C_{4v}(C_4)/C_1$	70	1.1..	...1.	...1.	1.1..
40	$C_{4v}(C'_{2v})/C_1$	70	1.1..	...1.	1	.11..
41	D'_{2d}/C_1	70	1.1..	1	1.1..	1
42	$D'_{2d}(C_{2v})/C_1$	70	1.1..	1	.11..	...1.
43	$D'_{2d}(S_4)/C_1$	70	1.1..	1	...1.	.11..
44	$D'_{2d}(D'_2)/C_1$	70	1.1..	1	1	1.1..
45	$C_i(C_1)$	35	11233	.....	.....	11233
46	$C_{2h}(C_2)/C_i(C_1)$	36	11211	.....	.....	11211

Tab. C.25 (continued)

No	O_h^* /	No cl	Γ^+_q 12345	Γ^+_u 12345	Γ^-_q 12345	Γ^-_u 12345
47	$C_{2h}(C_s)/C_i(C_1)$	37	11211	.....	.....	...22
48	$C'_{2h}(C'_2)/C_i(C_1)$	38	1.112	.....	.....	1.112
49	$C'_{2h}(C'_s)/C_i(C_1)$	38	1.112	.....	.....	.1121
50	$S_6(C_3)/C_i(C_1)$	39	11.11	.....	.....	11.11
51	$D_{2h}(C_{2v})/C_i(C_1)$	41	112..	.....	.....	...11
52	$C_{4h}(C_4)/C_i(C_1)$	40	1.11.	.....	.....	1.11.
53	$C_{4h}(S_4)/C_i(C_1)$	40	1.11.	.....	.....	.11.1
54	$D'_{2h}(D'_2)/C_i(C_1)$	42	1.1.1	.....	.....	1.1.1
55	$D'_{2h}(C'_{2v})/C_i(C_1)$	42	1.1.1	.....	.....	.111.
56	$D'_{2h}(C''_{2v})/C_i(C_1)$	43	1.1.1	.....	.....	...11
57	$D_{3d}(D_3)/C_i(C_1)$	44	1...1	.....	.....	1...1
58	$D_{3d}(C_{3v})/C_i(C_1)$	44	1...1	.....	.....	.1.1.
59	$D_{4h}(C_{4v})/C_i(C_1)$	45	1.1..	.....	.....	...1.
60	$D_{4h}(D'_{2d})/C_i(C_1)$	45	1.1..	.....	.....	1
61	C_i	35	11233	.....	11233	.....
62	C_{2h}/C_i	36	11211	.....	11211	.....
63	$C_{2h}(C_i)/C_i$	37	11211	.....	...22	.....
64	C'_{2h}/C_i	38	1.112	.....	1.112	.....
65	$C'_{2h}(C_i)/C_i$	38	1.112	.....	.1121	.....
66	S_6/C_i	39	11.11	.....	11.11	.....
67	$D_{2h}(C_{2h})/C_i$	41	112..	.....	...11	.....
68	C_{4h}/C_i	40	1.11.	.....	1.11.	.....
69	$C_{4h}(C_{2h})/C_i$	40	1.11.	.....	.11.1	.....

Tab. C.25 (continued)

No	O_h^* /	No	Γ^+_q 12345	Γ^+_u 12345	Γ^-_q 12345	Γ^-_u 12345
70	D'_{2h}/C_i	42	1.1.1	.....	1.1.1	.....
71	$D'_{2h}(C_{2h})/C_i$	42	1.1.1	.....	.111.	.....
72	$D'_{2h}(C'_{2h})/C_i$	43	1.1.1	.....	...11	.....
73	D_{3d}/C_i	44	1...1	.....	1...1	.....
74	$D_{3d}(S_6)/C_i$	44	1...1	.....	.1.1.	.....
75	$D_{4h}(C_{4h})/C_i$	45	1.1..	.....	...1.	.....
76	$D_{4h}(D'_{2h})/C_i$	45	1.1..	.....	1	.....
77	C_1^*	35	11233	11233	.....	.....
78	C_2^*/C_1^*	36	11211	11211	.....	.....
79	C_s^*/C_1^*	37	11211	...22	.....	.....
80	C'^*_2/C_1^*	38	1.112	1.112	.....	.....
81	C'^*_s/C_1^*	38	1.112	.1121	.....	.....
82	C_3^*/C_1^*	39	11.11	11.11	.....	.....
83	C''^*_{2v}/C_1^*	41	112..	...11	.....	.....
84	C_4^*/C_1^*	40	1.11.	1.11.	.....	.....
85	S_4^*/C_1^*	40	1.11.	.11.1	.....	.....
86	D'^*_2/C_1^*	42	1.1.1	1.1.1	.....	.....
87	C'^*_{2v}/C_1^*	42	1.1.1	.111.	.....	.....
88	C''^*_{2v}/C_1^*	43	1.1.1	...11	.....	.....
89	D_3^*/C_1^*	44	1...1	1...1	.....	.....
90	C_{3v}^*/C_1^*	44	1...1	.1.1.	.....	.....
91	C_{4v}^*/C_1^*	45	1.1..	...1.	.....	.....
92	D'^*_{2d}/C_1^*	45	1.1..	1	.....	.....

Tab. C.25 (continued)

No	O_h^* /	No cl	Γ^+q 12345	Γ^+u 12345	Γ^-q 12345	Γ^-u 12345
93	C_i^*	28	11233	.....	.....	.....
94	C_{2h}^*/C_i^*	29	11211	.....	.....	.....
95	$C_{2h}'^*/C_i^*$	30	1.112	.....	.....	.....
96	S_6^*/C_i^*	31	11.11	.....	.....	.....
97	C_{4h}^*/C_i^*	32	1.11.	.....	.....	.....
98	$D_{2h}'^*/C_i^*$	33	1.1.1	.....	.....	.....
99	D_{3d}^*/C_i^*	34	1...1	.....	.....	.....
100	D_2	18	112..	112..	112..	112..
101	D_4/D_2	19	1.1..	1.1..	1.1..	1.1..
102	$D_4(D_2)/D_2$	19	1.1..	1.1..	.11..	.11..
103	D_{2d}/D_2	19	1.1..	.11..	1.1..	.11..
104	$D_{2d}(D_2)/D_2$	19	1.1..	.11..	.11..	1.1..
105	$D_{2h}(D_2)$	16	112..	.....	.....	112..
106	$D_{4h}(D_4)/D_{2h}(D_2)$	17	1.1..	.....	.....	1.1..
107	$D_{4h}(D_{2d})/D_{2h}(D_2)$	17	1.1..	.....	.....	.11..
108	D_{2h}	16	112..	.....	112..	.....
109	D_{4h}/D_{2h}	17	1.1..	.....	1.1..	.....
110	$D_{4h}(D_{2h})/D_{2h}$	17	1.1..	.....	.11..	.....
111	D_2^*	16	112..	112..	.....	.....
112	D_4^*/D_2^*	17	1.1..	1.1..	.....	.....
113	D_{2d}^*/D_2^*	17	1.1..	.11..	.....	.....
114	D_{2h}^*	10	112..	.....	.....	.....
115	D_{4h}^*/D_{2h}^*	11	1.1..	.....	.....	.....

Tab. C.25 (continued)

No	O_h^* /	No cl	Γ^+q 12345	Γ^+u 12345	Γ^-q 12345	Γ^-u 12345
116	T	9	11...	11...	11...	11...
117	O	8	1....	1....	1....	1....
118	O(T)	8	1....	1....	.1...	.1...
119	T_d	8	1....	.1...	1....	.1...
120	T_d(T)	8	1....	.1...	.1...	1....
121	T^*(T)	8	11...	.....	.....	11...
122	T_h	8	11...	.....	11...	.....
123	T^*	8	11...	11...	.....	.....
124	$O_h(T_d)$	2	1....	.....	.....	.1...
125	O_h(O)	2	1....	.....	.....	1....
126	$O_h(T_h)$	2	1....	.....	.1...	.....
127	O_h	2	1....	.....	1....	.....
128	T_d^*	2	1....	.1...	.....	.....
129	O^*	2	1....	1....	.....	.....
130	T_h^*	2	11...	.....	.....	.....
131	O_h^*	1	1....	.....	.....	.....

In the three-terms symbol at the first and at the second positions are the symmetry groups of the two phases, while Core H' = Ker Γ_α separates the active symmetry breaking irrep from the permutational representation for each of the cases. The symbol of the magnetic structure is shown on the right side, through the vectors $\vec{F}=\vec{S}_1+\vec{S}_2+\vec{S}_3+\vec{S}_4$, $\vec{A}=\vec{S}_1-\vec{S}_2-\vec{S}_3+\vec{S}_4$. The components of $\vec{F}$ and $\vec{A}$ transform according to the respective active representation, i.e. they play the role of order parameter.

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Received by Publishing Department
on June 6, 1990.

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E17-90-389

Перестановочные магнитные точечные группы
и их применение в теории фазовых
переходов Ландау
Транзитивные перестановочные представления

Выведены и табулированы все транзитивные перестановочные представления магнитных точечных групп /индуцированные из тривиального представления их подгрупп/. Результаты применяются в симметрийном анализе магнитных фазовых переходов и при выводе перестановочных магнитных точечных групп.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1990

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E17-90-389

Permutational Magnetic Point Groups and
Their Application in the Landau Theory
of Phase Transitions
Transitive Permutational Representations

All transitive permutational representations for the magnetic point groups have been constructed. Their application in the symmetry analysis of magnetic phase transitions is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1990