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T.M.Mishonov

FLUCTUATIONAL TORQUE FOR HIGH TEMPERATURE SUPERCONDUCTORS IN STRONG MAGNETIC FIELD

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1. Introduction

As is well-known high-temperature superconductors H.T.S. have a strong anisotropy connected with the weak binding of different conducting CuO-layers [1-3]. In the CuO-plane the anisotropy created by the orthorombic splitting is not large [4,5]. In this sense, the CuO-planes (a-b-planes) are "easy" planes [6,7], and the effective mass m_{CuO} of a Cooper pair in them is much lower than the effective mass m_c in the c-direction perpendicular to the CuO plane. For the ratio of these masses

$$\frac{m_c}{m_{CuO}} = \frac{H_{c2}^{a}\ H_{c2}^{b}}{(H_{c2}^{c})^2}$$

experimental data [8] for $Y_1Ba_2Cu_3O_{7-\delta}$ give 25÷30; see for the discussion [9]. But for the $Bi_{2.2}CaSr_{1.9}Cu_2O_{8+\delta}$ crystal [10] this ratio obtained by measuring the upper critical field in principal directions of the crystal H_{c2}^{a}, H_{c2}^{b}, H_{c2}^{c} is considerably larger, $(1÷2)\times10^3$.

Anisotropy of the mass tensor **m** of Cooper pairs for noncubic superconductors is an old idea [11,12], which is successfully used for the description of the transverse magnetization $M_\perp$ and the torque Θ of the Abrikosov lattice in H.T.S. [13], for the angular (θ,φ) dependence of the H_{c2} [14-17], and for other thermodynamic characteristics of anisotropic superconductors (for introduction into the problem see, for example, papers [18-29]).

The upper critical field is one of the most important characteristics of superconductors. The temperature dependence $H_{c2}(T)$, for example, determines GL coherence lengths $\xi_a(0), \xi_b(0), \xi_c(0)$ along the principal axes of the crystal. But unfortunately, for the H.T.S., the determination of $H_{c2}(T)$ by resistivity measurements is difficult due to the depinning of the flux [30] and strongly fluctuational phenomena [31-32]. Care is therefore needed in determining coherence lengths from "H_{c2}" measurements [17]. The nonlinear dependence of $H_{c2}(T)$ provokes some considerations [33-36] of strong fluctuations or even modifications of the classical [37,38] GL model. This problem is widely discussed in the literature [39,40], but some recent clear experiments give, nevertheless, the linear temperature dependence for the lower [42] $H_{c1}(T)$ and upper [4] $H_{c2}(T)$ critical fields, as for usual BCS-GL superconductors.

In some cases, as for example the temperature dependence of the a-b-plane penetration depth [41] $\lambda(T)$ in the $Y_1Ba_2Cu_3O_{7-\delta}$ epitaxial films, the comparison of the H.T.S. experimental data with the BCS weak-coupling theory is remarkable.

The main aim of this work is to suggest a new contact-free method for the determination of $H^n_{c2}(T)$ of anisotropic H.T.S. by the measurement of the FT. More precisely, the method gives the inverse function $T_{c2}(\mathbf{H})$, the critical temperature as a function of the external magnetic field $\mathbf{H}$ ($\mathbf{n}=\mathbf{H}/H$).

An experimental method must be based on a simple physical picture, and in our case this simplicity arises due to the thermodynamic (gaussian) nature of superconductive fluctuations at temperatures $T > T_{c2}(\mathbf{H})$. Under these conditions, in the normal phase, there appear, by fluctuations, decaying Cooper pairs with a negligible interaction. They move and are rotated by the magnetic field mainly in the CuO-planes, and therefore, their magnetic moment $\mathbf{M}$ is oriented mainly in the c-direction. As a result, the anisotropy of the material creates a FT

$$\underline{\Theta} = \mathbf{H} \times \mathbf{M}, \qquad \Theta = H\, M_\perp,$$

which aspires to turn "easy" CuO-planes parallel to the magnetic field, as is shown in fig. 1.

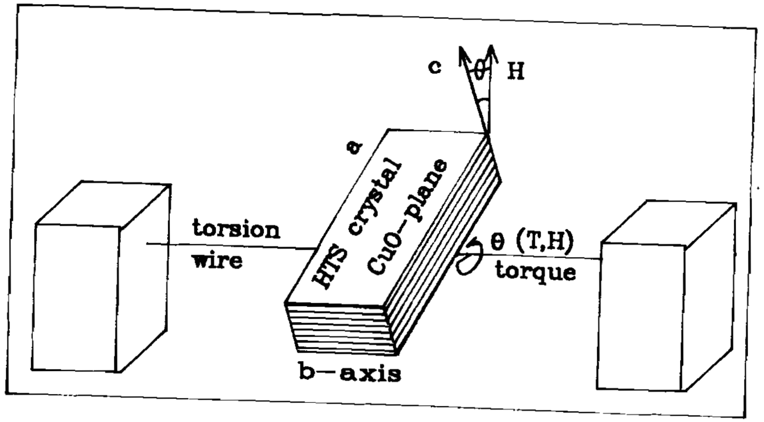

Figure 1. A principle scheme of a torsional magnetometer similar to the one used for measurements of the magnetization of a two-dimensional electron gas [43]. The superconductor specimen is affixed to the torsion wire and θ is the angle between the magnetic field and c-axis.

In the normal region the gaussian integral for the partition function can be easily obtained, and in the framework of the GL theory a simple analytical expression for the Gibbs free energy G is derived.

Thermodynamic quantities such as the heat capacity, diamagnetic moment, and the diamagnetic susceptibility are expressed in terms of derivatives of G. For example, FT is a derivative with respect to angular variables which describe the orientation of the specimen with respect to the magnetic field. Near to the phase curve $H^n_{c2}(T)$ the FT has a singular behaviour.

The main idea of the proposed method for the determination of $T_{c2}(\mathbf{H})$ consist in the extrapolation of this high-temperature dependence from the gaussian region to the critical one. A similar method has been used [44] for the determination of the BCS critical temperature T_c of thin film superconductors with the help of the fluctuational Aslamasov-Larkin conductivity. By that method the experimental data processing does not require a investigation of the critical region, where the theoretical picture is very complicated due to disorder, defects of a crystal, and due to interaction of different fluctuational modes.

2. Model

Our starting point is the Gibbs free energy functional of the anisotropic GL model

$$G\{\psi, A\} = \langle\psi|\hat{H}|\psi\rangle + (b/2)\int|\psi|^4 d^3x + \int(\mathrm{rot}\underline{A} - \underline{H})^2\, d^3x \,/\, 8\pi\,, \tag{1}$$

where

$$\hat{H} = (\hat{p} - qA/c_1)\,(2\,\mathbf{m})^{-1}\,(\hat{p} - qA/c_1) + a(T),$$

$$\mathbf{m} = \begin{pmatrix} m_a & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_c \end{pmatrix}, \quad \hat{p} = -i\hbar\nabla, \quad q = 2e,$$

$$a(T) = a(0)\tau, \quad \tau \equiv (T - T_c)/T_c,$$

and c_1 is the light velocity. Here q is the Cooper pair charge, and the coordinate system (a, b, c) is chosen so that the Cooper pair effective mass tensor $\mathbf{m}$ is diagonal.

In the normal region $T > T_{c2}(\mathbf{H})$ outside of the narrow band near to the phase curve fluctuations of the order parameter are small, the diamagnetic screening is negligible, and the magnetic field rot**A** coincides with the external magnetic field **H**. The second term in (1) which represents the interaction of different fluctuational modes is also negligible in this case, and the free energy is expressed through a gaussian integral and the determinant of the operator $\hat{H}$; see for a introduction the nice review of fluctuations in superconductors [45].

$$\exp(-G(T,\mathbf{H})/k_B T) = \int\int D\psi\, D\psi^* \exp(-\langle\psi|\hat{H}|\psi\rangle/k_B T)$$

$$= \det(\pi k_B T/\hat{H}). \qquad (2)$$

The spectrum of the "hamiltonian" $\hat{H}$ is given by the well-known expressions for the cyclotron resonance in the effective mass approximation [46]

$$E_{n,p_\parallel} = \hbar\omega_c(n + \tfrac{1}{2}) + p_\parallel^2 / 2m_\parallel + a(T), \qquad (3)$$

where: $n=0,1,2,\dots,\quad -\infty < p_\parallel < +\infty,\quad \omega_c = qH/m_\perp c_l,$

$m_\perp \equiv (\det(\mathbf{m})/m_\parallel)^{1/2} = m_*/g(\mathbf{n}),$

$m_* = (\det\mathbf{m})^{1/3},\quad m_{CuO} = (m_a m_b)^{1/2},$

$\mathbf{n} = \mathbf{H}/H = (s_\theta c_\varphi, s_\theta s_\varphi, c_\theta),$

$s_\theta \equiv \sin\theta,\ c_\varphi \equiv \cos\varphi,\ \dots$

$m_\parallel \equiv \mathbf{n}\,.\mathbf{m}\,.\mathbf{n} = m_* g^2(\mathbf{n}),$

$g(\theta,\varphi) = (\mathbf{n}\,.\,\underline{\mu}\,.\,\mathbf{n})^{1/2} = [(\mu_a c_\varphi^2 + \mu_b s_\varphi^2)s_\theta^2 + \mu_c c_\theta^2)]^{1/2},$

$\underline{\mu} = \mathbf{m} / m_*,\quad \det\underline{\mu} = 1.$

Here (θ,φ) are spherical angles of orientation of an external magnetic field in the coordinate system (a,b,c) connected with main axes of the tensor **m** and the crystal.

Each of these eigenvalues is $N_\perp$ time degenerated

$$N_\perp = S_\perp H/\phi_0, \qquad (4)$$

where: $\phi_0 = 2\pi\hbar c_l/q$ is the fluxon, $S_\perp$ is an area of the specimen perpendicular to the magnetic field.

The upper critical field $H^n_{c2}(T)$ can be obtained from the condition of vanishing of the ground state ($n = 0$, $p_\parallel = 0$) energy of the spectrum (3). From this condition

$$\tfrac{1}{2} + a(T)/\hbar\omega_c = 0,$$

after elementary substitutions we easily obtain formulae [47]:

$$H^n_{c2}(T) = H^n_{c2}(0)\,(-\tau),\quad 0 < (-\tau) \ll 1, \qquad (5)$$

where

$H^n_{c2}(0) \equiv H^*_{c2}(0)/g(\mathbf{n}) = -T_c(\partial H^n_{c2}(T) / \partial T)\big|_{T = T_c},$

$H^*_{c2}(0) \equiv \phi_0/2\pi\xi_*^2(0),$

$\xi_*(0) \equiv \left[\xi_a(0)\xi_b(0)\xi_c(0)\right]^{1/3},$

$\hbar^2/2m_i\xi_i^2(0) \equiv a(0),\quad i = a,b,c,$

$\xi_i(0) = \xi_*(0)/(\mu_i)^{1/2}.$

This angular dependence is in good agreement with the experiment, and the data [8] for $Y_1Ba_2Cu_3O_{7-\delta}$ give [9]

$H^*_{c2}(0) = 370$ T, $\quad \xi_*(0) = 9.3$ Å,

$\mu_a = 0.4,\quad \mu_b = 0.3,\quad \mu_c = 8.8.$

3. Calculation

The substitution of (4) and (3) into (2) gives

$$G(T,\mathbf{H}) = k_B T_c N_\perp \sum_{n=0}^{\infty} L_\parallel \int_{-\infty}^{+\infty} \frac{dp_\parallel}{2\pi\hbar} \ln\left(E_{n,p_\parallel} / \pi k_B T_c\right). \qquad (6)$$

Here $L_\parallel$ is the length of the specimen in the direction of the magnetic field. The geometrical form, of course, does not matter, and the result depends only on the volume $V = L_\parallel S_\perp$ of the specimen. Only the temperature dependence of $a(T) \propto \tau$ is relevant for the critical behaviour, and therefore in (6) the temperature T is replaced by T_c.

Let us introduce the notation

$$k \equiv (p_\parallel^2 / 2m_\parallel\hbar\omega_c)^{1/2},$$

$$x \equiv \frac{1}{2} + \frac{a(T)}{\hbar\omega_c}.$$

In the argument of the logarithm in (6) we separate the irrelevant for the critical behaviour multiplier $\hbar\omega_c/\pi k_B T_c$

$$\ln\left(E_{n,p_\parallel} / \pi k_B T_c\right) = \ln(n + x + k^2) + \ln\hbar\omega_c/\pi k_B T_c.$$

The last term in this equality adds an irrelevant (although infinite) constant to the free energy and we will neglect it in the further analysis. It is useful also to introduce dimensionless magnetic fields

$$h \equiv H/H^{n}_{c2}(0) = gh^{\bullet}, \qquad h^{\bullet} \equiv H/H^{\bullet}_{c2}(0).$$

Thanks to the equality

$$k_B T_c N_{\perp}(L_{\parallel}/\hbar)(2m_{\parallel}\hbar\omega_c)^{1/2} = E_0 h^{3/2}/2^{1/2}\pi ,$$

$$E_0 \equiv k_B T_c V/\xi_a(0)\xi_b(0)\xi_c(0),$$

which is checked by an elementary substitution, the Gibbs free energy takes the form

$$G(T,H) = (E_0/2^{1/2}\pi)h^{3/2}f(x), \tag{7}$$

where

$$f(x) \equiv \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} \ln(x + n + k^2)\, dk/2\pi .$$

The obtained function $f(x)$ contains ultraviolet (UV) divergencies, and for their regularization we will use a method similar to the zeta-function method [48].

Let

$$\zeta(\nu,x) \equiv \sum_{n=0}^{\infty} 1/(x+n)^{\nu}, \qquad x > 0, \qquad \nu > 1$$

be the Hurwitz zeta-function [49] defined for $\nu < 1$ by analytical continuation in the variable ν.

In our further thermodynamical analysis we will use the formulae:

$$\frac{d}{dx}\zeta(\nu,x) = -\nu\zeta(\nu+1,x) \tag{8a}$$

$$\zeta(\nu,x) = \begin{cases} x^{-\nu} & 0 < x \ll 1 \quad (8b)\\ (2^{\nu}-1)\zeta(\nu), & x = 1/2 \quad (8c)\\ \dfrac{1}{(\nu-1)x^{\nu-1}} - \dfrac{1}{24x^{\nu+1}}, & x \gg 1 \quad (8d)\end{cases}$$

$$\zeta(\nu) \equiv \sum_{n=1}^{\infty} 1/n^{\nu} = \zeta(\nu,1), \qquad \nu > 1, \tag{8e}$$

$$\zeta(\nu) = 2^{\nu}\pi^{\nu-1}\sin(\pi\nu/2)\,\Gamma(1-\nu)\,\zeta(1-\nu), \tag{8f}$$

$$\zeta(-1/2,1/2) = 0.060888. \tag{8g}$$

The equality

$$f(x) = \zeta(-1/2,x) + Ax + B \tag{9}$$

can be easily checked after double x-differentiation of the definition in (7) using (8a). Our method for UV regularization of (7) consists in the neglect of infinite, but irrelevant for the critical behaviour constants A and B in (9).

For the Gibbs free energy we finally get

$$G(T,H) = \left(\frac{1}{2^{1/2}\pi}\right)k_B T_c \frac{V}{\xi_a(0)\xi_b(0)\xi_c(0)} h^{3/2}\zeta(-1/2,x), \tag{10}$$

where

$$x = \frac{1}{2} + \frac{\tau}{2h} = t_H/2h$$

$$= \frac{1}{2}\left[(T - T_{c2}(H))/H\right]\left[-\partial H^{n}_{c2}(T)/\partial T\right]\Big|_{T=T_c},$$

$$t_H \equiv (T - T_{c2}(H)) / T_c .$$

The latter formulae are verified directly by substituting the definition of x, and using the linear temperature dependence of the upper critical field. In the dimensionless variables the phase curve $H^{n}_{c2}(T)$ can be found from the condition

$$x(\tau,h) = 0,$$

and has the obvious solution

$$h_{c2}(\tau) = (-\tau) \quad \text{or} \quad \tau_{c2}(h) = -h, \qquad \text{for} \quad |\tau|, h \ll 1.$$

Formula (10) is a central result of our consideration. This formula has the same functional form as in the "spherical cow approximation" [50], i.e. as for a usual isotropic type-II BCS superconductor [51]. In the next section, we will present expressions obtained by this formula for the FT in different physical conditions and mathematical variants of (8).

4. Fluctuational torque

Let the H.T.S. single crystal is oriented as is shown in fig.1. The b-axis of the crystal is parallel to the torsion wire and $\varphi = 0$. Under torsion the magnetic field turns in the (a,c)- plane and the torsion angle coincides with θ. In this case,

$$\Theta_b(\theta,h^{\bullet},\tau) = -(\partial G / \partial \theta)_{\tau,h^{\bullet}} . \tag{11}$$

Angular variables appear in G only through the function $g(\theta,\varphi)$ and therefore it is useful to express the magnetic field in (10) through the angle-independent dimensionless magnetic field $h^{\bullet}$

$$G(T,H) = (E_0/2^{1/2}\pi)\,(h^{\bullet}g)^{3/2}\,\zeta(-1/2,1/2 + \tau/2h^{\bullet}g), \tag{12}$$

$$g = (\mu_a\sin^2\theta + \mu_c\cos^2\theta), \qquad \varphi = 0.$$

4.1. H_{c2} DETERMINATION

Near the phase curve $0 < t_H \ll h$ and $x \ll 1$. In this case we can use in (11,12) the approximative formula (8b). We get for the FT

$$\Theta_b = \frac{\pi}{2}(\mu_c-\mu_a)(Vk_BT_c)\,\xi_*(0)\,(H/\phi_0)^2\,\sin(2\theta)/(t_H)^{1/2}. \qquad (13)$$

This law gives the possibility to determine the H_{c2} by measuring the FT. Experimentally, FT is determined by subtraction from the torque, its value at high temperatures and in the same magnetic field $\Theta_b(T \gg T_c, H)$. That is why a part of normal electrons cancels and so does irrelevant for the critical behavior part of superconducting fluctuations. This experimental procedure is analogous to the cancellation of constants A and B by UV regularization of (9). Formula (13) can be rewritten in the form

$$\left[\frac{\pi}{2}k_BT_c\,(\mu_c-\mu_a)\,(H/\phi_0)^2\,\sin(2\theta)\Big/\Big(\Theta_b(T,H)-\Theta_b(T\gg T_c,H)\Big)\right]^2$$

$$= \frac{t_H}{\xi_*^2(0)}, \qquad t_H = \frac{T - T_{c2}(H)}{T_c} = \tau + h. \qquad (14)$$

With the help of (14) the transition temperature $T_{c2}(H)$ can be determined by a linear extrapolation of the expression in the left-hand side versus the temperature. This field-cooling method is shown schematically in fig. 2 and 3.

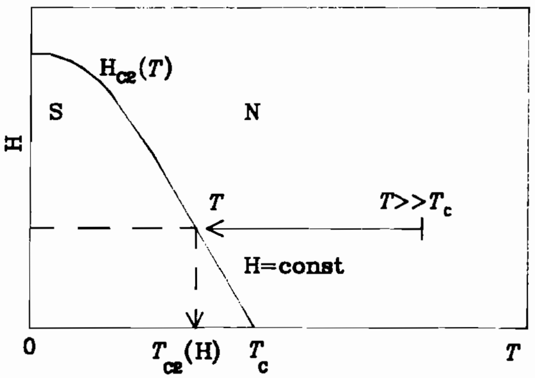

Figure 2. Determination of the upper critical field $H_{c2}(T)$ by FT measurements. Field-cooling of the specimen begins off a high temperature.

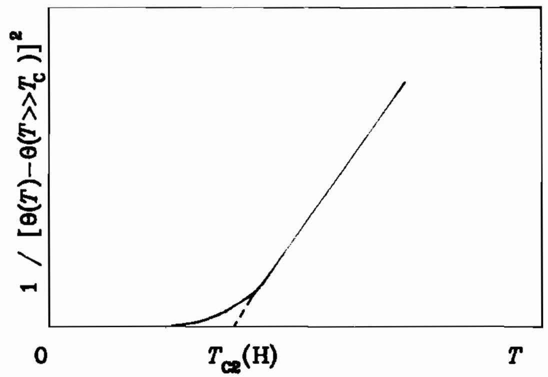

Figure 3. Critical temperature determination by extrapolation from the normal (N) high temperature region where only gaussian thermodynamical fluctuations of the ideal crystal are essential. The method is analogous to that described in [44,45].

If the magnetometer is calibrated in absolute units, then the coefficient in front of t_H in (14) gives the value of $(\mu_c-\mu_a)\xi_*(0)$. Such measurements are however more difficult.

4.2. STRONG MAGNETIC FIELDS ($T = T_c$)

Just at the critical temperature T_c the high field approximation $h \gg \tau$ is applicable. By the substitution $x = 1/2$ in (8c), from (8g) and (12) we obtain

$$G(T_c,H) = VT_cA_*\,\frac{2}{3}\,(Hg)^{3/2}, \qquad (15)$$

$$A_* = 3k_B\pi^{1/2}\,\zeta(-1/2,1/2)\,/\phi_0^{3/2},$$

$$3\pi^{1/2}\,\zeta(-1/2,1/2) = 0.32376.$$

It is interesting that $\xi_*(0)$ cancels and A_* is a constant general for all superconductors [53]. When the magnetic field is parallel to some of main axes, the c-axis for example, for the magnetic moment

$$M = -(\partial G/\partial H)_T$$

from (15) we get

$$- M / VT_c H^{1/2} = A_* \, g_c^{3/2}, \tag{16}$$

$$g_c^{3/2} = (\xi_a(0)\xi_b(0))^{1/2} / \xi_c(0) = (m_c/m_{CuO})^{1/2}.$$

For an isotropic material $g = 1$. As can be expected, the diamagnetic moment has a maximum when H is perpendicular to CuO-planes.

In this strong field regime we get for the FT from (11-12) and (15)

$$-\Theta_b / VT_c H^{3/2} = A_*(\mu_c-\mu_a)s_\theta c_\theta / (\mu_a s_\theta^2 + \mu_c c_\theta^2)^{1/4}. \tag{17}$$

Certain deviations from this law will give valuable information about the nonlocality of the Cooper pairs analogous to the well-known [54] deviation from (16) for conventional superconductors. For H.T.S. the nonlocality can be connected with the Josephson junction between CuO-layers and also with the Pippard type nonlocality for Cooper pairs propagating along these two-dimensional layers. It must be remembered that few per cent deviations [54] from (16) are observed even if the coherent length $\xi(0)$ is an order of magnitude smaller than the magnetic length

$$l_H \equiv (\hbar c_l/eH)^{1/2} = 256 \text{ Å} / (H(T))^{1/2}.$$

Therefore in strong magnetic fields $\simeq$ 5T it is possible to anticipate deviations from the power law (17) ($\Theta_b/H^{3/2}$ versus $\xi_{ab}(0)/l_H$) created by a Pippard type nonlocality in CuO-planes. In framework of BCS theory the Pippard type nonlocality of anisotropic superconductors is investigated in [55].

The advantage of torque measurements over SQUID measurements is the possibility to work with higher magnetic fields. In addition to this, the sensitivity of some torque magnetometers is higher [43] than that of best commercial SQUIDs.

4.3. WEAK FIELDS

At the end we get the anisotropy of the magnetic susceptibility $\underline{\chi}$ for a superconductor in a weak magnetic field ($h \ll \tau \ll 1$). In this case $x \simeq \tau/2h \gg 1$, and for the zeta-function we will use the asymptotic (8d) obtained by the Poisson-Euler summation. Thus, from (12) we get

$$G(T,\mathbf{H}) = \frac{4}{3} C_+ V \tau^{3/2} - V\mathbf{H}.\underline{\chi}.\mathbf{H}/2,$$

where [56,57]

$$C_+ = k_B T_c / 8\pi\xi_a(0)\xi_b(0)\xi_c(0),$$

$$\underline{\chi} = -(\pi/6)\,(k_B T_c/\phi_0^2)\,\underline{\mu}\,\xi_*(0)/\tau^{1/2}.$$

The anisotropy of the magnetic susceptibility follows the anisotropy of the Cooper pair mass tensor. For example, susceptibility is maximal when the magnetic field is parallel to the c-axis and Cooper pairs rotate in "easy" CuO-planes.

The diamagnetic moment in this case is

$$\mathbf{M} = \underline{\chi}.\mathbf{H},$$

and the torque can easily be found.

5. Discussion

Torque measurements are a well developed method in physics of magnetism [58]. These measurements have been long used for the investigation of anisotropy of superconductors [59].

On the other hand, investigations on fluctuations in H.T.S. are a vast field of activity. Here can be mentioned the specific heat [60], the excess electrical conductivity [61,62], the fluctuation induced diamagnetism [63], and the thermopower [64]. Fluctuations in strongly anisotropic H.T.S. are pronounced because of a small coherence length.

Therefore we may expect many experimental papers to appear on measurements of FT $\propto E_0$. For $Y_1Ba_2Cu_3O_{7-\delta}$

$$E_0/V = 1.50 \text{ J/cm}^3.$$

Observation of the Pippard type nonlocality in CuO-planes may be a more difficult problem. In particular, for the extremely layered material $Bi_{2.2}CaSr_{1.2}Cu_2O_{8+\delta}$ [65], it can be expected that main deviations from the local theory are caused by the Josephson junction between layers [66]

$$\hat{H} = \hat{p}_a^2/2m_a + \hat{p}_b^2/2m_b + l_c[1 - \cos(c\hat{p}_c^2/h)] + a(T),$$

$$l_c = (\hbar/c)^2/m_c.$$

For strong magnetic fields

$$\hbar(qH_c/m_{CuO}c_l) \gg l_c, \quad H_c = H\,|c_\theta|$$

a crossover [67,68] from a three-dimensional (3D) to a purely two-dimensional (2D) behaviour will be observed in this superconductor. The analytical result in our approach is obtained by the replacements:

$$\int dp_\parallel/2\pi\hbar \Rightarrow 1/c \quad \text{in (6)},$$

$$m_c \Rightarrow \infty, \quad \omega_c = qH_c/m_{CuO}c_l, \quad \text{in (3)}.$$

Thermodynamic quantities in this case are expressed through zeta-functions of integer ν. Such a 2D model is also appropriate for the

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