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PLASMA WAVES IN JOSEPHSON ARRAYS AND THIN SUPERCONDUCTING LAYERS

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Recent experimental and theoretical studies of arrays of Josephson junctions and superconducting layers have attracted great attention (see for example [1,2] and citations therein). Though such structures are conducting, one may wonder if there are existing plasma waves like those in other conducting media (plasma, metals, 2DEG and so on). The aim of this work is to obtain the excitation frequency dispersion law and to describe how they can be detected.

Let us consider a two-dimensional square lattice with a lattice constant a (recently used superconducting arrays contain $N \times N$ grains ($N \propto 1000$) [2-4]). Each grain has coordinates $\vec{x} = (x,y) = a(\ell_1, \ell_2)$, $\ell_{1,2} = 1,2,\ldots$. The quantities that are attributed to a grain are the Josephson's phase $\theta(\vec{x})$ and the electrical potential $\phi(\vec{x})$. The links between the grains are equal Josephson junctions with critical current I_c and normal state resistance R_N. Each link is labeled with two vectors $(\vec{x},\vec{n})$, $\vec{x}$ being the starting grain coordinates and $\vec{n}$ being the direction vector $\vec{n} = (a,\ 0);\ (0,\ a)$. We will denote current through the link which is directed toward x with $I_x(\vec{x})$ and similiarly for y direction $I_y(\vec{x})$ (notion $\vec{I}$ means that the link is directed in $\vec{n}$ direction).

Let a plane wave external field with electrical potential $\phi^{ex}(\vec{x},t) = \phi_0^{ex} \exp[\,i(\vec{k},\vec{x} - \omega t)]$ $(e^* \phi_0^{ex} \ll h\omega \ll \Delta, \lambda = 2\pi/k \gg a)$ perturbate our system. We will use resistively shunted model to describe Josephson junctions [5,6]. The equations of the model are (t is the time):

$$\vec{I}(\vec{x}) = I_c \sin(\theta(\vec{x}+\vec{n},t) - \theta(\vec{x},t)) + (\phi^{ex}(\vec{x}+\vec{n},t) - \phi^{ex}(\vec{x},t))/R, \quad (1)$$

$$h \frac{d}{dt}(\theta(\vec{x}+\vec{n}) - \theta(\vec{x})) = e^*(\phi^{ex}(\vec{x},t) - \phi^{ex}(\vec{x}+\vec{n},t)), \quad (2)$$

where $e^* = 2e$ is the charge of the Cooper pair, R is the resistance between grains and h is Plank's constant. In the long wave limit the phase differences θ between adjacent grains are small and we can replace the $\sin\theta$ in (1) with θ. In this limit $\vec{I}(\vec{x}) \ll I_c$ and the resistively shunted model [5] §2.3 gives correct description of the Josephson junctions if we take appropriate value for R (equal to $10 \div 20\, R_N$ for tunnel junctions, or R_N for SNS junctions and for resistively shunted junctions /5 chapter 2).

From (1) and (2) it is easy to obtain the current caused by the external field in the long wave limit:

$$\vec{I}(x) = \frac{e^* I_c}{h\omega}\left(1 - \frac{i\omega}{\omega_c}\right)\vec{k}\cdot\vec{n}\,\phi^{ex}(x,t),$$

where $\omega_c = e^* I_c R/h$. If we introduce the surface density of the current $\vec{j}(\vec{x},t) = \vec{I}(\vec{x})/a$ we will have:

$$\vec{j}(\vec{x}) = \sigma_{Dr}\,\vec{E}^{ex}, \quad \vec{E}^{ex} = -\nabla\phi^{ex}, \quad \sigma_{Dr} = i\,\frac{e^* I_c}{h\omega}\left(1 - \frac{i\omega}{\omega_c}\right),$$

where σ_{Dr} is Drude's conductivity of the array (we name it Drude's conductivity because of the same ω - dependence of the first term as the high frequency response of free particles) and $\vec{E}^{ex}$ is the external electric field. Taking into account the charge conservation law $\nabla\vec{j} = -\partial_t\rho$ (ρ is charge density) for the polarizability Π of the system ($\rho = \Pi\phi^{ex}$) we obtain $\Pi = -i\sigma_{Dr}k^2/\omega$.

In order to obtain self consistent description of the system we have to calculate the potential caused by the given charge density $\rho(\vec{x},t) = \rho_0\exp[i(\vec{k}\cdot\vec{x} - \omega t)]$. We will consider the general case in which there is ideal conducting plane parallel to the array at a distance d (see fig.1) like the ground plane is the integral Josephson schemes /7/ (see fig.1) (this can be metal layer with negligible resistivity or what is better superconductor). In this case result is /8/:

$$\phi = \frac{4\pi}{k(\epsilon_1 + \epsilon_2\,\mathrm{cth}\,kd)}\,\rho = U\rho,$$

where $\epsilon_{1,2}$ are dielectric constants of adjacent dielectrics and U is the Fourier transform of the Coulomb potential of the charge in this structure (we are neglecting the retardation effects and because of the long wave limit the near neighbour capacitance effects). In the dielectric formalism /9/ the renormalized conductivity of the system is $\sigma = \sigma_{Dr}/\kappa$, where $\kappa = 1 - \Pi U$. The dispersion equation of the plasmons is $\kappa = 0$, so we obtain the main result (neglecting dissipation):

$$\omega_{p\ell}^2(k) = 4\pi e^* I_c k/(\epsilon_1 + \epsilon_2\,\mathrm{cth}\,kd)h. \qquad (3)$$

From experimental point of view only the kinetic inductance per square $L_\square$ /2/ is essential for two dimensional plasmons.

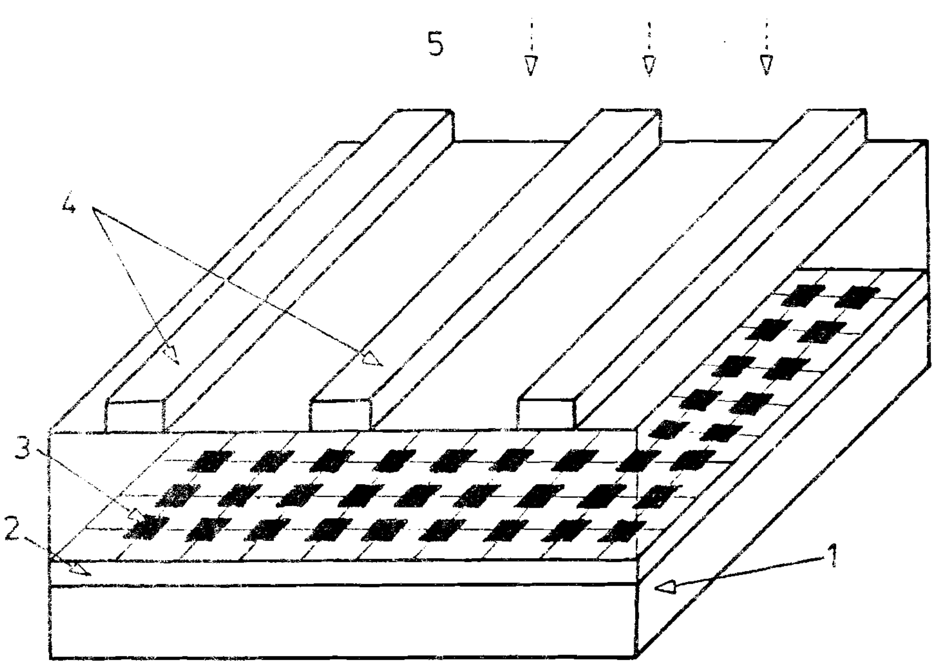

Fig. Scheme of the structure for observation of Josephson plasmons. 1 - metal layer; 2 - oxide layer; 3 - Josephson array; 4 - metal grating; 5 - microwave irradiation. This scheme is similar to those used in /26/ for observation of two-dimensional plasmons in Si-inversion layer.

For Josephson arrays /2,3,10/ $L_\square = h/e^* I_c$. Depending on the geometry of the system we obtain the well-known from plasmons in 2DEG square root /11-14/ dispersion law:

$$\omega_{p\ell}^2(k) = 2\pi L_\square^{-1}\,k/\epsilon_m \quad \text{for} \quad d \gg \lambda, \qquad (4)$$

and acoustic /15,16/ dispersion law:

$$\omega_{p\ell}(k) = k(4\pi L_\square^{-1}\,d/\epsilon_2)^{1/2} = v_{ac}k \quad \text{for} \quad d \ll \lambda. \qquad (5)$$

In this form formulae are applicable not only for Josephson arrays but for thin superconducting films.

If we account the dissipation in first order, the plasmon frequency acquires imaginary part that has to be much lower than its real part:

$$\mathrm{Im}\,\omega_{p\ell}(k)/\mathrm{Re}\,\omega_{p\ell}(k) = \omega_{p\ell}(k)/2\omega_c = 1/\omega_{p\ell}(k)\tau \ll 1,$$

where $\tau = 2\omega_c/\omega^2(k)$ is the relaxation time. To hold this inequality we have to: i) make ω_c as high as possible; ii) make k as low as possible; iii) make $I_c \propto L_{\square}^{-1}$ as low as possible.

i) The "characteristic" frequency (our definition is different from ordinarily used [5,6] $\omega_c = e^* I_c R_N/\hbar$) may vary many orders of magnitude: from $10 \div 20\,\Delta/\hbar$ for best tunnel junctions, through $\Delta/\hbar$ for junctions SNS with short thickness of normal metal layer, to $10^{-5}\Delta/\hbar$ for resistively shunted tunnel junctions and SNS junctions with long thickness of normal metal bridge. So it is preferable to use tunnel junctions or SNS junctions with short thickness of the normal metal layer.

ii) Hydrodynamical approach and finite size effects lead to restrictions $a \ll \lambda \ll L$ (L is characteristic size of the array) so appropriate number of the strips of the grating is $N^{1/2}$.

iii) In lowering I_c we are restricted by thermodynamic fluctuations which originate the Kosterlitz-Thouless transition in two-dimensional arrays [2,17,18] $I_c \gg 2e^* k_B T_{KT}/\hbar\pi$, where T_{KT} is the temperature of the Kosterlitz-Touless transition (for elementary introduction see [19]).

In the case of acoustic plasmon we have additional lowering in $\omega_{p\ell}(k)$ by factor $\sqrt{dk}$, see (4), (5).

As illustration let take following parameters $I_c = 1\mu A$, $\omega_c/2\pi = 1\,THz$ for Josephson junctions and 1 mm graiting spacing. In this case the kinetic inductance is $L_{\square} = 0.6nH$ and the frequency of the square root plasmon is $\omega_{p\ell}(k)/2\pi = 12.5GHz$. The group velocity is much lower than velocity of the light which confirms the neglecting of retardation effects. The dissipation is negligibly small which can be seen from $\omega_{p\ell}(k)\tau = 161$.

As another illustration let us consider the high T_c films made by Fiory and others [20]. The kinetic inductance is expressed by the London's penetration depth δ as [20] $L = 4\pi\delta_L^2/d_s c^2$, where d_s is thickness of the film. So for plasmon frequency we have (c is the light velocity):

$$\omega_{p\ell}(k) = \frac{c}{\delta_L}\sqrt{\frac{d_s k}{2\epsilon_m}} .$$

In order to neglect the retardation effects the following inequality must be hold $\lambda \ll 16\pi\epsilon_m\delta_L^2/d_s$. So for typical values of those films [20] $\delta_L = 150$ nm, $d_s = 50$ nm, $\epsilon_m = 10^2$ (ϵ_m is half of $SrTiO_4$ dielectric constant) we receive $\lambda \ll 1$ mm. For $\lambda = 100$ μm the plasmon frequency will be $\hbar\omega_{p\ell} = 5.2$ meV. This frequency is less than the gap frequency $2\Delta = 3.2 k_B T_c = 25$ meV.

It is interesting to look at the main result (3) in Feynman's two level picture [21]. In this approach each site has wave function $\psi(\vec{x})$. The equations for the wave functions are:

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{x}) = A \sum_{\vec{n}} \psi(\vec{x}+\vec{n}) + \psi(\vec{x}-\vec{n}) ,$$

where A is the transition amplitude for a Cooper pair to hope from one site to its neighbour. As it is known in such a system there are freely propagating particles (in the effective mass approximation) with effective mass $m^* = \hbar^2/Aa^2$. If we interpret the modulus of order parameter $|\psi|^2$ in the framework of the Ginsburg-Landau theory as namber of Cooper pairs grain, the critical current can be expressed by the surface density $n = |\psi|^2/a^2$ in the form $I_c = e^* n a^2 A/\hbar$. Substituting in this formula $A = \hbar^2/m^* a^2$ we obtain $L_{\square}^{-1} = n e^{*2}/m^*$. This is the well-known expression for the gas of free particles. Putting this formula in the expression for the plasmon frequency we receive the well-known formula for 2DEG plasmons [11-14].

The result of this work - the existence of dynamic excitations (plasmons) in Josephson arrays returns us to many years ago work of F. and H.London [22]. They use dynamic equations describing motion of charged particles without friction to explain the Meissner effect. Lately Landau (1941) [23] § 44 explains the Meissner effect merely by use of existence of order parameter and gauge invariance. On intuitive grounds is also Josephson derivation of his effect [24] describing dynamic tunneling of Cooper pairs. Josephson effect can also be described in terms of Ginsburg-Landau theory see [23] § 50. To do this one has to step a bit over initial assumptions of the Ginsburg-Landau theory (especially for ac Josephson effect). This step means adding gauge invariance (a dynamic effect) to static Ginsburg-Landau theory. Feynmans picture [21] also gives correct result although it is based on intuitive assumptions too. The rigorous microscopical analysis [25] (on the basis of BCS) confirms this two level picture.

The result of this work again shows the complete coincidence between intuitive description of the Londons, Josephson, Feynman and rigorous description in the framework of Ginsburg-Landau and BCS. In terms of Ginsburg-Landau theory the Josephson effect is described with following term in the free energy functional:

$$A \sum_{\vec{n},\vec{x}} \psi(\vec{x})\,\psi(\vec{x}+\vec{n})$$

which in the long wave limit transforms into:

$$\int_{\vec{x}} (|\nabla \psi(\vec{x})|^2 / 2m^*)\, d^3x \,.$$

This may be called simply "kinetic energy" of quantum mechanical particles. This is the origin of the kinetic inductance in the plasmon's dispersion law. For example for cubic array $\Omega^2_{p\ell} = 4\pi n^{3D} e^{*2}/m^*$ where we can substitute $n^{3D}/m^* = I_c/e^* \hbar a$ to obtain $\Omega^2_{p\ell} = 4\pi I_c e^*/\hbar a$. So we suggest that it may be possible for 3D plasmons to be detected. For example this can be done on bad enough High-Tc ceramics (one should ensure that $k_B T \ll \hbar\Omega_{p\ell} \ll \Delta$). So the photon acquires mass as dynamic effect - the idea in gauge theories with spontaneous symmetry breaking, returns back in the physics of the superconductivity where it came from.

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