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Kh.O.Abdulloev*, M. Agüero, A.V.Makhankov,
V.G.Makhankov, Kh.Kh.Muminov*

GENERALIZED SPIN COHERENT STATES AS A TOOL TO STUDY QUASICLASSICAL BEHAVIOUR OF THE HEISENBERG FERROMAGNET

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*Tajik State University, Dushanbe, USSR

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INTRODUCTION

The most popular model to study ferromagnets is the Heisenberg model. It allows one to investigate behaviour of quantum systems reducing into the e.v. problem. For studying quasiclassical behavior of the systems, and for example, collective excitations (magnons) over ground state, one should proceed from the quantum level of description to a quasiclassical one, and such a transition should be done very carefully. Thus the problem to formulate a consistent reduction procedure appears.

In fact this procedure consists in choosing trial functions (i.e. some basis) which can be used for averaging the Hamiltonian. It is natural to choose for that coherent states (CS) since these states are the most classical and minimized the uncertainty relation (according to Perelomov [1]).

Conventional coherent states (the so-called Glauber CS) are constructed on the Heisenberg-Weyl group and can be utilized in the case when the Hamiltonian is given in terms of Bose-operators only. The most familiar way to treat spin (or pseudospin) Hamiltonians in such a way is applying the Holstein-Primakoff transformations (HPT). Then the (2s+1)-dimensional state space enlarges infinitely due to appearing of the so-called nonphysical states. Moreover radicals these transformations contain give rise to asymptotic quasiclassical series to appear as a result of operators ordering. These series must be truncated.

More natural in studying spin Hamiltonians is to use coherent states constructed on the spin operators of the group SU(2) (the so-called SCS). Such states for arbitrary spin S, are those corresponding to points of the coset space SU(2s+1)/SU(2s)*U(1). It is not necessary in this case to perform bosonization of the Hamiltonian since the latter as well as corresponding CS are constructed via the same operators. It is namely due to this fact generalized SCS are the most adequate tool to study spin (magnetic) and quasispin (finite-level) models.

1. MATRIX OF THE GENERALIZED SPIN COHERENT STATES

Dimension of parameter manifold of spin coherent states M is given by representation

$$|\Psi> = \sum_{k=1}^{2j+1} S_k \, |S, -S + K>$$

which gives us 2j+1 complex constants (or 4j+2 real constants). Since vector $|\Psi>$ is defined up to the arbitrary phase θ we can put it equal to zero. Another additional condition is the normalization of $|\Psi>$, viz.,

$$<\Psi|\Psi> = 1$$

so that

$$\dim M = 4j + 2 - 1 - 1 = 4j \, .$$

In what follows we will treat the spin Hamiltonian of the form

$$\hat{H} = -J \sum_j \{ \hat{\vec{S}}_j \hat{\vec{S}}_{j+1} + \delta \hat{S}^z_j \hat{S}^z_{j+1} \} \qquad (1)$$

i.e. the Heisenberg Hamiltonian with uniaxial anisotropy in the nearest neighbour approximation. We will also take into consideration (if necessary) the terms of one-ion anisotropy

$$-[\hat{S}^z_j]^2 - \alpha [\hat{S}^x_j]^2 \, . \qquad (2)$$

Operators we use to write the Hamiltonian are the generators of the SU(2) group, and as such for any representation they can be expressed through the generators of the corresponding group SU(2j+1). It means that for any spin j Hamiltonian $\hat{H}$ can be written in terms of the generators of the group Su(2j+1).

This promts us possible coherent states which can be utilized as the corresponding trial functions. For each j besides the SU(2) CS one can construct the so-called generalized spin coherent states (GSCS) on the group SU(2s+1) in the Perelomov sense/1/. Choosing as a vacuum state the vector of the most minimal weight we define the group SU(2s)⊗U(1) as the isotopy group, then GSCS correspond to the points of the coset space SU(2s+1)/SU(2s)⊗U(1) the so-called complex projective space CP^{2s}.

One can easily calculate the dimension of this space $\tilde{M}$

$$\dim \tilde{M} = (2j+1)^2 - 1 - (2j)^2 + 1 - 1 = 4j$$

and find that dimensions of $\tilde{M}$ and M coincide.

We construct GSCS on CP^{2s} as follows

$$|\Psi> = e^{\sum_i^{2j} [\zeta_i \hat{T}_i^+ - \bar{\zeta}_i \hat{T}_i^-]} |0> = \qquad (3)$$

$$= \cos|\zeta| \{ |0> + \Psi_1 |1> + \ldots + \Psi_{2j} |2j> \} ,$$

where

$$|\zeta|^2 = \sum_{i=1}^{2j} |\zeta_i|^2, \quad |\zeta| < \pi/2, \quad \Psi_i = \frac{\zeta_i}{|\zeta|} \tan|\zeta| , \qquad (4)$$

and $\hat{T}^{\pm}$ are the generators of faithful representation of the group SU(2j+1) and given by the formulae

$$\hat{T}^+ = \begin{bmatrix} 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 1 \\ \ldots & \ldots & \ldots & \vdots \\ 0 & 0 & \ldots & 0 \end{bmatrix} \qquad \hat{T}^- = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \vdots \\ 1 & 1 & \ldots & 0 \end{bmatrix} .$$

We denote through $|i>$ the vector with unity on the i+1 line from below and zeros otherwise.

From (4) it follows

$$\cos|\zeta| = [1 + |\Psi|^2]^{-1/2}$$

and finally

$$|\Psi> = [1 + |\Psi|^2]^{-1/2} \{ |0> + \sum_{i=1}^{2j} \Psi_i |i> \} . \qquad (5)$$

The system (5) of GSCS is the first point in the consequence of the systems which are related to more higher representations of the SU(2j+1) group. As a result the construction assumes the form of upper triangle matrix

$$
\begin{matrix} CP^1 \\ CP^2 \\ CP^3 \\ \vdots \\ CP^{2j} \end{matrix}
\begin{bmatrix} s = 1/2, & 1, & 3/2, & \cdots \\ & 1, & \cdots & \cdots \\ & & 3/2, & \cdots \\ & & & \vdots \\ & & & j \end{bmatrix} = S . \qquad (6)
$$

Each point of this matrix is the set of GSCS, and naturally we can call it the GSCS matrix. We note that its diagonal elements are given by the fundamental representations of the corresponding group and are the most natural (complete) basis for the spin system of a given S.

II. THE GLAUBER COHERENT STATES AND THE HAMILTONIAN BOSONIZATION

The Glauber CS are the states constructed on the Heisenberg-Weyl group, which algebra is the algebra of bose-operators a, a^+.

In order to use these states as trial functions one should first bosonize the Hamiltonian (1). The most wide-spread way to do that is to apply the Holstein-Primakoff transformations

$$
\hat{S}_j^+ = \sqrt{2s}\sqrt{1 - \frac{a_j^+ a_j}{2s}} \cdot a_j ,
$$

$$
\hat{S}_j^- = \sqrt{2s}\, a_j^+ \sqrt{1 - \frac{a_j^+ a_j}{2s}} , \qquad (7)
$$

$$
\hat{S}_j^z = S - a_j^+ a_j .
$$

The radicals in (7) one must regard as corresponding operator series. Had been ordered, these series become asymptotic series which truncated parts allow one to take into account quantum corrections. The procedure of truncation makes us to do an additional careful symmetry analysis of the Hamiltonian and the ground state /2/.

We consider the following operator of the spin angle

$$
\cos\theta = S^z/s = 1 - \frac{a\,a}{s} . \qquad (8)
$$

Let the value of S increase while keeping θ fixed (in the quasiclassical approximation) then new operators b and b^+ are more appropriate

$$
b^+ = a^+/\sqrt{s}, \quad b = a/\sqrt{s} \qquad (9)
$$

with the commutation relation

$$
[b\, b^+] = \frac{1}{s} I . \qquad (10)
$$

Whereby operators b and b^+ may be regarded as c-numbers in the limit $s \gg 1$ (with $0(s^{-1})$ accuracy).

In /2/, a classical Hamiltonian was obtained via averaging the quantum Hamiltonian (1) over Glauber CS

$$
|a\rangle = e^{-\frac{1}{2}|a|^2} e^{a a^+} |0\rangle \qquad (11)
$$

after its bosonization.

Up to terms $0(|a|^2)$ it reads

$$
H_6 = \frac{J}{a_0}\int [2\delta s|a|^2 + A|a|^4 + \frac{1}{16 s^2}|a|^6 + s a_0^2 |a_x|^2 - \frac{a_0^2}{2} A *
$$

$$
*(|a|_x^2)^2 + \frac{a_0^2}{4}(\bar{a}^2 a_x^2 + \bar{a}_x^2 a^2) + \frac{a_0^2}{16 s}|a|^2(|a|_x^2)^2]\, dx , \qquad (12)
$$

where $A = -\delta + \frac{1}{8s} + \frac{1}{32s^2}$ and a_0 is the lattice spacing.

In classical limit (when only the main terms in series in 1/s are left) one gets.

$$
H_6 = s^2 a_0 J \int \{(\Delta|\beta|^2 + |\beta_x|^2)(1 - \frac{|\beta|^2}{2}) + \frac{1}{4}(|\beta|_x^2)(1 + \frac{|\beta|^2}{4} + 2\delta)\}dx \qquad (13)
$$

with $\delta = \frac{a_0^2}{2}\Delta$..

To identify quantum terms of order of 1/s in the Hamiltonian we proceed to variable

$$
|\beta|^2 = |a|^2/s \qquad (14)
$$

so that

$$
\cos\theta = 1 - |\beta|^2 . \qquad (15)
$$

In the classical limit one can derive the total Hamiltonian via summarizing the series in $|\beta|^2$ *.

*This Hamiltonian after notation coincides with one obtained in /3/.

$$H = s a_0 J \int \left[\Delta|\beta|^2 + |\beta_x|^2)\left(1 - \frac{|\beta|^2}{2}\right) + \frac{1}{4}(|\beta^2|_x)^2 \left\{ \frac{1 - \frac{1}{4}|\beta|^2}{1 - \frac{1}{2}|\beta|^2} + 2\delta \right\} \right] dx. \qquad (16)$$

When expanded up to the $0(|\beta|^6)$ terms it naturally coincides with (13).

In [2] the truncated Hamiltonian was shown to describe correctly easy-axis magnons $(\Delta > 0)$ but it gives a wrong result even for easy-plane magnon dispersion formula $(\Delta < 0)$. Let us look at this. Consider for that the dispersion of easy-plane magnons in the framework of (16). Make a substitution

$$\beta = 1 + \eta \qquad (17)$$

which means the expansion in the vicinity of the easy-plane classical vacuum state and keeping terms of the second order in η we come to

$$H = \frac{1}{2} s a_0^2 J \int \{-\Delta(\eta + \bar\eta)^2 + |\eta_x|^2 + (\frac{3}{4} + \delta)(\eta_x + \bar\eta_x)^2\} dx. \qquad (18)$$

This Hamiltonian generates the following linear equation

$$i\dot\eta = -\Delta(\eta + \bar\eta) - \frac{5}{2}\eta_{xx}(1 + \frac{4}{5}\delta) - \frac{3}{2}\bar\eta_{xx}(1 + \frac{4}{3}\delta).$$

Along with complex conjugate equation it gives the dispersion

$$\omega = |k|\sqrt{(1+\delta)k^2 + |\Delta|}. \qquad (19)$$

Hamiltonian (13) gives rise to

$$\omega_6 = |k|\sqrt{(\frac{7}{5} + \delta)k^2 + |\Delta|}. \qquad (20)$$

From (19) and (20) it follows that Hamiltonian (9) correctly describes the dispersion of easy-plane magnons with accuray 7% for large $k \gg |\Delta|$. .

The Holstein-Primakoff series is often truncated to allow for the $0(|\beta|^2)$ terms. In this approximation we have

$$i\dot\eta = -\Delta(\eta + \eta) - \eta_{xx}(1 + \delta) - \eta_{xx}(1 + 2\delta) \qquad (21)$$

or

$$\omega = |k|\sqrt{(\frac{3}{4} + \delta)k^2 + |\Delta|}, \qquad (22)$$

i.e. the dispersion a bit worse than (20) in the large k limit.

From (20) and (22) we see that H_4 and H_6 well describe even easy-plane magnon dispersion in the region of small $k \ll \ll |\Delta|$ and then the classical easy-plane vacuum as well.

It was shown in [2] that if in bosonizing (1) via HPT one takes into account the quantum ground state (vacuum) symmetry then the truncated $0(|\beta|^6)$ series gives correct result. In fact, let us direct the quantization axis OZ along the spontaneous magnetization axis lying in the easy-plane, then we obtain

$$H_{ep} = s^2 a_0 J \int \{|\tilde\beta_x|^2 - \frac{\Delta}{2}(|\beta|^2 - \frac{\tilde\beta^2 + \tilde\beta^2}{2})(1 - \frac{|\tilde\beta|^2}{2})\} dx, \qquad (23)$$

whereby

$$i\dot{\tilde\beta} = \tilde\beta_{xx} + \frac{\Delta}{2}(\tilde\beta - \bar{\tilde\beta}) - \frac{\Delta}{2}|\tilde\beta|^2\tilde\beta + \frac{\Delta}{8}\tilde\beta^3 + \frac{3}{8}\Delta|\tilde\beta|^2\bar{\tilde\beta}$$

which when expanded near the ground state, $|\tilde\beta| = 0$, gives

$$\omega = |k|\sqrt{k^2 + \Delta}. \qquad (24)$$

The latter is just the same as (19) if $\delta \ll 1$. The condition $\delta \ll 1$ is also necessary in order to derive the Landau-Lifshits equation starting from (1). The function β and spin angle θ are coupled with the relation following from (19)

$$\beta = \sqrt{2}\sin\frac{\theta}{2}\,e^{i\phi}. \qquad (25)$$

We proceed now to estimate "quantum" corrections of the order of 1/s to both easy-axis and easy-plane vacua. The Hamiltonian (12) gives for classical vacua the equation

$$\beta\{|\delta|(1 - |\beta|^2) + \frac{1}{8s}|\beta|^2(1 + \frac{3}{4}|\beta|^2)\} = 0. \qquad (26)$$

The equation (26) possesses two solutions

1. $\beta = 0$,

which corresponds to $S^z = S(1 - |\beta|^2) = S$, i.e. to the easy-axis vacuum. Here quantum corrections vanish.

$$2.\ -|\delta|(1 - |\beta|^2) + \frac{1}{8s}|\beta|^2 + (1 + \frac{3}{4}|\beta|\ \ ) = 0 \qquad (27)$$

Assuming $8s\delta \gg 1$ we have $(\delta < 0)$

$$|\beta|^2 = 1 - \frac{7}{32}\frac{1}{|\delta|s} \qquad (28)$$

whereby in the easy-plane case "quantum corrections spoil even the classical vacuum $|\beta|^2 = 1$ and spin $\vec{S}$ goes out from the easy plane the more the less value of $|\delta|S$ is.

So we can formulate quasiclassical condition

$$\frac{32}{7}|\delta|S \gg 1, \qquad (29)$$

Nearly the same we come to in the case of easy-axis model if we take into account magnon interaction. The latter as it follows from (13) and (26) is given at small $|\beta|^2 \ll 1$ by the formula

$$H_{int} \sim (-\delta + \frac{1}{8s})|\beta|^4 . \qquad (30)$$

It means when $8s < 1$ the interaction term changes the sign and magnon-magnon attraction is replaced by their repulsion. Such an effect does not occur in the framework of the exact solvable S = 1/2 quantum Heisenberg chain and in the model obteined by averaging over SCS. So we come to the conclusion: quasiclassical condition for the easy-axis model again is

$$8s\delta \gg 1. \qquad (31)$$

If we direct the quantization axis along the spontaneous magnetization axis, we obtain

$$\hat{H} = -J\sum_j \{S^x_j S^x_{j+1} + \rho S^y_j S^y_{j+1} + S^z_j S^z_{j+1}\}. \qquad (32)$$

Having ordered and averaged it (up to $O(1/s^2)$) becomes

$$H = H_0 + H_s$$

$$H_0 = \frac{\delta s^2 J}{a_0}\int dx \{(1 - \frac{|\beta|^2}{2})(\frac{\beta^2 + \bar{\beta}^2}{2} - |\beta|^2)\}, \qquad (33)$$

$$H_s = \frac{s^2 J}{a_0}\int dx \{1 + \frac{1}{4s} + \frac{|\beta|^2}{2}(1 + \frac{15}{8}\frac{1}{s}) + \frac{|\beta|^4}{4}\} \times$$

$$\times[(1 + \frac{\delta}{2})\frac{|\beta|^4}{8s} - \frac{\delta}{2}\frac{|\beta|^2(\beta^2 + \bar{\beta}^2)}{16}].$$

Consider terms not higher than $O(1/s)$ then

$$H_s = \frac{s^2}{a_0}\int dx |\delta|(|\beta|^2 - \frac{\beta^2 + \bar{\beta}^2}{2})(1 - \frac{|\beta|^2}{2}(1 + \frac{1}{8s})),$$

or after substitution

$$\beta = |\beta| e^{i\phi}$$

we get

$$(\bar{\beta}^2 + \beta^2)/2 = \beta^2 \cos 2\phi,$$

$$\mathcal{H} = \frac{2s^2}{a_0}|\delta||\beta|^2 \sin^2\phi\,(1 - \frac{|\beta|^2}{2}(1 + \frac{1}{8s})) \geq 0, \qquad (34)$$

whereby $\mathcal{H} = \mathcal{H}_{min}$ when $|\beta| = 0$. "Quantum corrections vanish in this case as well.

Conclusion

Nonvanishing "quantum" corrections for the easy-plane vacuum is an artifact viz., the result of unappropriate application of HPT.

Hamiltonian (33) leads to the correct dispersion

$$\omega = |k|\sqrt{(1 + \delta)k^2 + |\Delta|}. \qquad (35)$$

All this tells in favour of necessity to do a careful analysis of symmetry of the quantum ground state and the Hamiltonian before performing HPT.

III. SPIN COHERENT STATES AND THE LANDAU-LIFSHITS EQUATION

Spin coherent states or generalized CS of the SU(2) group (see [1]) were used to study quasiclassical behaviour of the Heisenberg ferromagnetic in a number of papers. Following Perelomov we write SCS in the form

$$|\Psi\rangle = (1 + |\Psi|^2)\, e^{-j\psi \hat{J}^+} |0\rangle, \qquad (36)$$

where $|0\rangle = |j, -j\rangle$ is the vacuum. In this case complex function Ψ performs the stereographic projection of the sphere S^2 onto the complex plane

$$\Psi = \tan\frac{\theta}{2} e^{i\phi}. \qquad (37)$$

In what follows spin operators will be denoted by $\hat{J}^\circ$ and $\hat{J}^\pm$ and their classical descendants by J°, $J^\pm$, i.e.

$<\hat{J}^{\circ}> = J^{\circ}, \quad <\hat{J}^{\pm}> = J^{\pm}$

calculating the correlators

$$<\Psi|\hat{J}^{\circ}|\Psi> = (\hat{J}^{\circ}) + \frac{1}{2j}<\hat{J}^{+}><\hat{J}^{-}>,$$
$$<\Psi|\hat{J}^{+}\hat{J}^{-}|\Psi> = <\hat{J}^{+}><\hat{J}^{-}> + \frac{1}{2j}(<\hat{J}^{\circ}> - j)^2, \quad (38)$$
$$<\Psi|\hat{J}^{-}\hat{J}^{+}|\Psi> = <\hat{J}^{+}><\hat{J}^{-}> + \frac{1}{2j}(<\hat{J}^{\circ}> + j)^2$$

one can see that pair correlators as well as three-points ones are expressed through J° and $J^{\pm}$. Therefore we introduce

$$\vec{S} = <\Psi|\hat{\vec{J}}|\Psi> \quad (39)$$

the classical spin vector.

Since spin operators commute in neighbour lattice sites. The GCS of the total lattice is just a direct product of separate sites

$$|\Psi> = \prod_j |\Psi>_j . \quad (40)$$

On the average of the Hamiltonian (1) with the help of (40) we loose all 0(1/s) quantum corrections and arrive at the classical model

$$H_{cl} = 2s^2 a_0 J \int \frac{|\Psi_x|^2 + \Delta|\Psi|^2}{(1+|\Psi|^2)^2} dx + \text{const} . \quad (41)$$

This Hamiltonian yields stereographic projection of the Landau-Lifshits equation if $\delta \ll 1$, since in getting it we disregard the term

$$\delta \int (S_x^z)^2 dx . \quad (42)$$

We also stress that in the easy-axis model minimum of the functional (41) is reached when $\Psi = 0$ or in terms of SCS the vector

$$|\Psi_0> = \prod_\ell <j, -j>_\ell \equiv \prod_\ell |0>_\ell \quad (43)$$

generated by the classical vacuum agrees with an exact quantum easy-axis ground state at any S. When $\Delta < 0$, the Hamiltonian (41) minimum is defined by the equation $|\Psi|^2 = 1$ and the corresponding vector (SCS) is now

$$|\Psi\ \ > = \prod_\ell 2^{-\ell} e^{-\hat{S}_\ell^{+}} |j, -j>_\ell . \quad (44)$$

This state structure depends substantially on j (SU(2) representation). Had been expanded in pure spin states, this assumes the forms

$$|\Psi_0>_j = \frac{1}{2^j} \sum_{\ell=0}^{2j} \sqrt{C_\ell^{2j}}\, |j, -j+1> \quad (45)$$

with C_ℓ^{2j} being the binomial coefficients and $|\Psi_0> = \prod_j |\Psi_0>_j$. The state (45) contains all 2j+1 spin states with probability for each

$$W = \frac{1}{2^{2j}} C_\ell^{2j} . \quad (46)$$

The state (44) is not an exact quantum easy-plane ground state and approximates to it to terms 0(1/s).

The Hamiltonian (41) gives for the easy-plane magnon dispersion

$$\omega = |k| \sqrt{(1+\delta)k^2 + |\Delta|} \quad (47)$$

which is none but the dispersion (35), obtained via the HPT procedure at $S = \infty$.

We can now infer that when $S = \infty$ the Glauber CS system and that of SU(2) group coincide (after compactification of the former). We should also underline here that SU(2) SCS can be applied to average only quantum Hamiltonians with very small anisotropy $\delta \ll 1$ for the greater δ the more significant difference in symmetries of the Hamiltonian and the SCS image (manifold).

If the Hamiltonian contains one-ion anisotropy terms of the type (2), e.g.

$$H_{\delta_1} = -\delta_1 (\hat{S}_j^z)^2 . \quad (48)$$

then averaged via SCS Hamiltonian gives again the Landau-Lifshitz equation (when $\delta_1 \ll 1$) but with the following renormalized anisotropy

$$\tilde{\delta} = \delta + \delta_1 (1 - \frac{1}{2s}). \quad (49)$$

Averaged via HPT procedure Hamiltonian gives the same equation with $\tilde{\delta} = \delta + \delta_1$, plus an additional term linear in S viz.

$\delta_1(S^z - S)$, which can be thought of as the one corresponding to an external magnetic field directed along OZ axis.

IV. THE S = 1 MAGNETIC MODEL AND SU(3) COHERENT STATES

Now we focus on the spin S=1 model which is related to the second element of the first line and the first element of the second line of the matrix (6).

Corresponding CS for S = 1 read

$$|\Psi\rangle = \frac{1}{1+|\Psi|^2}\{|0\rangle + \sqrt{2}\,\Psi\,|1\rangle + \Psi^2|2\rangle\} \tag{50}$$

in the SU(2) version, and

$$|\zeta\rangle = (1+|\zeta|^2)^{-1/2}\{|0\rangle + \zeta_1|1\rangle + \zeta_2|2\rangle\} \tag{51}$$

in the SU(3) version.

The S = 1 quantum system lives in four-dimensional parameter manifold of spin states, so that SU(2) averaged Hamiltonian governs behaviour of the system in its two-dimensional section

$$\zeta_2 = \frac{1}{2}\zeta_1^2, \quad \zeta_1 = \sqrt{2}\,\Psi\,. \tag{52}$$

The spin operators in this case are of the forms

$$\hat{S}^+ = \begin{Bmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{Bmatrix}, \quad \hat{S}^- = \begin{Bmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{Bmatrix}, \quad \hat{S}^z = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{Bmatrix}. \tag{53}$$

Upon calculating their averages and correlators we have

$$\langle\hat{S}^+\rangle = \sqrt{2}\,\frac{\bar{\zeta}_1 + \zeta_1\bar{\zeta}_2}{1+|\zeta|^2}, \quad \langle\hat{S}^-\rangle = \overline{\langle\hat{S}^+\rangle}, \quad \langle\hat{S}^z\rangle = \frac{|\zeta_2|^2 - 1}{1+|\zeta|^2}, \tag{54}$$

$$\langle(\hat{S}^+)^2\rangle = 2N\bar{\zeta}_2, \qquad \langle(\hat{S}^-)^2\rangle = 2N\zeta_2, \tag{55}$$

and $N = (1+|\zeta|^2)^{-1}$. Whereby the Casimir operator is

$$\langle\hat{C}^2\rangle = \frac{1}{2}(\langle\hat{S}^+\hat{S}^-\rangle + \langle\hat{S}^-\hat{S}^+\rangle) + \langle(\hat{S}^z)^2\rangle = 2\,. \tag{56}$$

Then

$$\begin{aligned} &\langle\hat{S}^+\hat{S}^-\hat{S}^+\rangle = N\,2\sqrt{2}(\bar{\zeta}_1 + \zeta_1\bar{\zeta}_2) \equiv \overline{\langle\hat{S}^-\hat{S}^+\hat{S}^-\rangle} = 2\langle\hat{S}^+\rangle\,, \\ &\langle(\hat{S}^+)^2\hat{S}^-\rangle = N\,2\sqrt{2}\,\zeta_1\bar{\zeta}_2 = \langle\hat{S}^+(\hat{S}^-)^2\rangle = \langle\hat{S}^z\hat{S}^+\rangle = \langle\hat{S}^-\hat{S}^z\rangle\,, \\ &\langle\hat{S}^-(\hat{S}^+)^2\rangle = N\,2\sqrt{2}\,\bar{\zeta}_1 = \langle(\hat{S}^-)^2\hat{S}^+\rangle = -2\langle\hat{S}^+\hat{S}^z\rangle = -2\langle\hat{S}^z\hat{S}^-\rangle\,, \\ &\langle(\hat{S}^\pm)^3\rangle = 0\,, \quad \langle(\hat{S}^z)^3\rangle = \langle\hat{S}^z\rangle\,. \end{aligned} \tag{57}$$

The following relations can be found from (57)

$$\begin{aligned} &\langle(\hat{S}^+)^2\hat{S}^-\rangle + \langle\hat{S}^-(\hat{S}^+)^2\rangle = 2\langle\hat{S}^+\rangle \\ &\langle\hat{S}^z\hat{S}^+\rangle - \langle\hat{S}^+\hat{S}^z\rangle = \langle\hat{S}^+\rangle\,. \end{aligned} \tag{58}$$

All triple correlators are expressed through the pair correlators and the averages. The same holds for the higher correlators.

Relations (57) and (58) enable to write all the averages and pair correlators as linear combinations of any of these, for example through the following

$$\langle\hat{S}^+\rangle, \quad \langle\hat{S}^z\rangle\,, \quad \langle\hat{S}^+\hat{S}^z\rangle, \quad \langle\hat{S}^-\hat{S}^z\rangle\,. \tag{59}$$

One can also obtain the identity

$$\begin{aligned} S^2 + \{&\langle\hat{S}^-\hat{S}^z\rangle\langle\hat{S}^+\hat{S}^z\rangle + \langle\hat{S}^z\hat{S}^-\rangle\langle\hat{S}^z\hat{S}^+\rangle + \langle\hat{S}^-\hat{S}^-\rangle\langle\hat{S}^+\hat{S}^+\rangle + \\ &+ (1 - \langle\hat{S}^z\hat{S}^z\rangle)^2\} = 1, \end{aligned} \tag{60}$$

where we use the notation

$$\vec{S} = \langle\hat{\vec{S}}\rangle\,. \tag{61}$$

The identity (60) replaces the conventional one, $\vec{S}$ = const and hence leads to breaking down the conservation law of spin vector modulus, so that

$$\partial_t S^2 \neq 0\,, \quad \partial_x S^2 \neq 0\,. \tag{62}$$

Instead we have conservation of the sum of S^2 and of a bilinear combination of pair correlators.

Let us consider the influence of (62) on the quasiclassical behaviour of S=1 magnetic (1). We begin with a simplest isotropic model ($\delta = 0$). Upon averaging (1) via SU(2) CS one obtaines the Landau-Lifshitz Hamiltonian

$$\mathcal{H}_{cl} = \frac{Ja_o}{2} \int (\vec{S}_x)^2 dx. \tag{63}$$

Averaging via SU(3) CS gives

$$\mathcal{H}_{cl} = \frac{J}{a_o} \int \{ [\langle \hat{S}^- \hat{S}^z \rangle \langle \hat{S}^+ \hat{S}^z \rangle + \langle \hat{S}^z \hat{S}^- \rangle \langle \hat{S}^z \hat{S}^+ \rangle + \langle \hat{S}^+ \hat{S}^+ \rangle \langle \hat{S}^- \hat{S}^- \rangle + (1 - \langle \hat{S}^z \hat{S}^z \rangle)^2] + \frac{a_o^2}{2} (\vec{S}_x)^2 \} dx. \tag{64}$$

This Hamiltonian contains additional terms of the quadrupole moment nature. Thus starting from isotropic quantum Hamiltonian we arrive at the classical one with some effective anisotropy.

To begin with we consider classical vacuum states, i.e. we find minima of the integrand of (64).

Easy-axis model. The classical vacuum

$$\zeta_1 = \zeta_2 = 0 \tag{65}$$

agrees with the SU(2) model and the quadrupole term vanishes.

Easy-plane model. The classical vacuum

$$|\zeta_1| = \sqrt{2}, \quad |\zeta_2| = 1 \tag{66}$$

is the same as in the SU(2) model and again the quadrupole term equals zero. The vectors (CS) describing these vacua also are the same in both models,

$$|\Psi\rangle_{ea} = \prod_j |0\rangle_j, \tag{67}$$

$$|\Psi\rangle_{ep} = \prod_j \frac{1}{2} \{ |0\rangle + \sqrt{2} |1\rangle + |2\rangle \}. \tag{68}$$

There are analogous results for vacuum states when S = 3/2 and averaging goes via SU(4) CS.

We can thus suppose that the classical vacuum states for arbitrary S lie in the SU(2) section of the whole 4S-dimensional spin state manifold.

We proceed now to discuss dispersion relations of the modes allowed in the S = 1 system.

For the case $\delta > 0$ we obtain

$$\omega_1 = k^2 a^2 - 2\delta, \tag{69}$$

$$\omega_2 = -4(1+\delta). \tag{70}$$

The first relation coincides with the SU(2) one and corresponding low frequency waves of the field ζ_1 can propagate in the system. In addition, in the system also there are the high frequency oscillations of the field ζ_2 with dispersion (70), and then quadrupole term does not vanish

$$\mathcal{H} = 4 |\zeta_2|^2 \tag{71}$$

and the system leaves the SU(2) section. The dispersion relations for the easy-plane model are

$$\omega_- = \frac{4}{9-\delta} k^2 a^2 (k^2 a^2 + 2|\delta|), \tag{72a}$$

$$\omega_+ = 4(9 + |\delta|), \tag{72b}$$

The first formula is equivalent to the well-known Bogolubov dispersion but renormalized by the coupling with the second mode. The latter corresponds to the high frequency oscillations of the field ζ_2 and again due to excitation of this mode the system goes out the SU(2) section and quadrupole contribution does not vanish

$$\mathcal{H} \propto \frac{1}{4} |\eta_2 - \sqrt{2}\eta_1|^2 \neq 0.$$

Taking into account $0(|\zeta|^4)$ nonlinear interactions of the modes in the easy-axis model we arrive at two coupled equations which solutions

$$\zeta_1 = \eta_1 e^{i\theta_1} \quad \text{and} \quad \zeta_2 = \eta_2 e^{i\theta_2} \tag{73}$$

are related to each other as

$$\eta_2 = \eta_1^2 / (2 + 2\delta + \omega) \tag{74}$$

and $\theta_2 = 2\theta_1$. Then the equation for the first component assumes the forms

$$a_o^2 \eta_{1xx} + (2\delta + \omega)\eta_1 + 2\delta\eta_1^3 = 0, \tag{75}$$

i.e. the conventional nonlinear Shrödinger equation.

This equation possesses many-soliton solutions and plane wave propagating through the system is unstable against its decay into the set of solitons.

Solutions (73) and those of (75) show that the high frequency oscillation are captured by the low frequency waves and together they form solution like solutions such that the quadrupole contribution of such composite solutions vanishes unlike that of the linear waves.

CONCLUSION

For the Heisenberg model studied SU(2) section may be regarded as classical attractor in the spin phase space (manifold) of the initial easy-axis quantum system, for both the vacuum states and for the nonlinear (soliton) "trajectories" in the vicinity of the vacuum. It may be of interest to underline once more that had adjusted two modes form nonlinear stationary waves (solitons) which tend to the SU(2) attractor. Once again mysterious properties of solitons?

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Абдуллоев Х.О. и др. E17-89-800
Обобщенные спиновые когерентные состояния как инструмент изучения квазиклассического поведения магнетика Гейзенберга

Для описания квазиклассического поведения ферромагнетика Гейзенберга использованы обобщенные когерентные состояния, определенные на группах Гейзенберга-Вейля, SU(2) и SU(3). Сравнение полученных результатов выявило некоторые замечательные свойства изучаемой модели. Показано, в частности, что использование преобразований Холштейна-Примакова до усреднения по глауберовым когерентным состояниям приводит к появлению асимптотических рядов, которые должны быть оборваны. Для изучения магнетиков со спином S=1 наиболее адекватным инструментом являются когерентные состояния группы SU(3), а SU(2) сечение полного спинового фазового пространства оказывается классическим аттрактором для нелинейных фазовых траекторий.

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Abdulloev Kh.O. et al. E17-89-800
Generalized Spin Coherent States as a Tool to Study Quasiclassical Behavour of the Heisenberg Ferromagnet

To describe quasiclassical behaviour of the Heisenberg ferromagnet generalized coherent states defined on the Heisenberg-Weyl, SU(2) and SU(3) groups are used. The results thereby derived are compared to show some remarkable features of the model studied. In particular SU(2) section of the total spin phase space turns out to be a classical attractor for phase trajectories.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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