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L.H.Lan, A.S.Shumovsky, Tran Quang, V.Buzek

AMPLITUDE-SQUARED SQUEEZING
IN COLLECTIVE RESONANCE FLUORESCENCE

In recent years, the problem of squeezing becomes the central problem of quantum optics[1-3 and refs. therein]. The squeezed light has been generated in some laboratories[4-7] and applications of the squeezed light have been widely discussed[8-11].

In the papers[12,13] Hong and Mandel introduced a higher-order squeezed state to the (2N)th order if the expectation value of the (2N)th power of the difference between a field quadrature component and its mean value is less than it would be in a coherent state. The higher-order squeezed states defined by Hong and Mandel are nonclassical states, and they are examined in a number of processes such as resonance fluorescence[13], second harmonic generation[13,14] and Jaynes - Cummings model[15].

In recent works[16,17] Hillery has introduced squeezing of the square of the field amplitude which is also nonclassical effect and is examined in harmonic generation[16,17] and two-atom resonance fluorescence[43]. In present work we consider the amplitude-squared squeezing in collective resonance fluorescence in an intense external field. It is shown that contrary to the single-atom case, the amplitude-squared squeezing occurs in collective resonance fluorescence. The amplitude-squared squeezing is present only in the mixture of two sidebands but not in separate spectrum components or in the whole fluorescence field. It is shown that the collective effects increase the degree of the amplitude-squared squeezing. The influence of the thermal reservoir on the degree of squeezing is also discussed.

Let N two-level atoms be concentrated in a region small compared to the wave-length of all the relevant radiation modes (Dicke model). The atoms are driven by a single-mode coherent field of the frequency ω_L and coupled to a reservoir containing all modes of the radiation. In treating the external field classically and using the Markov and rotating wave approximation for describing the coupling of the system with the thermal reservoir, one arrives at the following master equation for the reduced atomic density operators[18] :

$$\frac{\partial \rho}{\partial t} = -i \left[\frac{\delta}{2} (J_{22} - J_{11}) + G (J_{12} + J_{21}) , \rho \right]$$
$$- \frac{\gamma}{2} (\bar{n} + 1) (J_{21} J_{12} \rho - J_{12} \rho J_{21} + H.C.) \qquad (1)$$
$$- \frac{\gamma}{2} \bar{n} (J_{12} J_{21} \rho - J_{21} \rho J_{12} + H.C.) \equiv L\rho ,$$

where $\delta = \omega_{21} - \omega_L$ is the detuning of the laser frequency from the atomic resonance frequency ω_{21} ($\omega_{21} = \omega_2 - \omega_1$; $\hbar \equiv 1$); $G = - d_{21} E_0$ is the resonant Rabi frequency; γ is the spontaneous transition rate from the excited level $|2\rangle$ to the ground state $|1\rangle$;

$$\bar{n} = \bar{n} (\omega_{21}) = [exp(\omega_{21}/kT) - 1]^{-1} \qquad \text{is the}$$

mean proton number in the broad-band thermal field at frequency ω_{21} :

$$J_{ij} = \sum_k |i\rangle_k \langle j| \qquad (i,j = 1,2) \qquad (2)$$

are the collective operators (angular momenta) of the atomic system.

It is useful to note that the master equation (1) can be applied to the case of many Rydberg atoms in a low-Q cavity providing γ is replaced by the appropriate cavity damping constant ($kg^2/(k^2 + \delta_c^2)$), where k is the decay constant of the cavity, g is the atom-cavity mode coupling constant and δ_c is the cavity detuning[19] and $\bar{n}$ is taken at the frequency of the single-cavity mode.

For further uses we introduce the Schwinger representation [21,22] for collective operators

$$J_{ij} = C_i^+ C_j \qquad (i,j = 1,2) \qquad , \qquad (3)$$

where the operators C_i and C_i^+ obey the boson commutation relations

$$[\, C_i \,,\, C_j^+ \,] = \delta_{ij}$$

and can be treated as annihilation and creation operators for atoms populating the level $|i>$. We make the canonical (dressing) transformation

$$C_1 = Q_1 \cos G + Q_2 \sin G \qquad ,$$
$$C_2 = - Q_1 \sin G + Q_2 \cos G \qquad , \qquad (4)$$

where $tg\, 2G = 2G/\delta$. One can show that in the case of an intense external field

$$\Omega = (\tfrac{1}{4}\delta^2 + G^2)^{1/2} \gg N\gamma,\ \bar{n}\gamma \qquad (5)$$

the secular approximation [20-21] is justified and the master equation (1) reduces to

$$\frac{\partial \tilde{\rho}}{\partial t} = - i\Omega\,[\,R_3\,,\,\rho\,] + B\,(2R_3\,\tilde{\rho}\,R_3 - R_3^2\,\tilde{\rho} - \tilde{\rho}\,R_3^2\,)$$
$$+ X_1\,(2R_{12}\,\tilde{\rho}\,R_{21} - R_{21}R_{12}\,\tilde{\rho} - \tilde{\rho}\,R_{21}R_{12}\,)$$
$$+ X_2\,(2R_{21}\,\tilde{\rho}\,R_{12} - R_{12}R_{21}\,\tilde{\rho} - \tilde{\rho}\,R_{12}R_{21}\,) \quad , \qquad (6)$$

with $\tilde{\rho} = U \rho U^+$ where U is the unitary operator representing the canonical transformation (4)

$$B = \gamma(\bar{n} + \tfrac{1}{2})\sin^2 G \cos^2 G \qquad , \qquad (7)$$

$$X_1 = \frac{\gamma}{2}(\bar{n}+1)\cos^4 G + \frac{\gamma}{2}\bar{n}\sin^4 G \qquad , \qquad (8)$$

$$X_2 = \frac{\gamma}{2}(\bar{n}+1)\sin^4 G + \frac{\gamma}{2}\bar{n}\cos^4 G \qquad , \qquad (9)$$

$$R_3 = R_{22} - R_{11} \qquad , \qquad (10)$$

$R_{ij} = Q_i^+ Q_j$ $(i,j = 1,2)$ are the collective operators of the dressed atoms.

The reduced master (6) has the exact stationary solution

$$\tilde{\rho}_{st} = Z^{-1} \sum_{n_1 = 0}^{N} X^{n_1}\,|\,n_1 > < n_1\,| \qquad , \qquad (11)$$

where

$$X = \frac{X_1}{X_2} = \frac{(\bar{n}+1)\cos^4 G + \bar{n}\sin^4 G}{\bar{n}\cos^4 G + (\bar{n}+1)\sin^4 G} \qquad , \qquad (12)$$

$$Z = \frac{X^{N+1} - 1}{X - 1} \qquad , \qquad (13)$$

the state $|n_1 >$ is the eigenstate of the operators R_{11} and $R_{11} + R_{22}$. For later use, we introduce the characteristic function

$$\chi_{R_{11}} = Tr\,\{\, e^{i\zeta R_{11}}\,\tilde{\rho}_{st}\,\} = < e^{i\zeta R_{11}} >_s$$
$$= Z^{-1} \cdot \frac{Y^{N+1} - 1}{Y - 1} \qquad (14)$$

where $Y = X \cdot e^{i\zeta}$.

All the statistical moments of dressed atoms of the form $< R_{11}^q >_s$ can be found from the characteristic function $\chi_{R_{11}}(\zeta)$ and have the following form:

$$< R_{11}^q >_s = \frac{\partial^q}{\partial(i\zeta)^q}\,\chi_{R_{11}}(\zeta)\,\Big|_{i\zeta = 0} \quad . \qquad (15)$$

Further we study the amplitude-squared squeezing in the fluorescence field having the following form:

$$E^+(t,\vec{r}) = E_{free}^+(\vec{r},t) + \psi(\vec{r})\,J_{12}(t - |\vec{r}|/c) \cdot$$
$$exp\,[\,-i\omega_L(t - |\vec{r}|/c)\,] \qquad , \qquad (16)$$

where $\psi(\vec{r})$, is the geometrical factor, $E^+_{free}(\vec{r},t)$ is the free part of the fluorescence field. The collective operator J_{12} takes the following form

$$J_{12} = \sin G \cos G \, R_3 + \cos^2 G \, R_{12} - \sin^2 G \, R_{21} , \quad (17)$$

where the operators $-\sin^2 G \, R_{12}$, $\sin G \cos G \, R_3$ and $\cos^2 G \, R_{21}$ can be considered as operators-sources of Mollow's spectrum components centered at frequencies $\omega_L - 2\Omega$, ω_L, $\omega_L + 2\Omega$ and for simplicity we denote these operators by S_{-1}, S_0 and S_{+1}, respectively [21].

The quadrature components of the square of the field amplitude are defined as:

$$E_1 = \frac{1}{2} (E^{+2} + E^{-2}) ,$$

$$E_2 = \frac{-i}{2} (E^{+2} - E^{-2}) .$$

It is easy to show that

$$[E_1 , E_2] = 2iC (E^- E^+ + \frac{1}{2} C) , \quad (18)$$

where $\quad C = [E^+ , E^-] .$

The variances of the quadrature components E_1 and E_2 satisfy the following relation;

$$\langle (\Delta E_i)^2 \rangle = \langle : (\Delta E_i)^2 : \rangle + C (\langle E^- E^+ \rangle + \frac{C}{2}) . \quad (19)$$

$$(i = 1, 2)$$

From eqs.(18),(19) one finds the condition for the amplitude-squared squeezing, i.e. squeezing of the quadrature components E_1 or E_2 in the form

$$\langle : (\Delta E_i)^2 : \rangle \quad < 0 \qquad i = 1 \text{ or } 2 . \quad (20)$$

By using the relations (16) and (17) one can show that the amplitude square squeezing is absent in the whole fluorescente field as well as in spectrum components S_i $(i = -1, 0, +1)$ taken separately. As for ordinary squeezing [21], the amplitude-squared squeezing occurs in the mixture of the two sidebands S_{-1} and S_{+1}. By replacing the operators J_{12} in eq.(16) by

$$\tilde{J}_{12} = \cos^2 G \, R_{12} - \sin^2 G \, R_{21} ,$$

one finds the normal-ordered variances of the quadrature components E_1 and E_2 in the following form:

$$\langle : (\Delta E_{1,2})^2 : \rangle = \frac{\psi^2}{2} (\cos^4 G \pm \sin^4 G) \cdot$$
$$\cdot (\cos^4 G \langle R^2_{21} R^2_{12} \rangle_s \pm \sin^4 G \langle R^2_{12} R^2_{21} \rangle_s) , \quad (21)$$

where

$$\langle R^2_{21} R^2_{12} \rangle_s = \langle R^4_{11} \rangle_s - (2N-4) \langle R^3_{11} \rangle_s +$$
$$+ (N^2 - 7N + 5) \langle R^2_{11} \rangle_s + (3N^2 - 7N + 2) \langle R_{11} \rangle_s$$
$$+ 2N^2 - 2N , \quad (22)$$

$$\langle R^2_{12} R^2_{21} \rangle_s = \langle R^4_{11} \rangle_s - 2(N+2) \langle R^3_{11} \rangle_s +$$
$$+ (N^2 + 5N + 5) \langle R^2_{11} \rangle_s - (N^2 + 3N + 2) \langle R_{11} \rangle_s \quad (23)$$

and the statistical moments of the form $\langle R^q_{11} \rangle_s$ $(q = 1, 2, 3, 4)$ can be found according to eq.(15). In eq.(21), the delayed time contribution has been ignored in the stationary limit. The free part E^+_{free} in relation (16) is ignored and it is easy to show that this approximation is satisfied in the case of $N \gg \bar{n}$. It is easy to see from eq.(21) that the amplitude-squared squeezing is directly connected with the correlation between the photons of the separate spectrum components S_{-1} and S_{+1} which are described by the quantities $\langle R^2_{12} R^2_{21} \rangle_s$ and $\langle R^2_{21} R^2_{12} \rangle_s$ respectively.

For the one-atom case one can use the operator relation

$$R_{ij} \, R_{i'j'} = R_{ij'} \, \delta_{i'j} \quad (i,j,i',j' = 1,2) \,,$$

then eq.(21) reduces to

$$\langle : (\Delta E_{1,2})^2 : \rangle = 0 \quad ,$$

thus the amplitude-squared squezing is absent for the single-atom resonance fluorescence . The amplitude-squared squeezing is also absent for the case of exact resonance.

The behaviour of the relative normal-ordered variance

$\langle : (\Delta E_2)^2 : \rangle / (N^2 \psi^2)$ as a function of the parameter

$\cos^2 \varphi$ for the case of $\bar{n} = 0$ and for various numbers of atoms

is shown in Fig.1. As is seen in Fig. 1, the enhancement of the

degree of amplitude-squared squeezing occurs when the number of
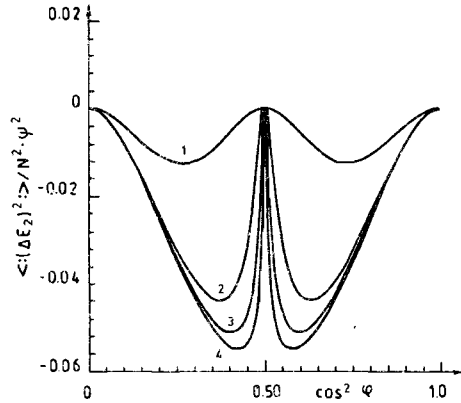
atoms increases.

Fig.1. Relative normal-ordered variance $\langle : (\Delta E_2)^2 : \rangle / (N^2 \psi^2)$

as a function of the parameter $\cos^2 \varphi$ for $\bar{n} = 0$.

The curves (1)-(4) correspond $N = 2, 20, 50, 100$.

The influence of the thermal reservoir on the amplitude-squared squeezing is shown in Fig.1 and 2 where the relative

normal-ordered variance $\langle : (\Delta E_2)^2 : \rangle / (N^2 \psi^2)$ is plotted as

a function of the parameter $\cos^2 \varphi$ for $N = 50$ and for various

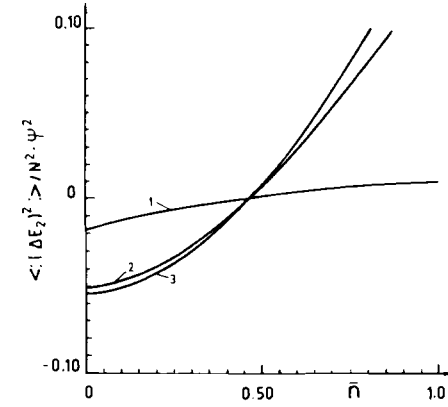values of the thermal field intensity (Fig.2) or as a function of

Fig.2. Relative normal-ordered variance $\langle : (\Delta E_2)^2 : \rangle / (N^2 \psi^2)$

as a function of the parameter $\bar{n}$ for $\cos^2 \varphi$ = 0.4.

The curves (1)-(3) correspond $N = 2, 50, 100$.

the parameter $\bar{n}$ for the case of $\cos^2 \varphi$ = 0.4 and for various

numbers of atoms. As is seen from Figs. 1,2, the thermal field

strongly affects the amplitude-squared squeezing in the mixture

of two sidebands of Mollow's triplet. For the case of

$\omega_{21} / 2\pi \approx 10 \, GHz$, $T = 2 \, {}^{\circ}K$ we have $\bar{n} = 0.1$ and a large

squeezing can be possible.

Summaring this work we note that the amplitude-squared

squeezing is investigated for collective resonance fluorescence.

Contrary to the single-atom case, where the amplitude-squared

squeezing is absent, the amplitude-squared squeezing occurs in

the mixture of two sidebands but not in separate spectrum compo-

nents. The degree of amplitude-squared squeezing is enhanced

with the number of atoms increases. The influence of the thermal

field on the degree of amplitude-squared squeezing is also consi-

dered in this work .

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Лан Л.Х. и др.   E17-87-619
Амплитудно-квадратное сжатие в коллективной
резонансной флуоресценции

Рассмотрено амплитудно-квадратное сжатие по определению Хиллери/17/ в коллективной резонансной флуоресценции. В отличие от случая с одним атомом амплитудно-квадратное сжатие существует в смеси двух крайних компонент коллективного флуоресцентного поля. Показано увеличение степени амплитудно-квадратного сжатия, когда число атомов возрастает. Исследовано влияние теплового поля на амплитудно-квадратное сжатие.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Amplitude-Squared in Collective Resonance
Fluorescence

The amplitude-squared squeezing, defined by Hillery/17/ in collective resonance fluorescence is considered. Contrary to the single-atom case the amplitude-squared squeezing occurs in the mixture of sidebands of the collective fluorescence field. The enhancement of the degree of amplitude-squared squeezing is shown when the number of atoms increases. The influence of the thermal field on the amplitude-squared squeezing is also discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.