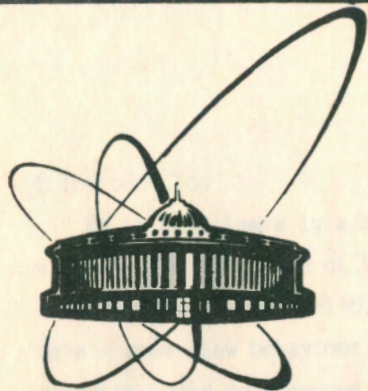


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NONLINEAR CVC AND CRITICAL CURRENT  
OF SUPERCONDUCTIVE GLASS MODEL

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## 1. Introduction

Up to date there is a number of experimental papers dealing with the investigation of the current-voltage characteristics (CVC) of the high- $T_c$  oxides /1-6,15-19/. The most interesting of them have a power-law behaviour  $V \sim (I - I_c)^a$  with specific temperature and magnetic field dependences of the exponent  $a$  and critical current  $I_c$  (see e.g. /1-6/). To treat these peculiarities some explanations, namely: i) 2D Kosterlitz-Thouless (KT) transition /2/, ii) collective fluxoid motion /3-5/, iii) 3D Josephson medium /1,6/ have been suggested.

On the other hand, in the frame of the superconductive glass (SG) model, pioneered in /7/ and then advanced in /8-12/, a rather successful description of the nonequilibrium (nonergodic) properties of high- $T_c$  superconductors was achieved. In a somewhat program paper /12/ the connection between the SG model and the phenomenological critical state model (including the flux creep) is discussed. It is stressed in /12/ that the glassy picture of high- $T_c$  oxides is restricted to a near  $T_c$  temperature region.

In the present paper we consider via the SG model a nonlinear power-law CVC as well as temperature-field dependences of a critical supercurrent. As it will be shown these dependences behave rather like 3D intergranular systems /6/ than 2D systems with a topological KT transition /2/. In conclusion we discuss possible relations of the long-time ( low-frequency ) behaviour of the Josephson spin correlator ( in the SG model ) with the type of a nonlinear CVC for systems containing intragranular weak links.

## 2. Power-law CVC of the SG model

As is well known /8,9/ the Hamiltonian of the SG model in the pseudospin representation has a form

$$\hat{H} = -\text{Re} \sum_{ij} J_{ij} S_i^* S_j, \quad (1)$$

where

$$J_{ij} = J(T) \exp(iA_{ij}), \quad S_i = \exp(if_i),$$

$$A_{ij} = \pi H(x_i + x_j)(y_i - y_j) / \phi_0, \quad \phi_0 = \hbar c / 2e. \quad (2)$$

The model (1) describes the interaction between superconductive clusters (with phases  $f_i$ ) via Josephson junctions (with an energy  $J(T)$ ) on a 2-D disordered lattice (with cluster coordinates  $(x_i, y_j)$ ) in a frustrated external magnetic field  $H = (0, 0, H)$ . So, as usually (see, however, /10/), we have neglected the shielding current effects. The field is normal to the ab-plane where a glass-like picture of high- $T_c$  oxides is established. Since the Hamiltonian (1) contains only the tunneling of the Cooper pairs, the linear (Ohmic) contribution to the CVC is absent. So, as in paper /13/ we shall use the supercurrent  $I_S \sim I - V/R_N$  and not the total current  $I$  (this distinction is inessential, however, when  $I_S \gg I_N$ ), and the critical current  $I_c$  (we are dealing with) is a decoupling current, built up from the maximum Josephson current of individual junctions.

Following /13/ for the supercurrent density operator (on the square lattice with a side  $d$ ) we have

$$j_S = 2ie \sum_{i,j} J_{ij} S_i^* S_j r_{ij} / \hbar d^2, \quad (3)$$

where  $r_{ij} = r_i - r_j$  is the distance between clusters. In our case  $r_i = (x_i, y_i, 0)$ .

When an electric field  $E$  is applied to the system (1), the coupling energy  $J_{ij}(T, H)$  is renormalized in a way:  $J_{ij} \rightarrow J_{ij} \exp(i\omega_{ij}t)$ , where  $\omega_{ij} = 2eEr_{ij} / \hbar$ .

According to the linear response theory /14/ the thermal averaging of the Fourier transform of the supercurrent density  $j_S(\omega)$  has a form

$$\langle j_S(\omega) \rangle = 4e^2 \int_0^\infty dt \cos(\omega t) \sum_{i,j} J_{ij} e^{-i\omega_{ij}t} \langle S_i^*(t) S_j(0) \rangle (r_{ij} E) r_{ij} / \hbar^2 d^2. \quad (4)$$

In order to obtain from (4) the mean value of the supercurrent density in the SG model (1), one needs to perform averaging over random cluster coordinates  $(x_i, y_j)$  ( by the Gaussian  $P(x_i, y_j) = \exp(-x_i^2/2\sigma - y_j^2/2\sigma) / 2\pi\sigma$ , where  $\sigma$  is the projected area of superconductive loops with a uniform phase ) /8/. Choosing , for simplicity,  $E = (E, 0, 0)$ , after the configurational averaging one gets for the supercurrent

$$I_S \equiv d \langle j_S^X(\omega) \rangle = A \int_0^\infty dt \cos(\omega t) \exp(-4e^2 E^2 \sigma t^2 / \hbar^2) D(t) E, \quad (5)$$

where

$$J(T, H) = J(T) / (1 + H^2 / H_0^2)^{1/2}, \quad A = 8e^2 \sigma J(T, H) N / \hbar^2 d, \quad H_0 = \phi_0 / 2\sigma,$$

$$D(t) = \sum_{i,j} \langle S_i^*(t) S_j(0) \rangle / N.$$

So, the form of CVC (5) ( as well as the magnetization in the SG model /8/ ) is determined by the behaviour of the correlator  $D(t)$ . As is shown in /9,11/ the long-time ( low-frequency ) singularities of the correlator  $D(t)$  define glassy properties of the SG model and observable experimental peculiarities. In the present paper we restrict ourselves to the simplest Ansatz of this kind . Namely , we take a power-like relaxation law /9/

$$D(t) = L + D_0 (t/\tau)^{-\alpha}, \quad (6)$$

where

$$\alpha = 1/2 - 3L/2D_0, \quad \tau = \hbar^2 N / 4e^2 RT.$$

Here,  $L(T,H)$  is the order parameter in the SG model (1) ( studied in detail in /8,9,11/ ),  $N$  is the number of superconducting clusters, and  $R$  is the normal state resistance between two grains. Remember /9/ that the order parameter  $L(T,H)$  is the solution of the equation :  $L^3 - L^2 + L - (1 - T^2/T_c^2(H)) = 0$ , where the phase boundary  $T_c(H)$  ( separating the nonergodic state from the ergodic one ) was described in detail in /8/. Thus , the field dependence of the  $L(T,H)$  is governed by the function  $T_c(H)$  . In the range of validity of the glass-like picture /12/ we have  $D_0 \sim 1$ ,  $L \sim 1 - T/T_c$  , and  $0 < \alpha \leq 1/2$  ( see /9/ ). According to (6) and (5) one gets a power-law CVC

$$V/V_0 = (I_s - I_c)^a / I_0, \quad (7)$$

where

$$a = 1/\alpha, \quad V = Ed, \quad V_0 = \hbar d / 2e\tau^{1/2}, \quad I_0 = AD_0 F(\alpha) \hbar / 2e\tau^{1/2},$$

$$I_c = I_0 F(0) L / F(\alpha) D_0, \quad F(\alpha) = \int_0^\infty dx x^{-\alpha} \exp(-x^2). \quad (8)$$

So, in view of (7) and (8) the temperature and field dependences of the exponent  $a(T,H)$  and the critical current  $I_c(T,H)$  are governed by the order parameter  $L(T,H)$  ( and via the Josephson energy  $J(T,H)$  (5)). Figure 1 shows the calculated by (6)-(8) behaviour of  $a(T)$  versus  $T/T_c$  at  $H=0$ . It is seen that  $a(T_c) = 2$  ( in agreement with the critical long-time behaviour of the correlator  $D(t)$  /9/ ) and increases linearly when the reduced temperature  $t = T/T_c$  decreases ( cf. with a similar behaviour reported for high- $T_c$  oxides /1,5,6/ ). As is shown in /9/ , the nonergodicity parameter  $L$  decreases with increasing magnetic field  $H$  . In

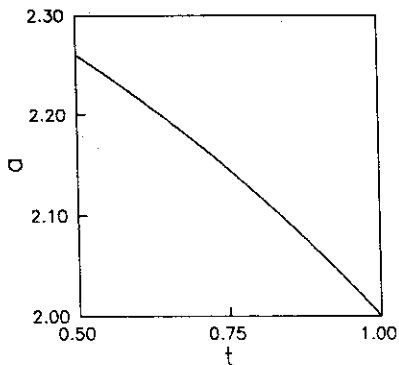


Fig. 1: The temperature dependence of the CVC exponent  $a(T)$  versus  $t=T/T_c$ .

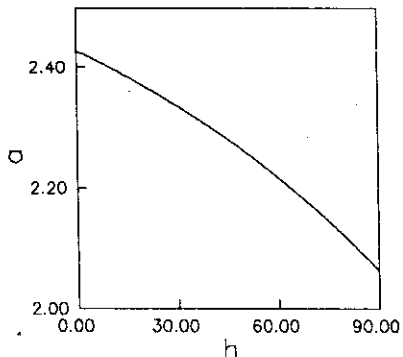


Fig. 2: The magnetic field dependence of the CVC exponent  $a(H)$  versus  $h=H/H_0$  at  $t=T/T_c=0.6$ .

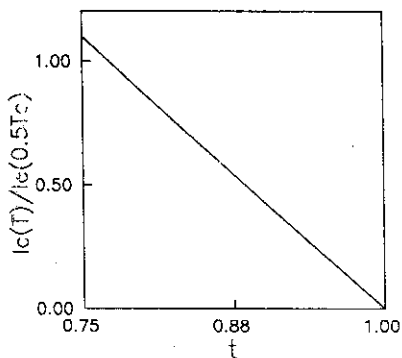


Fig. 3: The behaviour of reduced critical current  $I_c(T)/I_c(0.5T_c)$  versus reduced temperature  $t=T/T_c$ .

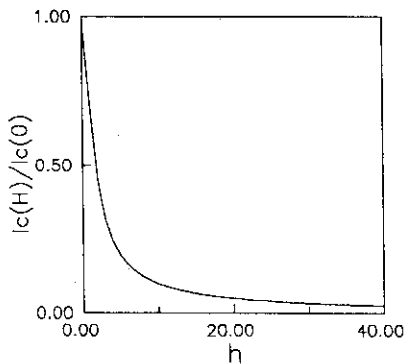


Fig. 4: The behaviour of reduced critical current  $I_c(H)/I_c(0)$  versus reduced magnetic field  $h=H/H_0$ .

particular, in the region of strong frustration ( $H \gg H_0$ ) this leads to the following behaviour of the exponent  $a(H) \sim 2 \times (H_0/H)^2$ . Figure 2 shows the behaviour of the  $a(H)$  versus reduced field  $h = H/H_0$  at the reduced temperature  $t = T/T_c = 0.6$ . Such a behaviour really correlates with some observations [3,4].

Let us now turn to the critical current  $I_c(T, H)$ . As it follows from (8) the temperature behaviour of the  $I_c$  near  $T_c$  is determined by the critical behaviour of the parameter  $L(T, H)$  and has an asymptotic form  $I_c \sim (1 - T/T_c)$  (see Fig.3). A similar dependence of the critical current for high- $T_c$  oxides is discussed in [5,6,13,16-19]. The magnetic field dependence of the critical current is influenced by two factors. They are: i) the coupling energy  $J(T, H)$  (5), and ii) the nonergodicity parameter  $L(T, H)$  [9]. The main contributor is the Josephson energy. In the case of weak fields ( $H \ll H_0$ ) from (5) and (8) it follows that:

$$I_c(H) = I_c(0)(1 - H^2/H_0^2). \quad (9)$$

In the opposite case of strong fields ( $H \gg H_0$ ) one gets:

$$I_c(H) = I_c(0)H_0/H. \quad (10)$$

The behaviour of the reduced critical current  $I_c(H)/I_c(0)$  versus reduced applied field  $h = H/H_0$  is illustrated in Fig.4. It is seen that the observable forms of the  $I_c(H)$  [4,5,17,18,20] are in qualitative agreement with the predictions of the SG model (1).

### 3. Discussion

To better understand the described above features in the behaviour of the CVC in the real experimental situations, let us make some estimates for typical values of the current  $I_0$  and the voltage  $V_0$ . In view of  $D_0 \sim 1$ ,  $F(\alpha) \sim 1$ , and  $d \sim N\alpha^{1/2}$  for  $J \sim 50K$  and  $R \sim 5 \Omega$ , from (6) and (8) one gets:

$$I_0 \sim 4eJ/h \sim 10^{-9} \text{A} , \quad V_0 \sim 2eRT_c/h \sim 10^{-3} \text{V} . \quad (11)$$

Following the experimental data /1-6,15-19/ , this is the range of parameters where the obtained above peculiarities in a power-law CVC behaviour were registered .

It is known , however , that there are more complex than a power-like one types of CVC ( e.g. logarithmic-like /21/ ) . Not pretending to give a unique description of all the observable laws,

we would like to emphasize the following circumstances. As is well-known /7-9,11,12/ , the most prominent feature of the glassy behaviour is the existence of a hierarchy of long-time relaxation laws ( including the aging effects /22/ ). If , for instance , instead of the power-law Ansatz (6) for the correlator  $D(t)$  ( used in the present paper ) we took a logarithmic one /9/ , the above described procedure would lead us to the logarithmic CVC /21/. So , it is possible that the existence of different types of CVC reflects complex dynamical ( low-frequency ) processes of relaxation in these materials . To verify such a hypothesis one needs to make the measurements of both the long-time relaxation law ( e.g. of the remanent magnetization ) and the corresponding CVC for the same sample but at different initial conditions ( e.g. for different waiting times ) .

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