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TIME EVOLUTION OF VARIANCES OF QUADRATURE OPERATORS IN A TWO-MODE BOSON SYSTEM

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One of the most interesting and actively developing trends in physics concerns an investigation of Bose-field collective states (squeezed, sub-Poissonian, etc.), their statistical and fluctuation properties. Interest in the stated problems is caused, in the first place, by increasing accuracy of quantum measurements including detecting of a signal below the shot noise level [1,2], and creation of optical low-noise communication systems as well as elaboration of optical computers [3]. At the same time, a common treatment of a problem is of great interest in other fields of physics, in particular, in statistical mechanics, condensed-matter physics, nuclear physics, and quantum chromodynamics.

In theoretical consideration of quantum optics problems collective states are generated by diverse types of nonlinear interactions. In the simplest case, the system Hamiltonian has a bilinear form with respect to Bose operators. This fact allows one to get the exact solutions with the help of the Bogolubov canonical transformation [4]. For instance, in ref. [5] the quadratic single-mode Hamiltonian of the form

$$H = \hbar f_1 a^+ a + \hbar f_2 a^+ a^+ + \hbar f_2^* aa + \hbar f_3 a^+ + \hbar f_3 a \qquad (1)$$

has been considered, where f_i are numerical parameters which might depend on time. For the Hamiltonian (1) the existence of a squeezed state has been shown. In ref. [6] the two-mode problem with the interaction Hamiltonian of the form

$$H = \hbar G a^+ b^+ + \text{h.c.} \qquad (2)$$

has been considered. With the help of this Hamiltonian the parametric generation process has been described. In that paper, the pair coherent states have been introduced and statistical fluctuation properties have been examined. The possibility for squeezing of quantum fluctuations and violation of the Cauchy-Schwarz inequality was also demonstrated (see also [7]).

In the present paper, we investigate the time dependence of the quadrature operator variances characterizing the quantum fluctuations in the more general two-mode model problem with the Hamiltonian having the form

$$H = \hbar\omega_1 c_1^+ c_1 + \hbar\omega_2 c_2^+ c_2 + i\hbar g_1 (c_1^+ c_2 - c_2^+ c_1) - i\hbar g_2 (c_1^+ c_2^+ - c_1 c_2). \quad (3)$$

With the purpose of diagonalizing the Hamiltonian we introduce new operators

$$a_k = \sum_{m=1}^{2} \left(A_m^k c_m + B_m^k c_m^+ \right) , \quad (4)$$

where the coefficients A_m^k and B_m^k are determined from the equations of motion

$$[a_k , H] = \hbar\Omega_k a_k \quad (5)$$

and from the fulfilment of the commutation relations

$$\begin{aligned} &[a_n , a_m^+] = \delta_{nm} , \\ &[a_n , a_m] = [a_n^+ , a_m^+] = 0 . \end{aligned} \quad (6)$$

The straightforward diagonalization of the Hamiltonian (3) leads to the result

$$H = \hbar\Omega_1 \left(a_1^+ a_1 + \frac{1}{2} \right) + \hbar\Omega_2 \left(a_2^+ a_2 + \frac{1}{2} \right) , \quad (7)$$

where

$$\Omega_k^2 = \frac{\omega_1^2 + \omega_2^2}{2} + g_1^2 - g_2^2 + (-1)^{k+1} \frac{\Delta}{2} , \qquad k = 1,2 ,$$

$$\Delta = \left[(\omega_1^2 - \omega_2^2)^2 + 4(\omega_1^2 + \omega_2^2)(g_1^2 - g_2^2) + 8\omega_1\omega_2(g_1^2 + g_2^2) \right]^{1/2} . \tag{8}$$

It is worth noticing that a special case of the Hamiltonian (3) is the model Hamiltonian for the single-particle motion in the field of the rotating anisotropic oscillator, which has firstly been analysed in ref.[8]. The solution of equations (5,6) allows us to determine the transformation coefficients A_m^k and B_m^k by the following relations:

$$V_1^k = \Omega_k \frac{\Omega_k^2 - \omega_2^2 - (g_1^2 - g_2^2)}{\omega_2(g_1 + g_2)^2 + \omega_1(\Omega_k^2 - \omega_2^2)} Y_1^k , \tag{9}$$

$$V_2^k = i\ \Omega_k \frac{\omega_1(g_1 + g_2) + \omega_2(g_1 - g_2)}{\omega_2(g_1 - g_2)^2 + \omega_1(\Omega_k^2 - \omega_2^2)} V_1^k , \tag{10}$$

$$Y_2^k = \Omega_k \frac{\Omega_k^2 - \omega_1^2 - (g_1^2 - g_2^2)}{\omega_1(g_1 + g_2)^2 + \omega_2(\Omega_k^2 - \omega_1^2)} V_2^k , \tag{11}$$

$$|Y_1^k|^2 = (-1)^{k+1} \frac{1}{\Omega_1^2 - \Omega_2^2} \frac{\omega_1(\Omega_k^2 - \omega_2^2) + \omega_2(g_1 + g_2)^2}{\Omega_k} , \tag{12}$$

where

$$Y_m^k = A_m^k + B_m^k , \quad V_m^k = A_m^k - B_m^k . \tag{13}$$

Time evolution of the operators $a_k(t)$ represented by the relation

$$a_k(t) = a_k(0)\ e^{-i\Omega_k t} \tag{14}$$

gives a possibility of finding the time dependence for the operators c_m. In accordance with the definition (4) we have

$$c_m(t) = \sum_{k=1}^{2} \left(\overset{*}{A}{}^k_m \, a_k(t) - B^k_m \, a^+_k(t) \right) =$$
$$= \sum_{k=1}^{2} \left(\alpha_{mn}(t) c_n(0) + \beta_{mn}(t) c^+_n(0) \right) , \tag{15}$$

where

$$\alpha_{mn}(t) = \sum_{k=1}^{2} \left(\overset{*}{A}{}^k_m \, A^k_n \, e^{-i\Omega_k t} - B^k_m \, \overset{*}{B}{}^k_n \, e^{i\Omega_k t} \right) , \tag{16}$$

$$\beta_{mn}(t) = \sum_{k=1}^{2} \left(\overset{*}{A}{}^k_m \, B^k_n \, e^{-i\Omega_k t} - B^k_m \, \overset{*}{A}{}^k_n \, e^{i\Omega_k t} \right) . \tag{17}$$

The amplitudes $\alpha_{mn}(t)$, $\beta_{mn}(t)$ obey the relation

$$\sum_{n=1}^{2} \left(|\alpha_{mn}(t)|^2 - |\beta_{mn}(t)|^2 \right) = 1 \tag{18}$$

We introduce the time-dependent quadrature operators for the fields $c_m(t)$ in the usual way

$$Q_m(t) = \frac{c^+_m(t) + c_m(t)}{2} , \quad P_m(t) = i \, \frac{c^+_m(t) - c_m(t)}{2} . \tag{19}$$

Then, for the variances of these quadrature operators we obtain

$$\left. \begin{array}{l} \langle(\Delta Q_m)^2\rangle \\ \langle(\Delta P_m)^2\rangle \end{array} \right\} = \frac{1}{4} \left[1 + 2 \langle(\Delta c^+_m(t) c_m(t))\rangle \pm 2 \, \mathrm{Re} \, \langle(\Delta c_m(t))^2\rangle \right], \tag{20}$$

where

$$\langle(\Delta c_m^+(t)c_m(t))\rangle \equiv \langle c_m^+(t)c_m(t)\rangle - \langle c_m^+(t)\rangle\langle c_m(t)\rangle =$$
$$= \sum_{n=1}^{2} \Big[\; |\beta_{mn}(t)|^2 + \Big(\; |\alpha_{mn}(t)|^2 + |\beta_{mn}(t)|^2 \; \Big) \; \langle(\Delta c_m^+(0)c_m(0))\rangle + $$
$$+ 2 \text{ Re } \Big(\; \alpha_{mn}(t)\overset{*}{\beta}_{mn}(t) \; \langle(\Delta c_m(0))^2\rangle \; \Big) \Big] \;, \tag{21}$$

$$\langle(\Delta c_m(t))^2\rangle \equiv \langle c_m^2(t)\rangle - \langle c_m(t)\rangle^2 =$$
$$= \sum_{n=1}^{2} \Big[\; \alpha_{mn}(t)\beta_{mn}(t) + 2 \; \alpha_{mn}(t)\beta_{mn}(t) \; \langle(\Delta c_m^+(0)c_m(0))\rangle + $$
$$+ \alpha_{mn}^2(t) \; \langle(\Delta c_m(0))^2\rangle + \beta_{mn}^2(t) \; \langle(\Delta c_m^+(0))^2\rangle \; \Big] \;. \tag{22}$$

It follows from equations (20-22) that the quadrature operator variances for the fields c_1 and c_2 depend on the choice of their initial states over which an averaging is performed. On the other hand, the dependence of the quadrature operator variances on the values α_{mn}, β_{mn} leads to their nontrivial dependence on the coupling constants g_1 and g_2 of the fields c_1 and c_2. Thus, there arises a possibility of studying the time dependence of the quadrature operator variances both on the choice of initial states and on the character of interaction of the fields in the two-mode system. In the present paper, we consider only the following three cases in choosing the initial states (t=0) of the fields c_1 and c_2:

a) both the fields c_1 and c_2 are assumed to be in the coherent states

$$|\; \alpha_m \;\rangle = D(\alpha_m)|\; 0 \;\rangle \;, \tag{23}$$

where

$$D(\alpha_m) = \exp\Big(\; \alpha_m \; c_m^+(0) - \overset{*}{\alpha}_m \; c_m(0) \; \Big) \;,$$

and for which the following relations are valid:

$$\langle c_m(0)\rangle = \alpha_m \quad , \quad \langle c_m^+(0)\rangle = \overset{*}{\alpha}_m \quad ,$$
$$\langle c_m^+(0) c_m(0)\rangle = |\alpha_m|^2 \quad , \quad \langle c_m^2(0)\rangle = \alpha_m^2 \quad , \tag{24}$$

so that

$$\langle(\Delta c_m^+(0) c_m(0))\rangle = \langle(\Delta c_m(0))^2\rangle = 0 \quad ; \tag{25}$$

b) the squeezed vacuum states are chosen as initial states for each of the fields in consideration

$$|\ \Phi_m\ \rangle = S(\zeta_m)|\ 0\ \rangle \quad , \tag{26}$$

where

$$S(\zeta_m) = \exp\left(\frac{1}{2}\overset{*}{\zeta}_m\ c_m^2(0) - \frac{1}{2}\zeta_m\ c_m^{+2}(0)\right) \quad ,$$

$$\zeta_m = s_m\ e^{i\varphi_m} \quad ;$$

then we have

$$\langle c_m(0)\rangle = \langle c_m^+(0)\rangle = 0 \quad ,$$
$$\langle c_m^+(0) c_m(0)\rangle = \sinh^2 s_m \quad , \tag{27}$$
$$\langle c_m^2(0)\rangle = -\cosh s_m\ \sinh s_m\ e^{i\varphi_m} \quad ,$$

and, hence,

$$\langle(\Delta c_m^+(0) c_m(0))\rangle = \sinh^2 s_m \quad ,$$
$$\langle(\Delta c_m(0))^2\rangle = -\cosh s_m\ \sinh s_m\ e^{i\varphi_m} \quad ; \tag{28}$$

c) one of the fields is taken in the coherent state while the other is in the squeezed vacuum state.

The results of calculations for the time dependence of the quadrature operator variance $\langle(\Delta Q_2)^2\rangle$ of the field c_2

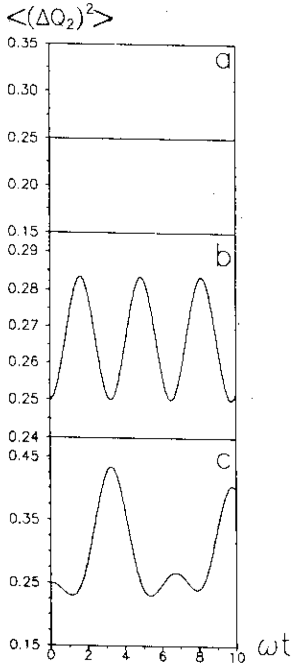

Fig.1. Time evolution of the quadrature operator variance $\langle(\Delta Q_2)^2\rangle$ for the system parameters $\omega_1 = = \omega_2 = \omega = 2$ and for diverse values of the strength constants g_1 and g_2: a) $g_1 = 1$, $g_2 = 0$; b) $g_1 = 0$, $g_2 = 0.5$; c) $g_1 = 1$, $g_2 = 0.5$. The initial states of each of the fields c_1 and c_2 are taken in the coherent states with $\alpha_1 = \alpha_2 = 1$.

are given in Figs. 1-3. For $g_1 \neq 0$ and $g_2 = 0$ the values of the variances do not change with time for each of the fields, i.e., the fields c_1 and c_2 remain in the coherent state (Fig.1a). At $g_1 = 0$ and $g_2 \neq 0$ the behaviour of the variances is noticeably changed; however, the minimal values of the variances (1/4) are related to the coherent states of the fields c_1 and c_2 (Fig. 1b). And only in the case when both types of interactions between the fields c_1 and c_2 are available ($g_1 \neq 0$, $g_2 \neq 0$),one can easily see a qualitative alteration in evolution of the system states: there exist certain time intervals when the coherent states of the fields c_1 and c_2 are "drawn" into the squeezed vacuum states (in Fig.1c it is just those time intervals where the variance values are less than 1/4). Note that the presence of both types of

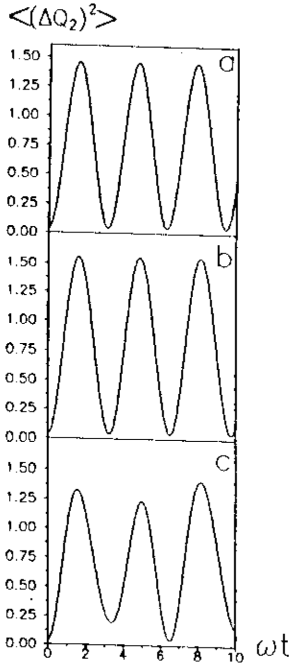

Fig.2. The same as in Fig.1. The squeezed vacuum states are chosen as the initial states for each of the fields with $s_1 = = s_2 =$ Arsh 1, $\phi_1 = \phi_2 = 0$.

interactions in the Hamiltonian (3) do not essentially affect the character of evolution of the system of the fields c_1 and c_2, being initially in the squeezed vacuum state, in comparison with the case when one of the coupling constants turns out to be equal to zero (see Fig.2). Nevertheless, in this case, one can see considerable reconstructions of the states of these fields. However, the situation is crucially changed when from the beginning one of the fields (say, c_1) has been prepared in the squeezed vacuum state and the other (c_2) – in the coherent state (see Fig.3). In this case the interaction of the type with $g_1 \neq 0$ and $g_2 = 0$ causes the "drawing" of the field c_2, being prepared in the coherent state, into the squeezed vacuum state (Fig.3a). This

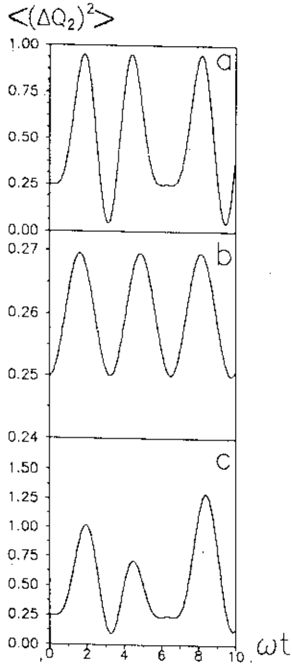

Fig.3. The same as in Fig.1. The field c_1 is taken in the squeezed vacuum state (s_1 = Arsh 1, $\phi_1 = 0$) and the field c_2 is in the coherent state ($\alpha_2 = 1$) as the initial states of the system.

effect doesn't happen with an interaction of the type $g_1 = 0$, $g_2 \neq 0$ (Fig.3b). The behaviour of the system evolution in the case of both types of interaction doesn't drastically differ from the case with an interaction of the type $g_1 \neq 0$, $g_2 = 0$ (see Fig.3c).

Thus, the consideration of the influence of two types of interactions in the Hamiltonian (3) on evolution of the two-mode system reveals the effects of "drawing" of the initially prepared coherent states into the squeezed vacuum states. In a forthcoming paper we intend to study time evolution of the correlation functions determining the statistical properties of the system considered.

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