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SQUEEZING OF THE SQUARE OF THE FIELD AMPLITUDE IN THE TWO-ATOM ONE-MODE MODEL WITH MULTIPHOTON TRANSITIONS

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Lately a remarkable amount of interest has been devoted to the study of the second order and higher-order squeezing in the one- and multiphoton Jaynes - Cummings models.[1-10] This interest was stimulated by the experiments of a Rydberg-atom maser which have allowed one to investigate the interaction of one or a few atoms with an electromagnetic field in a high-Q cavity.[11,12] One interesting type of a higher-order squeezing, namely squeezing of the squared field amplitude (SSFA), has recently been proposed by Hillery.[13] As has been shown by Yang and Zheng[14,15], the SSFA may be generated in the multiphoton Jaynes - Cummings model for the initial atom in the excited and the field in the coherent state or for the initial atom in the coherent state and the field in the vacuum state.

It is interesting to study what happens to SSFA in a cooperative system with the multiphoton interaction. This paper is devoted to examination of this problem on the basis of the simplest cooperative model consisting of two two-level atoms.

We consider two two-level atoms interacting with a single-mode radiation field in an ideal resonant cavity via m-photon--transition mechanism. The effective Hamiltonian for this system in the rotating wave approximation is

$$H = \hbar\omega a^{+}a + \sum_{f=1}^{2} \hbar\omega_0 R_f^{z} + \sum_{f=1}^{2} \hbar g (R_f^{+} a^{m} + R_f^{-} a^{+m}) . \quad (1)$$

Here $a^{+}(a)$ is the operator of creation (annihilation) of a photon, R_f^{z} and $R_f^{\pm}$ are the population inversion and transition operators of an f-th atom, ω and ω_0 are the frequencies of the field and atom, g is the atom-field coupling constant, and m is the photon multiple of transitions. The exact multiphoton resonance is assumed to occur: $\omega_0 = m\omega$.

To define the SSFA effect, we introduce the quantities [13]

$$Y_1 = (A^{+2} + A^2)/2 \;, \quad Y_2 = i\,(A^{+2} - A^2)/2 \;,$$

where $A = a\,e^{i\omega t}$ and $A^+ = a^+ e^{-i\omega t}$ are slowly varying operators.

The operators Y_1 and Y_2 correspond to the real and imaginary parts, respectively, of the field amplitude squared and obey the commutation relation

$$[Y_1, Y_2] = i\,(2N+1) \;,$$

where $N = A^+A$.

The uncertainty relation for these two quantities has the form

$$\Delta Y_1\, \Delta Y_2 \geqslant \langle N + \tfrac{1}{2} \rangle \;.$$

The SSFA state in Y_1 exists if

$$(\Delta Y_1)^2 < \langle N + \tfrac{1}{2} \rangle$$

and similarly for Y_2.

Following the papers [13,15] we can write the expressions of the SSFA in components Y_i as

$$Q_i = \frac{(\Delta Y_i)^2 - \langle N + \frac{1}{2} \rangle}{\langle N + \frac{1}{2} \rangle} \;, \quad i = 1,2 \;; \qquad (2)$$

and

$$Q_1 = \frac{1}{4}\, \frac{\langle A^{+4} + A^4 + 2A^{+2}A^2 \rangle - \langle A^{+2} + A^2 \rangle}{\langle N + \frac{1}{2} \rangle} \;, \qquad (3)$$

$$Q_2 = -\frac{1}{4}\, \frac{\langle A^{+4} + A^4 - 2A^{+2}A^2 \rangle - \langle A^2 - A^{+2} \rangle}{\langle N + \frac{1}{2} \rangle} \;. \qquad (4)$$

Let us investigate the SSFA generation in the model (I) for the initial field in the vacuum state. Of course, no spontaneous emission squeezing will be produced if both the atoms are initially fully excited. However, the atoms can be prepared in coherent superpositions of an excited and a ground state, and subsequent squeezing is possible.[4,6,15] We consider the following three cases where analytical solutions for SSFA can be obtained:

Case 1. Let the atoms be prepared in a superposition state such as

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}} \cos\frac{\alpha}{2} \left(|+,-;0\rangle + |-,+;0\rangle \right) + e^{-i\beta} \sin\frac{\alpha}{2} |-,-;0\rangle \;,$$

$$0 \leqslant \alpha \leqslant \pi \;, \qquad 0 \leqslant \beta \leqslant 2\pi \;.$$

Here $|\Theta,\varphi;n\rangle$ are eigenstates of the free hamiltonian $H_0 = \hbar\omega a^+ a + \sum_{j=1}^{2} \hbar\omega_0 R_j^z$:

$$|\Theta,\varphi;n\rangle = |\Theta\rangle_{A_1} |\varphi\rangle_{A_2} |n\rangle_F \;,$$

$\Theta, \varphi = +,-$, $n = 0,1, \ldots$. The sign "+" corresponds to an excited and "-" to a ground state of an atom, $|n\rangle_F$ is the Fock state of a field.

The interaction couples the states $|+,-;0\rangle$, $|-,+;0\rangle$ with the state $|-,-;m\rangle$ but the zero quantum state $|-,-;0\rangle$ is decoupled and does not involve. The time-evolved wave function has the form

$$|\Psi(t)\rangle = \cos\frac{\alpha}{2}\left\{R(t)\left(|+,-;0\rangle + |-,+;0\rangle\right) + F(t)|-,-;m\rangle\right\} + e^{i(m\omega t-\beta)}\sin\frac{\alpha}{2}|-,-;0\rangle\,, \tag{5}$$

where

$$R(t) = \frac{1}{\sqrt{2}}\cos\left(\sqrt{2(m!)}\,gt\right), \tag{6}$$

$$F(t) = -i\sin\left(\sqrt{2(m!)}\,gt\right).$$

Using equations (5) and (6) one can easily obtain for the expectation values of the operators $\langle A^2\rangle$, $\langle A^4\rangle$, $\langle A^+A\rangle$ and $\langle A^{+2}A^2\rangle$

$$\langle A^2\rangle = -\frac{i}{2}\sqrt{m(m-1)}\,\sin\alpha\,\sin\left(\sqrt{2(m!)}\,gt\right)e^{i\beta}\,\delta_{m,2}\,,$$

$$\langle A^4\rangle = -\frac{i}{2}\sqrt{m(m-1)(m-2)(m-3)}\,\sin\alpha\,\sin\left(\sqrt{2(m!)}\,gt\right)e^{i\beta}\,\delta_{m,4}\,, \tag{7}$$

$$\langle A^+A\rangle = m\cos^2\frac{\alpha}{2}\,\sin^2\left(\sqrt{2(m!)}\,gt\right),$$

$$\langle A^{+2}A^2\rangle = m(m-1)\cos^2\frac{\alpha}{2}\,\sin^2\left(\sqrt{2(m!)}\,gt\right).$$

Substituting equations (7) into formulae (3) and (4) we obtain

$$Q_1 = Q_2 = \frac{\cos^2\frac{\alpha}{2}\sin^2\left(\sqrt{2(m!)}\,gt\right)m(m-1)}{1 + 2m\cos^2\frac{\alpha}{2}\sin^2\left(\sqrt{2(m!)}\,gt\right)}\,, \tag{8}$$

for $m \neq 2,4$,

$$Q_1 = \frac{2\cos^2\frac{\alpha}{2}\sin^2(2gt)\left[1 - 2\sin^2\frac{\alpha}{2}\sin^2\beta\right]}{1 + 4\cos^2\frac{\alpha}{2}\sin^2(2gt)}\,, \tag{9}$$

$$Q_2 = \frac{2\cos^2\frac{\alpha}{2}\sin^2(2gt)\left[1 - 2\sin^2\frac{\alpha}{2}\cos^2\beta\right]}{1 + 4\cos^2\frac{\alpha}{2}\sin^2(2gt)}\,,$$

for $m = 2$,

$$Q_1 = \frac{2\sin(\sqrt{48}\,gt)\left[6\cos^2\frac{\alpha}{2}\sin(\sqrt{48}\,gt) + \frac{\sqrt{6}}{2}\sin\alpha\sin\beta\right]}{1 + 8\cos^2\frac{\alpha}{2}\sin^2(\sqrt{48}\,gt)}\,, \tag{10}$$

$$\frac{2\sin(\sqrt{48}\,gt)\left[6\cos^2\frac{\alpha}{2}\sin(\sqrt{48}\,gt) - \frac{\sqrt{6}}{2}\sin\alpha\sin\beta\right]}{1 + 8\cos^2\frac{\alpha}{2}\sin^2(\sqrt{48}\,gt)}\,,$$

for $m = 4$.

The quantities Q_i in formulae (8)-(10) vary periodically at Rabi frequencies which are $\sqrt{2}$ times that for a single-atom radiation.[15] This result shows the collective character of the spontaneous emission from a system of two atoms. It is seen from equations (8)-(10) that SSFA may only occur for the two-photon or for four-photon processes. Using the numerical calculations one can easily investigate the time behaviour of the factors Q_1 and Q_2. Similar dependences have been presented in paper [15] for the case of a single atom radiation. We offer here only the maximum magnitude of squeezing in the component Y_1 (similarly, in Y_2). The maximum degree of SSFA (corresponding to the minimum value of Q_1) is about 13 % ($\alpha = 2.2, \beta = \pi/2$) for a two-photon and 39% ($\alpha = 2.7, \beta = -\pi/2$) for a four-photon transition system. These maximum magnitudes of SSFA are equal to the corresponding maximum degrees obtained in the case with a single atom.

Case 2. Let one atom be prepared in a coherent superposition state; while the other atom, in the ground state. Then, the initial wave function is

$$|\Psi(0)\rangle = \cos\frac{\alpha}{2}|+,-;0\rangle + e^{-i\beta}\sin\frac{\alpha}{2}|-,-;0\rangle,$$

$$0 \le \alpha \le \pi, \quad 0 \le \beta \le 2\pi.$$

The wave function at the time t is

$$|\Psi(t)\rangle = \cos\frac{\alpha}{2}\{R(t)|+,-;0\rangle + F(t)|-,+;0\rangle + P(t)|-,-;m\rangle\} + e^{i(m\omega t-\beta)}\sin\frac{\alpha}{2}|-,-;0\rangle, \qquad (11)$$

where

$$R(t) = \cos^2(g\sqrt{m!/2}\,t),$$
$$F(t) = -\sin^2(g\sqrt{m!/2}\,),$$
$$P(t) = -\frac{i}{\sqrt{2}}\sin(g\sqrt{2(m!)}\,t). \qquad (12)$$

For this case from equations (11) and (12) we have

$$\langle A^2\rangle = -\frac{i}{2\sqrt{2}}\sqrt{m(m-1)}\,\sin\alpha\,\sin(g\sqrt{2(m!)}\,t)\,\delta_{m,2}\,e^{i\beta},$$
$$\langle A^4\rangle = -\frac{i}{2\sqrt{2}}\sqrt{m(m-1)(m-2)(m-4)}\,\sin\alpha\,\sin(g\sqrt{2(m!)}\,t)\,\delta_{m,4}\,e^{i\beta}, \qquad (13)$$
$$\langle A^+A\rangle = \frac{1}{2}m\cos^2\frac{\alpha}{2}\sin^2(g\sqrt{2(m!)}\,t),$$
$$\langle A^{+2}A^2\rangle = \frac{1}{2}m(m-1)\cos^2\frac{\alpha}{2}\sin^2(\sqrt{2(m!)}\,t).$$

According to (3),(4) the factors Q_1 and Q_2 are then

$$Q_1 = Q_2 = \frac{\frac{1}{2}\cos^2\frac{\alpha}{2}\sin^2(g\sqrt{2(m!)}\,t)\,m(m-1)}{1+\cos^2\frac{\alpha}{2}\sin^2(g\sqrt{2(m!)}\,t)}, \qquad (14)$$

for m $\neq$ 2.4 ,

$$Q_1 = \frac{2\cos^2\frac{\alpha}{2}\sin^2(2gt)\left[\frac{1}{2} - \sin^2\frac{\alpha}{2}\sin^2\beta\right]}{1+2\cos^2\frac{\alpha}{2}\sin^2(2gt)}, \qquad (15)$$

$$Q_2 = \frac{2\cos^2\frac{\alpha}{2}\sin^2(2gt)\left[\frac{1}{2} - \sin^2\frac{\alpha}{2}\cos^2\beta\right]}{1+2\cos^2\frac{\alpha}{2}\sin^2(2gt)},$$

for m = 2 ,

$$Q_1 = \frac{\sin(\sqrt{48}gt)\left[6\cos^2\frac{\alpha}{2}\sin(\sqrt{48}gt)+\sqrt{3}\sin\alpha\sin\beta\right]}{1+4\cos^2\frac{\alpha}{2}\sin^2(\sqrt{48}gt)}, \tag{16}$$

$$Q_2 = \frac{\sin(\sqrt{48}gt)\left[6\cos^2\frac{\alpha}{2}\sin(\sqrt{48}gt)-\sqrt{3}\sin\alpha\sin\beta\right]}{1+4\cos^2\frac{\alpha}{2}\sin^2(\sqrt{48}gt)},$$

for m = 4.

By analogy with the previous case, the quantities Q_1 and Q_2 oscillate with a collective Rabi frequency $\sqrt{2(m!)}$, and SSFA states exist only if m = 2 and m = 4.

The maximum degree of SSFA is about 9% (α = 2.2) for two-photon and 38% (α=2.7) for four-photon processes. These values are smaller than the degrees that may be obtained in a one-atom two- and four-photon transition system (13% and 39%, respectively).

Thus, the presence of an unexcited atom leads to a decrease of the maximum degree of SSFA in spontaneous emission from the superposition state-excited atom.

Case 3. Let the initial state of the total system be

$$|\Psi(0)\rangle = \cos\frac{\alpha}{2}|+,+;0\rangle + e^{-i\beta}\sin\frac{\alpha}{2}|-,-;0\rangle, \quad 0\le\alpha\le\pi, \quad 0\le\beta\le2\pi.$$

At the time t, the wave function is

$$|\Psi(t)\rangle = e^{-i\omega_0 t}\cos\frac{\alpha}{2}\{R(t)|+,+;0\rangle + F(t)|-,-;2m\rangle + P(t)(|+,-;m\rangle+|-,+;m\rangle)\} + e^{-i(\beta-\omega_0 t)}\sin\frac{\alpha}{2}|-,-;0\rangle. \tag{17}$$

Here

$$R(t) = 1 - \frac{g^2 m!}{\Omega_R^2}\sin^2(\Omega_R t),$$
$$F(t) = -\frac{g^2\sqrt{(2m)!}}{\Omega_R^2}\sin^2(\Omega_R t), \tag{18}$$
$$P(t) = -i\frac{g\sqrt{m!}}{2\Omega_R}\sin(2\Omega_R t),$$

where

$$\Omega_R = g\sqrt{\frac{1}{2}\left[m! + \frac{(2m)!}{m!}\right]}.$$

For $\langle A^2\rangle$, $\langle A^4\rangle$, $\langle A^+A\rangle$ and $\langle A^{+2}A^2\rangle$ we obtain

$$\langle A^2\rangle = -\frac{g^2}{2\Omega_R^2}\sqrt{2m(2m-1)}\sqrt{(2m)!}\sin\alpha\sin^2(\Omega_R t)e^{i\beta}\delta_{m,1}, \tag{19}$$

$$\langle A^4\rangle = -\frac{g^2}{2\Omega_R^2}\sqrt{2m(2m-1)(2m-2)(2m-3)}\sqrt{(2m)!}\sin\alpha \times \sin^2(\Omega_R t)e^{i\beta}\delta_{m,2},$$

$$\langle A^+A\rangle = \frac{2mm!g^2}{\Omega_R^2}\cos^2\frac{\alpha}{2}\sin^2(\Omega_R t)\left\{1-\left[1-\frac{g^2(2m)!}{\Omega_R^2 m!}\right]\times\sin^2(\Omega_R t)\right\},$$

$$\langle A^{+2}A^2\rangle = \frac{2mg^2}{\Omega_R^2}\cos^2\frac{\alpha}{2}\sin^2(\Omega_R t)\left\{(m-1)m!\cos^2(\Omega_R t) + \frac{g^2}{\Omega_R^2}(2m-1)(2m)!\sin^2(\Omega_R t)\right\}.$$

For m > 2 one can find from equations (19) that $\langle A^2\rangle=\langle A^4\rangle=0$ and therefore, no SSFA exists.

For m = 1 the factors Q_1 and Q_2 are

$$Q_1 = \frac{\frac{8}{9}\cos^2\frac{\alpha}{2}\,\sin^4\left(\sqrt{\frac{3}{2}}gt\right)\left[1 - 2\sin^2\frac{\alpha}{2}\cos^2\beta\right]}{\frac{1}{2} + \frac{4}{3}\cos^2\frac{\alpha}{2}\,\sin^2\left(\sqrt{\frac{3}{2}}gt\right)\left\{1 + \frac{1}{3}\sin^2\left(\sqrt{\frac{3}{2}}gt\right)\right\}}\,, \tag{20}$$

$$Q_2 = \frac{\frac{8}{9}\cos^2\frac{\alpha}{2}\,\sin^4\left(\sqrt{\frac{3}{2}}gt\right)\left[1 - 2\sin^2\frac{\alpha}{2}\sin^2\beta\right]}{\frac{1}{2} + \frac{4}{3}\cos^2\frac{\alpha}{2}\,\sin^2\left(\sqrt{\frac{3}{2}}gt\right)\left\{1 + \frac{1}{3}\sin^2\left(\sqrt{\frac{3}{2}}gt\right)\right\}}\,.$$

For a one-photon transition the maximum degree of SSFA is about 16% (α=2.2). The occurrence of SSFA for m=1 is an interesting feature of a collective spontaneous radiation from a system of two atoms. For a one-atom radiation this effect is not possible.

For m = 2 we have

$$Q_1 = \frac{\frac{2}{7}\sin^2(\sqrt{7}gt)\left\{2\cos^2\frac{\alpha}{2}\left[1 + \frac{29}{7}\sin^2(\sqrt{7}gt)\right] - 3\sin\alpha\cos\beta\right\}}{\frac{1}{2} + \frac{8}{7}\cos^2\frac{\alpha}{2}\,\sin^2(\sqrt{7}gt)\left\{1 + \frac{5}{7}\sin^2(\sqrt{7}gt)\right\}}\,, \tag{21}$$

$$Q_2 = \frac{\frac{2}{7}\sin^2(\sqrt{7}gt)\left\{2\cos^2\frac{\alpha}{2}\left[1 + \frac{29}{7}\sin^2(\sqrt{7}gt)\right] + 3\sin\alpha\cos\beta\right\}}{\frac{1}{2} + \frac{8}{7}\cos^2\frac{\alpha}{2}\,\sin^2(\sqrt{7}gt)\left\{1 + \frac{5}{7}\sin^2(\sqrt{7}gt)\right\}}\,.$$

In that case the maximum magnitude of SSFA for two-photon transition is about 38% (α=2.7). It is three times of the degree that may be obtained in a one-atom two-photon transition system.[15] For one-photon transition the maximum value of SSFA is about 16% (α=2.2).

Thus, one can prepare the atoms in such a coherent superposition of excited and ground state as to increase the maximum degree of SSFA in comparison to a single-atom situation.

We have investigated the generation of squeezing of the square of the field amplitude in the two-atom one-mode model with multiphoton transitions. The case when the field is initially in the vacuum state together with an atomic superposition state has been examined. The time-depending SSFA factors have been calculated. The comparison of maximum degree of squeezing for two- and single-atom situations has been made. A more detailed investigation of the SSFA in the model considered, in particular for coherent and squeezed initial field states, will be a subject of a subsequent paper.

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Сжатие квадрата амплитуды поля в двухатомной одномодовой модели с многофотонными переходами

Исследовано сжатие квадрата амплитуды поля в двухатомной одномодовой модели с многофотонными переходами. Вычислены времязависящие факторы сжатия. Приведены условия для оптимального сжатия.

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Squeezing of the Square of the Field Amplitude in the Two-Atom One-Mode Model with Multiphoton Transitions

The squeezing of the square of the field amplitude in the two-atom one-mode model with multiphoton transitions is investigated. The time-dependent squeezing factors are calculated. The conditions for the optimum squeezing are shown.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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