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QUANTUM LIMITATIONS IN THE GLUON CONDENSATION PROBLEM OF QUANTUM CHROMODYNAMICS

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The specific feature of the problems of the strong interaction theory consists in the description of complicated interaction processes involving a great amount of particles and quantum fields. The states of a system thus arising are highly correlated, in other words, they are collective states. Consequently, the representation of final states of a system as free states of the corresponding particles and fields is a very rough approach that does not correspond to the physical nature of processes.

Since the correct determination of the collective vacuum is a very complicated mathematical problem, simple models in which the collective vacuum could be determined exactly seem to be interesting. Thereto, in the last few years much progress has been achieved in the investigation of collective states of an electromagnetic field interacting with the nonlinear media. In particular, photon states, in which strong correlations cause nonordinary statistical properties [1-3], have experimentally been obtained and theoretically described.

As it has been shown in ref.[3], transition to the collective state of the quantum field brings about alteration in the relation between variances of conjugated quantum values involved in the Heisenberg uncertainty relation. For example, a variance of the number of quanta, when passing from the Fock state to the collective state, increases whereas a variance of a phase decreases. Since in experiments of elementary particle physics one usually measures a number of particles (or energy), then it is necessary to correctly estimate its quantum uncertainty.

We shall clarify this idea using as an example the model describing the Bose-condensation of gluons in the SU(2)-gauge theory [4-6]. The effective Hamiltonian of this model has the form:

$$H_{eff} = \sum_{K,\lambda} \Big[(\omega_K + \mu_1(K))(a^+_{K\lambda} a_{K\lambda} + b^+_{K\lambda} b_{K\lambda}) + \varphi_1(K)(a^+_{K\lambda} b^+_{-K\lambda} + a_{K\lambda} b_{-K\lambda}) + (\omega_K + \mu_2(K))\, c^+_{K\lambda} c_{K\lambda} + \frac{1}{2}\varphi_2(K)(c^+_{K\lambda} c^+_{-K\lambda} + c_{K\lambda} c_{-K\lambda}) \Big], \qquad (1)$$

where the effective fields $\mu_{1,2}(K)$, $\varphi_{1,2}(K)$ are determined self - consistently by the condition of an energy minimum; $a_{K\lambda}$ and $b_{K\lambda}$ are annihilation operators of colour-charged, and $c_{K\lambda}$ - colour-neutral gluons acting in the space of the perturbative QCD-vacuum

$$a_{K\lambda} \mid 0 >^{(K\lambda)}_{abc} = b_{K\lambda} \mid 0 >^{(K\lambda)}_{abc} = c_{K\lambda} \mid 0 >^{(K\lambda)}_{abc} = 0 , \qquad {}^{(K\lambda)}_{abc}< 0 \mid 0 >^{(K\lambda)}_{abc} = 1 \tag{2}$$

and obeying the Bose statistics:

$$[\, a_{K\lambda} , a^{+}_{K\lambda} \,] = [\, b_{K\lambda} , b^{+}_{K\lambda} \,] = [\, c_{K\lambda} , c^{+}_{K\lambda} \,] = 1 , \tag{3}$$

and all the rest commutators are equal to zero.

Since the Hamiltonian (1) is a quadratic form with respect to the Bose-operators, with the help of the Bogolubov canonical transformation [7] first introduced in the superfluidity theory and having the form:

$$\begin{pmatrix} \alpha_{K\lambda} \\ \beta^{+}_{-K\lambda} \end{pmatrix} = \begin{pmatrix} u_K & v_K \\ v_K & u_K \end{pmatrix} \begin{pmatrix} a_{K\lambda} \\ b^{+}_{-K\lambda} \end{pmatrix} , \qquad \begin{pmatrix} \gamma_{K\lambda} \\ \gamma^{+}_{-K\lambda} \end{pmatrix} = \begin{pmatrix} w_K & z_K \\ z_K & w_K \end{pmatrix} \begin{pmatrix} c_{K\lambda} \\ c^{+}_{-K\lambda} \end{pmatrix} , \tag{4}$$

it can be diagonalized [5]:

$$H = E_{vac} + \sum_{K,\lambda} \left[E_1(K) (\alpha^{+}_{K\lambda} \alpha_{K\lambda} + \beta^{+}_{K\lambda} \beta_{K\lambda}) + E_2(K) \gamma^{+}_{K\lambda} \gamma_{K\lambda} \right] , \tag{5}$$

where

$$E_{vac} = \sum_{K} \left\{ E_1(K) - (\omega_K + \mu_1(K)) + \frac{1}{2} \left[E_2(K) - (\omega_K + \mu_2(K)) \right] \right\} . \tag{6}$$

The parameters u_K, v_K, w_K, z_K, and $E_1(K)$, $E_2(K)$ are determined as eigenfunctions and eigenvalues from the systems of equations

$$\begin{cases} E_1(K)\, u_K = (\omega_K + \mu_1(K))\, u_K - \varphi_1(K)\, v_K \\ E_1(K)\, v_K = - (\omega_K + \mu_1(K))\, v_K + \varphi_1(K)\, u_K \\ u_K^2 - v_K^2 = 1 \end{cases} \tag{7a}$$

and

$$\begin{cases} E_2(K)\, w_K = (\omega_K + \mu_2(K))\, w_K - \varphi_2(K)\, z_K \\ E_2(K)\, z_K = - (\omega_K + \mu_2(K))\, z_K + \varphi_2(K)\, w_K \\ w_K^2 - z_K^2 = 1 \end{cases} \tag{7b}$$

and have the form

$$u_K^2 = \frac{1}{2} \left(\frac{\omega_K + \mu_1(K)}{E_1(K)} + 1 \right) , \quad v_K^2 = \frac{1}{2} \left(\frac{\omega_K + \mu_1(K)}{E_1(K)} - 1 \right) ;$$
$$w_K^2 = \frac{1}{2} \left(\frac{\omega_K + \mu_2(K)}{E_2(K)} + 1 \right) , \quad z_K^2 = \frac{1}{2} \left(\frac{\omega_K + \mu_2(K)}{E_2(K)} - 1 \right) ; \tag{8}$$
$$E_{1,2}(K) = \sqrt{ (\omega_K + \mu_{1,2}(K))^2 - \varphi^2_{1,2}(K) } .$$

It is not difficult to notice that the eigenstates of the Hamiltonian (5) are the Fock states for the Bose-fields $\alpha_{K\lambda}$, $\beta_{K\lambda}$, and $\gamma_{K\lambda}$ describing collective states of the system. As observable values are connected with the "real" fields $a_{K\lambda}$, $b_{K\lambda}$, and $c_{K\lambda}$, it is necessary to make relation between the eigenstates of the Hamiltonian (5) and the Fock states of the real fields. The latter are the complete set, and therefore, any state of the system can be expanded in a series over them. For simplicity, let us consider the vacuum state of the quasiparticle system with the Hamiltonian (5). Expanding it over the Fock basis states of the $a_{\kappa\lambda}$, $b_{\kappa\lambda}$, and $c_{\kappa\lambda}$ fields and bearing in mind the properties of the Fock states:

$$a_{\kappa\lambda} \mid n_{\kappa\lambda} > = \sqrt{n_{\kappa\lambda}} \mid n_{\kappa\lambda} - 1 > ,$$
$$a^{+}_{\kappa\lambda} \mid n_{\kappa\lambda} > = \sqrt{n_{\kappa\lambda} + 1} \mid n_{\kappa\lambda} + 1 > \tag{9}$$

(the same with respect to the $b_{\kappa\lambda}$ and $c_{\kappa\lambda}$ fields),we obtain

$$\mid 0 >^{(\kappa\lambda)}_{\alpha,\beta,\gamma} = \exp\left\{ - \frac{v_\kappa}{u_\kappa} a^{+}_{\kappa\lambda} b^{+}_{-\kappa\lambda} - \frac{z_\kappa}{w_\kappa} c^{+}_{\kappa\lambda} c^{+}_{-\kappa\lambda} \right\} \mid 0 >^{(\kappa\lambda)}_{a,b,c} , \tag{10}$$

where

$$\alpha_{\kappa\lambda} \mid 0 >^{(\kappa\lambda)}_{\alpha,\beta,\gamma} = \beta_{\kappa\lambda} \mid 0 >^{(\kappa\lambda)}_{\alpha,\beta,\gamma} = \gamma_{\kappa\lambda} \mid 0 >^{(\kappa\lambda)}_{\alpha,\beta,\gamma} = 0 ,$$
$$_{\alpha,\beta,\gamma}^{(\kappa\lambda)}< 0 \mid 0 >^{(\kappa\lambda)}_{\alpha,\beta,\gamma} = 1 . \tag{11}$$

Considering the problem of a gluon system, one usually tends to estimate the gluon condensation parameter (e.g. see [8]), which is in this case determined by the expression [5]

$$< G^2 > = {}_{\alpha,\beta,\gamma}< 0 \mid \frac{\alpha_s}{\pi} G^{a}_{\mu\nu} G^{a,\mu\nu} \mid 0 >_{\alpha,\beta,\gamma} \approx \tag{12}$$
$$\approx \frac{\alpha_s}{\pi} \sum_{\kappa,\lambda} \frac{w_\kappa}{V} {}^{(\kappa\lambda)}_{\alpha,\beta,\gamma}< 0 \mid a^{+}_{\kappa\lambda} b^{+}_{-\kappa\lambda} + a_{\kappa\lambda} b_{-\kappa\lambda} + \frac{1}{2} (c^{+}_{\kappa\lambda} c_{-\kappa\lambda} +$$
$$+ c_{\kappa\lambda} c_{-\kappa\lambda}) \mid 0 >^{(\kappa\lambda)}_{\alpha,\beta,\gamma} = - \frac{\alpha_s}{\pi} \sum_{\kappa,\lambda} \frac{w_\kappa}{V} [2 u_\kappa v_\kappa + w_\kappa z_\kappa] .$$

The measurable, from the quantum mechanical point of view, values of this parameter lie in the interval

$$[< G^2 > - \Delta , < G^2 > + \Delta] , \tag{13}$$

where

$$\Delta = \sqrt{<(\Delta G^2)^2>} = \sqrt{<(G^2)^2> - <G^2>^2} \tag{14}$$

is the uncertainty caused by the collective quantum properties of the system.

The straightforward calculations of the variance lead to the following result:

$$\Delta = \frac{\alpha_s}{\pi} \sum_{\kappa,\lambda} \frac{w_\kappa}{V} [(u^2_\kappa + v^2_\kappa) + \frac{1}{2} (w^2_\kappa + z^2_\kappa)] . \tag{15}$$

From the comparison of expressions (12) and (15) one can see the quantum uncertainty in measuring the gluon condensation parameter to exceed the mean value of this parameter. Therefore, from the point of view of quantum theory the value $<G^2>$ is badly measurable. An analogous result is also valid for the mean number of quanta of each gluon field.

Expression (15) yields the distribution width of measured values of the gluon condensation parameter. However, for the correct analysis of measurements the form of the distribution function is also needed.

Thus, the simplest model considered evidently demonstrates increase in the quntum uncertainty of a number of physical values when the collective state occurs. It should be noted that the higher the degree of nonlinearity of a process, i.e. the more various particles and fields participate in the interaction, the higher is this uncertainty [3]. Consequently, other problems of QCD need estimation of not only means of physical parameters but also the corresponding variances and distribution functions.

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