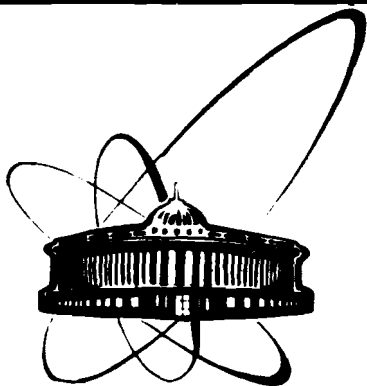


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COLLECTIVE DOUBLE RESONANCE
IN THE PRESENCE OF A THERMAL FIELD

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1. INTRODUCTION

Cooperative effects of a system of atoms interacting with common coherent fields and with the vacuum of radiation is a subject of continuing interest in the last years since Dicke first introduced the collective superradiance [1]. The process of superradiance and superfluorescence has received much theoretical and experimental attention [2-6 and refs. therein]; however, the superradiance in the frame of the Dicke model has been experimentally observed in the microwave range with using the Rydberg atoms [7] and in the radiowave range with using the system of nuclear magnetic moments [8].

In the last years, an extension of the Dicke model is widely used to study collective resonance fluorescence [9-11]. Collective Raman scattering [12,13] and collective double optical resonance [14,15]. In the works [14,15] the influence of the black-body radiation has been neglected. The effects of thermal fields are negligible for optical transitions even at normal temperature. However, when atoms are the Rydberg atoms and their transitions are the microwave transitions (where the Dicke model is justified) the effects of the thermal field become important at very low temperature ($T \sim 4^\circ\text{K}$). In this work we consider the influence of the thermal field on the spectral and statistical properties of the fluorescence field from collective double resonant processes.

II. BASIC MASTER EQUATION

The N three-level atoms (Fig.1), concentrated in a region small compared to the wavelength of all the relevant radiation

modes, interact with two resonant classical driving fields and with a thermal reservoir. In the boson representation of atoms [14] when each atomic level is compared with a boson variable, the master equation for the reduced atomic density operator ρ in the frame of rotating wave and Markovian approximation takes the following form (in interaction picture) [16]:

$$\frac{\partial \rho}{\partial t} = -i [H_{coh}, \rho] + \left. \frac{\partial \rho}{\partial t} \right|_A \equiv L \rho \quad (1)$$

where

$$H_{coh} = \Omega (\cos \alpha J_{12} + \sin \alpha J_{23} + H.C.) \quad (2)$$

$$\begin{aligned} \left. \frac{\partial \rho}{\partial t} \right|_A = & -\frac{1}{2} \chi_{21} (\bar{n}_1 + 1) (J_{21} J_{12} \rho - 2 J_{12} \rho J_{21} + \rho J_{21} J_{12}) \\ & -\frac{1}{2} \chi_{21} \bar{n}_1 (J_{12} J_{21} \rho - 2 J_{21} \rho J_{12} + \rho J_{12} J_{21}) \\ & -\frac{1}{2} \chi_{32} (\bar{n}_2 + 1) (J_{32} J_{23} \rho - 2 J_{23} \rho J_{32} + \rho J_{32} J_{23}) \\ & -\frac{1}{2} \chi_{32} \bar{n}_2 (J_{23} J_{32} \rho - 2 J_{32} \rho J_{23} + \rho J_{23} J_{32}), \end{aligned} \quad (3)$$

where χ_{21} and χ_{32} are radiative spontaneous transition rates for transitions $|2\rangle \rightarrow |1\rangle$ and $|3\rangle \rightarrow |2\rangle$, respectively; $\Omega = (\Omega_1^2 + \Omega_2^2)^{1/2}$ and $\tan \alpha = \Omega_2 / \Omega_1$, where Ω_1 and Ω_2 are the Rabi frequencies for the upper and lower atomic transitions respectively; $J_{ij} = C_i^\dagger C_j$ ($i, j = 1-3$) where C_i and C_i^\dagger satisfy the boson commutation relation

$$[C_i, C_j^\dagger] = \delta_{ij}$$

and can be treated as annihilation and creation operators for the atoms populated on the level $|i\rangle$;

$$\begin{aligned} \bar{n}_1 &= \bar{n}(\omega_{21}) = [\exp(\omega_{21}/kT) - 1]^{-1} \\ \bar{n}_2 &= \bar{n}(\omega_{32}) = [\exp(\omega_{32}/kT) - 1]^{-1} \end{aligned}$$

are the mean photon numbers in the broad-band thermal field provided by the reservoir at the atomic frequencies ω_{21} and ω_{32} respectively.

After performing the canonical (dressing) transformation

$$\begin{aligned} C_1 &= -\frac{1}{\sqrt{2}} \cos \alpha Q_1 - \sin \alpha Q_2 + \frac{1}{\sqrt{2}} \cos \alpha Q_3 \\ C_2 &= \frac{1}{\sqrt{2}} Q_1 + \frac{1}{\sqrt{2}} Q_3 \\ C_3 &= -\frac{1}{\sqrt{2}} \sin \alpha Q_1 + \cos \alpha Q_2 + \frac{1}{\sqrt{2}} \sin \alpha Q_3 \end{aligned} \quad (4)$$

the Liouville operator L appearing in equation (1) splits into the slowly varying part and terms oscillating at frequency $n\Omega$ ($n = 1-4$). In the case of intense external fields so that

$$\Omega \gg N \chi_{21}, N \chi_{32} \quad (5)$$

the secular approximation is justified [9,14] and master equation (1) reduces to

$$\begin{aligned} \partial \tilde{\rho} &= -i \Omega [R_3, \tilde{\rho}] - A (R_3^2 \tilde{\rho} - R_3 \tilde{\rho} R_3 + H.C.) \\ & - A (R_{31} R_{13} \tilde{\rho} + R_{13} R_{31} \tilde{\rho} - R_{31} \tilde{\rho} R_{13} - R_{13} \tilde{\rho} R_{31} + H.C.) \\ & - \chi_1 (R_{12} R_{21} \tilde{\rho} + R_{32} R_{23} \tilde{\rho} - R_{21} \tilde{\rho} R_{12} - R_{23} \tilde{\rho} R_{32} + H.C.) \\ & - \chi_2 (R_{21} R_{12} \tilde{\rho} + R_{23} R_{32} \tilde{\rho} - R_{12} \tilde{\rho} R_{21} - R_{32} \tilde{\rho} R_{23} + H.C.), \end{aligned} \quad (6)$$

where $\tilde{\rho} = U \rho U^\dagger$, U is the unitary operator representing the canonical transformation (4)

$$\begin{aligned} A &= \frac{1}{4} \chi_{21} \cos^2 \alpha (\bar{n}_1 + \frac{1}{2}) + \frac{1}{4} \chi_{32} \sin^2 \alpha (\bar{n}_2 + \frac{1}{2}), \\ \chi_1 &= \frac{1}{4} \chi_{21} \sin^2 \alpha (\bar{n}_1 + 1) + \frac{1}{4} \chi_{32} \cos^2 \alpha \bar{n}_2 \end{aligned} \quad (8)$$

$$\chi_2 = \frac{1}{4} \chi_{21} \sin^2 \alpha \bar{n}_1 + \frac{1}{4} \chi_{32} \cos^2 \alpha (\bar{n}_2 + 1) \quad (9)$$

$$R_3 = R_{33} - R_{11} \quad (10)$$

$R_{ij} = Q_i^\dagger Q_j$ ($i, j = 1-3$) are the collective operators of the dressed atoms. Since the transformation (4) is canonical the operators Q_i and Q_j^\dagger satisfy the boson commutation relation

$$[Q_i, Q_j^\dagger] = \delta_{ij}$$

so that

$$[R_{ij}, R_{i'j'}] = R_{ij} \delta_{i'j} - R_{i'j'} \delta_{ij'}$$

The exact stationary solution of equation (6) takes the form

$$\tilde{\rho} = Z^{-1} \sum_{P=0}^N X^P \sum_{M=0}^P |P, M\rangle \langle M, P| \quad (11)$$

where

$$X = \frac{x_2}{x_1} = \frac{\gamma_{21} \bar{n}_1 \sin^2 \alpha + \gamma_{32} (\bar{n}_2 + 1) \cos^2 \alpha}{\gamma_{21} (\bar{n}_1 + 1) \sin^2 \alpha + \gamma_{32} \bar{n}_2 \cos^2 \alpha} \quad (12)$$

$$Z = \frac{(N+1) X^{N+2} - (N+2) X^{N+1} + 1}{(X-1)^2} \quad (13)$$

The state $|P, M\rangle$ is an eigenstate of the operators $R = R_{11} + R_{33}$ (with an eigenvalue P), R_{11} (with an eigenvalue M) and

$$\hat{N} = R_{11} + R_{22} + R_{33} \quad (\text{with an eigenvalue } N).$$

The stationary density matrix (11) of the atomic system is dependent on the mean photon numbers \bar{n}_1 and \bar{n}_2 of the thermal field. Consequently, all the stationary expectation values of the atomic observables are dependent on the thermal reservoir. Only in the special case of $\gamma_{32} \cos^2 \alpha = \gamma_{21} \sin^2 \alpha$ the density matrix (11) is independent of the thermal field. By using the density matrix (11) one can calculate all the stationary expectation values of the atomic observables. Some of the results that will be needed for our further consideration are given in Appendix.

III. STEADY-STATE FLUORESCENCE SPECTRUM

In this section we investigate the spectral properties of the fluorescence field. We assume that $|\omega_{21} - \omega_{32}| \gg \Omega$

then, the steady-state spectra of fluorescence fields corresponding to the lower $|2\rangle \rightarrow |1\rangle$ and upper $|3\rangle \rightarrow |2\rangle$ atomic transitions are clearly separated and can be calculated as the Fourier transforms of the atomic correlation functions

$$\langle J_{21}(\tau) J_{12} \rangle_S = \lim_{t \rightarrow \infty} \langle J_{21}(t+\tau) J_{12}(t) \rangle \quad (14)$$

$$\langle J_{32}(\tau) J_{23} \rangle_S = \lim_{t \rightarrow \infty} \langle J_{32}(t+\tau) J_{23}(t) \rangle, \quad (15)$$

where $\langle \dots \rangle_S$ denotes an expectation value over the steady-state (11). First, let us consider the fluorescence spectrum according to the lower atomic transition $|2\rangle \rightarrow |1\rangle$.

By using the master equation (6) one can derive the equation for a mean value of the atomic operator Q

$$\begin{aligned} \frac{d}{dt} \langle Q \rangle = & -i\Omega \langle [Q, R_3] \rangle + \\ & + A \{ \langle [R_3, Q] R_3 \rangle + \langle R_3 [Q, R_3] \rangle \\ & + \langle [R_{31}, Q] R_{13} \rangle + \langle R_{31} [Q, R_{13}] \rangle \\ & + \langle [R_{13}, Q] R_{31} \rangle + \langle R_{13} [Q, R_{31}] \rangle \} + \\ & + X_2 \{ \langle [R_{21}, Q] R_{12} \rangle + \langle R_{21} [Q, R_{12}] \rangle \\ & + \langle [R_{23}, Q] R_{32} \rangle + \langle R_{23} [Q, R_{32}] \rangle \} \\ & + X_1 \{ \langle [R_{12}, Q] R_{21} \rangle + \langle R_{12} [Q, R_{21}] \rangle \\ & + \langle [R_{32}, Q] R_{23} \rangle + \langle R_{32} [Q, R_{23}] \rangle \}. \end{aligned} \quad (16)$$

In particular, the equation of motion for $\langle R_{ij}(t) \rangle$ can be derived from eq.(16) in following form:

$$\frac{d}{dt} \langle R_3(t) \rangle = -\gamma \langle R_3(t) \rangle + \frac{1}{2} \gamma \langle \{ N - R(t), R_3(t) \} \rangle, \quad (17)$$

$$\frac{d}{dt} \langle R_{32}(t) \rangle = (i\Omega - \gamma_1) \langle R_{32}(t) \rangle + \frac{1}{4} \gamma_c \quad (18)$$

$$\langle \{ N - 2R(t), R_{32}(t) \} \rangle = \frac{d}{dt} \langle R_{23}(t) \rangle^*$$

$$\begin{aligned} \frac{d}{dt} \langle R_{12}(t) \rangle &= (-i\Omega - \gamma_1) \langle R_{12}(t) \rangle \\ -\frac{1}{4} \gamma_c \langle \{ N - 2R(t), R_{12}(t) \} \rangle &= \frac{d}{dt} \langle R_{21}(t) \rangle^* \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{d}{dt} \langle R_{13}(t) \rangle &= (-2i\Omega - \gamma_2) \langle R_{13}(t) \rangle \\ -\frac{1}{2} \gamma_c \langle \{ N - R(t), R_{13}(t) \} \rangle &= \frac{d}{dt} \langle R_{31}(t) \rangle^* \end{aligned} \quad (20)$$

where

$$\begin{aligned} \gamma_0 &= \frac{1}{2} \gamma_{21} + \frac{1}{2} \gamma_{32} \sin^2 \alpha + \frac{1}{2} \gamma_{21} \bar{n}_1 (1 + \cos^2 \alpha) + \\ &+ \frac{1}{2} \gamma_{32} \bar{n}_2 (1 + \sin^2 \alpha) \end{aligned} \quad (21)$$

$$\begin{aligned} \gamma_1 &= \frac{1}{4} \gamma_{21} + \frac{1}{4} \gamma_{32} (1 + \cos^2 \alpha) + \frac{1}{4} \gamma_{21} \bar{n}_1 (2 + \sin^2 \alpha) \\ &+ \frac{1}{4} \gamma_{32} \bar{n}_2 (2 + \cos^2 \alpha) \end{aligned} \quad (22)$$

$$\begin{aligned} \gamma_2 &= \frac{1}{4} \gamma_{21} (2 + \cos^2 \alpha) + \frac{3}{4} \gamma_{32} \sin^2 \alpha + \\ &+ \frac{1}{2} \gamma_{21} \bar{n}_1 (1 + \cos^2 \alpha) + \frac{1}{2} \gamma_{32} \bar{n}_2 (1 + \sin^2 \alpha) \end{aligned} \quad (23)$$

$$\gamma_c = \frac{1}{2} (\gamma_{21} \sin^2 \alpha - \gamma_{32} \cos^2 \alpha) \quad (24)$$

$$\{ R_{ij}, R \} = R_{ij} R + R R_{ij} \quad (25)$$

In the single-atom case one can use the operator relation

$$R_{ij} R_{ke} = R_{ie} \delta_{kj} \quad (26)$$

and equations (17)-(28) take the form:

$$\frac{d}{dt} \langle R_3(t) \rangle = -\gamma_0 \langle R_3(t) \rangle \quad (27)$$

$$\frac{d}{dt} \langle R_{32}(t) \rangle = (i\Omega - \gamma_1) \langle R_{32}(t) \rangle = \frac{d}{dt} \langle R_{23}(t) \rangle^* \quad (28)$$

$$\frac{d}{dt} \langle R_{12}(t) \rangle = (-i\Omega - \gamma_1) \langle R_{12}(t) \rangle = \frac{d}{dt} \langle R_{21}(t) \rangle^* \quad (29)$$

$$\frac{d}{dt} \langle R_{13}(t) \rangle = (-2i\Omega - \gamma_2) \langle R_{13}(t) \rangle = \frac{d}{dt} \langle R_{31}(t) \rangle^* \quad (30)$$

Equations (27)-(30) are linear exact solvable. Applying the quantum regression theorem one can obtain the expression for the atomic correlation function of the kind (14) and show that the spectrum of the single-atom fluorescence (corresponding the lower transition $|2\rangle \rightarrow |1\rangle$) contains five components located at frequencies ω_{21} , $\omega_{21} \pm \Omega$ and $\omega_{21} \pm 2\Omega$ that have the linewidths γ_0 , γ_1 and γ_2 , respectively. It is shown in eqs. (21)-(23) that the linewidths of the spectrum components are expanded due to the presence of the thermal field. In this case the central (ω_{21}) intermediate ($\omega_{21} \pm \Omega$) and extreme sidebands ($\omega_{21} \pm 2\Omega$) have the integral intensities proportional to

$$I_0 \sim \frac{1}{4} \cos^2 \alpha \langle R_3^2 \rangle_S = \frac{1}{2} \cos^2 \alpha \frac{X}{2X+1} \quad (31)$$

$$I_{\pm 1} \sim \frac{1}{2} \sin^2 \alpha \langle R_{32} R_{23} \rangle_S = \frac{1}{2} \sin^2 \alpha \frac{X}{2X+1} \quad (32)$$

$$I_{\pm 2} \sim \frac{1}{4} \cos^2 \alpha \langle R_{31} R_{13} \rangle_S = \frac{1}{2} I_0 \quad (33)$$

It is easy to see from eqs. (31)-(33) and (12) that in the single-atom case integral intensities of the spectrum components are dependent on the thermal field without the point of $\sin^2 \alpha \gamma_{21} = \gamma_{32} \cos^2 \alpha$ where $X = 1$ for arbitrary values of \bar{n}_1 , \bar{n}_2 .

Further, we shall consider the collective case. One can see that eqs. (17)-(20) contain a product of operators that makes them

unsoluble. Only in the special case $\delta_{32} \cos^2 \alpha = \delta_{21} \sin^2 \alpha$ the parameters $\chi_1 = \chi_2$ and the terms with the products of operators vanish; then, all equations (17)-(20) reduce to the exact solvable linear differential equations.

For the common case, according to the works [10, 12, 14] we use the decorrelation scheme

$$\langle \{ R(t), R_{i,j}(t) \} \rangle = 2 \langle R \rangle_3 \langle R_{i,j}(t) \rangle \quad (34)$$

By using the density matrix (11) one can show that in the case of large N the decorrelation (34) yields a small error (with an order of $N^{-1/2}$) in the calculation of the steady-state fluorescent spectrum. Applying the decorrelation scheme (31), eqs. (17)-(20) have a simple exponential solution. Using the quantum regression theorem [17] one obtains the atomic correlation function (14) in the following form:

$$\begin{aligned} \langle J_{21}(\tau) J_{12} \rangle_3 &= \frac{1}{4} \cos^2 \alpha \langle R_3^2 \rangle_3 e^{-\Gamma_0 \tau} \\ &+ \sin^2 \alpha \langle R_{12} R_{21} \rangle_3 e^{-\Gamma_1 \tau} \cos(\Omega \tau) \\ &+ \frac{1}{4} \cos^2 \alpha \langle R_3^2 \rangle_3 e^{-\Gamma_2 \tau} \cos(2\Omega \tau), \end{aligned} \quad (35)$$

where

$$\Gamma_0 = \gamma_0 + \gamma_c (N - \langle R \rangle_3) \quad (36)$$

$$\Gamma_1 = \gamma_1 + \frac{1}{2} \gamma_c (N - 2 \langle R \rangle_3) \quad (37)$$

$$\Gamma_2 = \gamma_2 + \gamma_c (N - \langle R \rangle_3) \quad (38)$$

The expressions for $\langle R_{12} R_{21} \rangle_3$ and $\langle R_3^2 \rangle_3$ can be found in Appendix. It is easy to see that the steady-state spectrum of the fluorescence field, which is proportional to the Fourier transform of the atomic correlation function (35), contain five components at frequencies $\omega_{21} (S_0)$, $\omega_{21} \pm \Omega (S_{\pm 1})$ and

$\omega_{21} \pm 2\Omega (S_{\pm 2})$. All the spectrum components have the Lorentzian shape. The central component S_0 has the linewidth Γ_0 and intensity $I_0 \sim \frac{1}{4} \cos^2 \alpha \langle R_3^2 \rangle_3$; the intermediate sidebands $S_{\pm 1}$ have linewidths Γ_1 and intensities

$I_{\pm 1} \sim \frac{1}{2} \sin^2 \alpha \langle R_{12} R_{21} \rangle_3$; the extreme sidebands $S_{\pm 2}$ have the linewidths Γ_2 and intensities $I_{\pm 2} \sim \frac{1}{8} \cos^2 \alpha \langle R_3^2 \rangle_3$.

As for the case when the thermal field is absent, the fluorescence spectrum is symmetric.

From eqs. (36)-(38) one sees that the linewidths of the spectrum components contain the single-atom terms (γ_0, γ_1 and γ_2) and the collective terms (which are proportional to γ_c). For the case of $\delta_{21} \sin^2 \alpha = \delta_{32} \cos^2 \alpha$ the collective terms vanish and the linewidths of the spectrum components are the same as for the single-atom case. In this case, as has been noted above, the linewidths of the spectrum components are expanded due to the presence of the thermal fields and this expansion is rather large for the case of $\bar{n}_1, \bar{n}_2 \gg 1$. In this case, the parameter $\lambda = 1$ for arbitrary values of \bar{n}_1, \bar{n}_2 and, consequently, all the intensities of the spectrum components $I_0, I_{\pm 1}$ and $I_{\pm 2}$ are independent of the thermal field and have the superradiant behaviour ($\sim N^2$).

For the case of $\delta_{21} \sin^2 \alpha \neq \delta_{32} \cos^2 \alpha$ the density matrix (11) is dependent on the thermal field intensities \bar{n}_1, \bar{n}_2 , consequently, the single-atom part as well as the collective part of the spectrum linewidths are dependent on the thermal field. For a large number of atoms $N \gg 1, \bar{n}_1, \bar{n}_2$ the linewidths of the spectrum components take the form

$$\Gamma_0 \approx \begin{cases} \gamma_0 + |\gamma_c| N - 4 \chi_2 & \text{if } \delta_{21} \sin^2 \alpha > \delta_{32} \cos^2 \alpha \\ \tilde{\gamma}_0 & \text{if } \delta_{21} \sin^2 \alpha < \delta_{32} \cos^2 \alpha \end{cases} \quad (39)$$

IV. STATISTICAL PROPERTIES OF SPECTRUM COMPONENTS

In this section we consider the influence of the thermal field on the photon statistics of the spectrum components of the fluorescence field according to the lower atomic transition

$$|2\rangle \rightarrow |1\rangle + \gamma$$

It is easy to find from the canonical transformation (4) that

$$J_{21} = \frac{1}{2} \cos \alpha R_3 + \frac{1}{2} \sin \alpha (R_{32} + R_{21}) + \frac{1}{2} \cos \alpha (R_{13} - R_{31}) \quad (44)$$

The operators $\frac{1}{2} \cos \alpha R_{13}$, $\frac{1}{2} \sin \alpha R_{12}$, $\frac{1}{2} \cos \alpha R_3$, $\frac{1}{2} \sin \alpha R_{32}$ and $-\frac{1}{2} \cos \alpha R_{31}$ can be considered as operator sources of the spectrum components of the fluorescent field at frequencies $\omega_{21} - 2\nu$, $\omega_{21} - \nu$, ω_{21} and $\omega_{21} + 2\nu$ and $\omega_{21} + \nu$ respectively. For simplicity these operators will be denoted by S_1^+ , S_2^+ , S_3^+ and S_4^+ , respectively.

As Loudon [18], we define the degree of second-order coherence (normalized correlation function) of the spectrum components

$$S_i^+ \quad (i = 1, 2, 3, 4) \text{ in the form}$$

$$G_i^{(2)} = \frac{\langle S_i^+ S_i^+ S_i^+ S_i^+ \rangle}{\langle S_i^+ S_i^+ \rangle^2} \quad (i = 1, 2, 3, 4) \quad (45)$$

The normalized correlation function $G_i^{(2)}$ describes the photon statistics of the spectrum component S_i^+ .

By using the steady-state density matrix (11) one can find the normalized correlation functions $G_i^{(2)}$ in the form

$$G_i^{(2)} = \frac{\langle R_4^3 \rangle / \langle R_2^3 \rangle}{\langle R_4^2 \rangle / \langle R_2^2 \rangle} = \frac{3 \langle R_4^2 \rangle + 4 \langle R_2^2 \rangle + 8 \langle R_2^4 \rangle - 8 \langle R_4^2 \rangle}{3 \langle R_4^2 \rangle + 4 \langle R_2^2 \rangle + 8 \langle R_2^4 \rangle} \quad (46)$$

$$\left\{ \begin{aligned} J_2 &\approx \left\{ \begin{aligned} &R_2^2 - 4X^2 + 12|N\rangle \quad \text{if } R_2^2 \sin^2 \alpha > R_2^2 \cos^2 \alpha \\ &R_2^2 - 2X^2 + \frac{1}{2}|2\rangle |N\rangle \quad \text{if } R_2^2 \sin^2 \alpha < R_2^2 \cos^2 \alpha \end{aligned} \right. \quad (40) \\ & \quad \text{if } R_2^2 \sin^2 \alpha > R_2^2 \cos^2 \alpha \\ & \quad \text{if } R_2^2 \sin^2 \alpha < R_2^2 \cos^2 \alpha \end{aligned} \right. \quad (41)$$

$$\tilde{J}_2^0 = \tilde{J}_2 - 2X^2 = \tilde{J}_2 \cos^2 \alpha (\tilde{n}_2^+ + \frac{1}{2}) + \tilde{J}_2 \sin^2 \alpha < \tilde{J}_2 \quad (42)$$

$$\tilde{J}_2^2 = \tilde{J}_2 - 2X^2 = \tilde{J}_2 (\tilde{n}_2^+ + \frac{1}{2}) \cos^2 \alpha + \tilde{J}_2 (\tilde{n}_2^+ + \frac{1}{2}) \sin^2 \alpha < \tilde{J}_2 \quad (43)$$

It is seen from eqs. (39)-(43) that in the case of the collective part $\tilde{J}_2 \sin^2 \alpha > \tilde{J}_2 \cos^2 \alpha$ and $N \gg 1$, \tilde{n}_2^+ and demotes over the other parts of the line-widths. Consequently, in this case the spectrum line-widths are approximately independent of the thermal field intensities \tilde{n}_2^+ .

In this case all the integral intensities of the spectrum components I_0 , I_1^+ and I_2^+ are dependent on the thermal field intensities \tilde{n}_2^+ , \tilde{n}_2^- and are proportional to the number of atoms.

In the opposite case, $\tilde{J}_2 \sin^2 \alpha < \tilde{J}_2 \cos^2 \alpha$ and $N \gg 1$, \tilde{n}_2^+ , \tilde{n}_2^- ; the spectrum line-widths and intensities of the intermediate sidebands S_1^+ have analogous properties as in the previous case. Contrary to the intermediate sidebands, the line-widths of the central component S_0 and the extreme sidebands S_2^+ are less than the one-atom line-widths (see eqs. (42), (43), i.e. the collective narrowing of the one-atom line-widths of the central component S_0 and the extreme sidebands take place.

In this case the line-widths J_0 and J_2 broaden due to the thermal field and the intensities I_0 and I_2^+ have the super-radiant behaviour ($\sim N^2$).

$$G_{-1,-1}^{(2)} = \frac{\langle R_{12}^2 R_{21}^2 \rangle_S}{\langle R_{12} R_{21} \rangle_S^2} = G_{1,1}^{(2)} = \frac{4}{3} \frac{\langle R^4 \rangle_S - 2(N+2)\langle R^3 \rangle_S + (N^2+5N+5)\langle R^2 \rangle_S - (N^2+3N+2)\langle R \rangle_S}{(\langle R \rangle_S - \langle R^2 \rangle_S)^2} \quad (47)$$

$$G_{2,2}^{(2)} = \frac{\langle R_{31}^2 R_{13}^2 \rangle_S}{\langle R_{31} R_{13} \rangle_S^2} = G_{-2,-2}^{(2)} = \frac{6}{5} \frac{\langle R^4 \rangle_S + 4\langle R^3 \rangle_S + \langle R^2 \rangle_S - 6\langle R \rangle_S}{(\langle R^2 \rangle_S + 2\langle R \rangle_S)^2} \quad (48)$$

where the steady-state averages $\langle R^n \rangle_S$ can be found in Appendix.

For the single-atom case one finds

$$G_{0,0}^{(2)} = \langle R \rangle_S / \langle R^2 \rangle_S = \frac{2X+1}{2X} > 1 \quad (49)$$

$$G_{\pm 1, \pm 1}^{(2)} = G_{\pm 2, \pm 2}^{(2)} = 0 \quad (50)$$

It is easy to see from eqs. (47) and (12) that in the one-atom case the degree of second-order coherence of the central spectrum component S_0 is dependent on the thermal field intensities \bar{n}_1 , \bar{n}_2 without the point of $\chi_{21} \sin^2 \alpha = \chi_{32} \cos^2 \alpha$. However we note that as for the case when the thermal field intensities $\bar{n}_1 = \bar{n}_2 = 0$ [14], the central component S_0 has super-Poissonian photon statistics, i.e. $G_{0,0}^{(2)} > 1$, for all values of the parameter X . As is seen from eq.(48), the degree of second-order coherence of the sidebands $S_{\pm 1}$, $S_{\pm 2}$ is equal zero for arbitrary values of the thermal field intensities \bar{n}_1 , \bar{n}_2 , i.e. the sub-Poissonian photon statistics takes places for the sidebands.

In contrast with the single-atom case, the photon statistics of all the spectrum components in the collective case $N \geq 2$ is strongly dependent on the thermal field intensities \bar{n}_1 and \bar{n}_2 . The dependence of the normalized correlation functions $G_{0,0}^{(2)}$, $G_{\pm 1, \pm 1}^{(2)}$ and $G_{\pm 2, \pm 2}^{(2)}$ on the thermal field intensities \bar{n}_1 ,

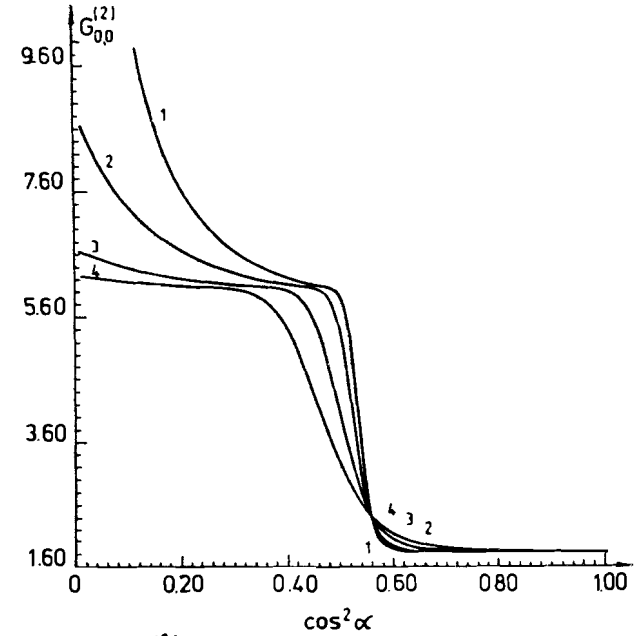


Fig. 1. Function $G_{0,0}^{(2)}$ against the parameter $\cos^2 \alpha$ for the case of $N = 50$; $\chi_{32} / \chi_{21} = 0.8$. The curves 1-4 correspond $\bar{n}_2 = 2\bar{n}_1 = 0; 0.3; 1; 2$.

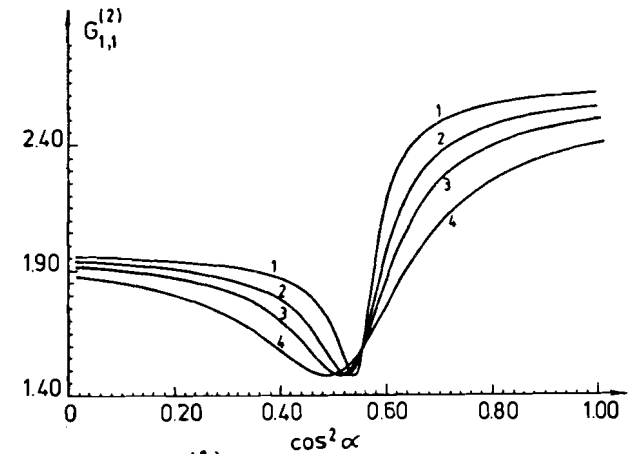


Fig. 2. Function $G_{1,1}^{(2)}$ against the parameter $\cos^2 \alpha$ for the case of $N = 50$; $\chi_{32} / \chi_{21} = 0.8$. The curves 1-4 correspond $\bar{n}_2 = 2\bar{n}_1 = 0; 0.5; 1; 2$.

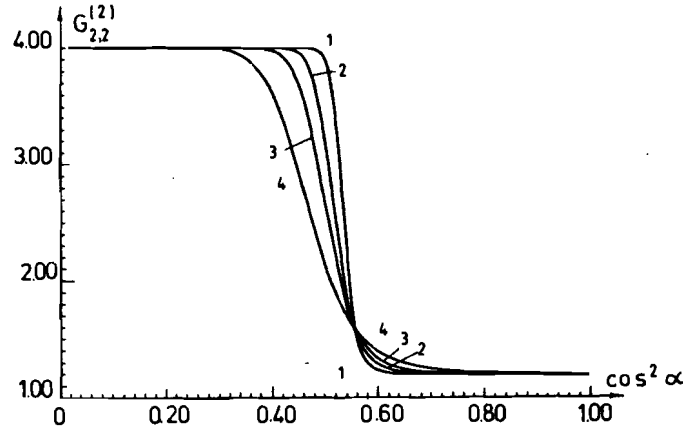


Fig. 3. Function $G_{2,2}^{(2)}$ against the parameter $\cos^2 \alpha$ for the case of $N = 50$; $\gamma_{32}/\gamma_{21} = 0.8$. The curves 1-4 correspond $\bar{n}_2 = 2\bar{n}_1 = 0; 0.5; 1; 2$.

\bar{n}_2 is shown in Fig. 1-3 where they are plotted as functions of the parameter $\cos^2 \alpha$ for various values \bar{n}_1 , \bar{n}_2 and for $\gamma_{32}/\gamma_{21} = 0.5$; $N = 50$. It is clear from Figs. 1-3 that except for the point of $\gamma_{21} \sin^2 \alpha = \gamma_{32} \cos^2 \alpha$, the thermal field plays an important role in the photon statistics of the spectrum components.

In a similar manner one can show that the fluorescence field corresponding to the upper atomic transition $|3\rangle \rightarrow |2\rangle$ has analogous spectral and statistical properties.

V. SUMMARY

We have considered the problem of collective double resonance in the presence of two coherent resonant external fields and of the thermal field reservoir. The stationary solution for the master equation in the secular approximation is given. The spectral and statistical properties of the fluorescence field are investigated. The influence of the thermal field on the spectral properties of the fluorescence field has been shown. The collective narrowing of some spectrum components has also been predicted. It is shown that contrary to the one-atom case the thermal field

plays an important role in determining the photon statistics of all the spectrum components.

APPENDIX

In this appendix we give the explicit expressions for the averages of the atomic operators over the steady-state (11). The parameters X and Z are given in formulas (12) and (13).

$$\langle R \rangle_S = Z^{-1} [N(N+1)X^{N+3} - 2(N+2)NX^{N+2} + (N+1)(N+2)X^{N+1} - 2X] / (X-1)^3 \quad (\text{A.1})$$

$$\langle R^2 \rangle_S = Z^{-1} [N^2(N+1)X^{N+4} - N(3N^2+6N-1)X^{N+3} + (N+2)(3N^2+3N-2)X^{N+2} - (N+1)^2(N+2)X^{N+1} + 4X^2 + 2X] / (X-1)^4 \quad (\text{A.2})$$

$$\langle R^3 \rangle_S = Z^{-1} [N^3(N+1)X^{N+5} - (4N^4+8N^3-3N^2+N)X^{N+4} + (6N^4+18N^3+3N^2-15N+8)X^{N+3} - (4N^4+16N^3+15N^2-9N-11)X^{N+2} + (N^4+5N^3+9N^2+7N+2)X^{N+1} - 8X^3 - 14X^2 - 2X] / (X-1)^5 \quad (\text{A.3})$$

$$\langle R^4 \rangle_S = Z^{-1} [N^4(N+1)X^{N+6} - (5N^5+10N^4-6N^3+4N^2-N)X^{N+5} + (10N^5+30N^4-4N^3-32N^2+42N-16)X^{N+4} - (10N^5+40N^4+24N^3-60N^2-24N+66)X^{N+3} + (5N^5+25N^4+36N^3-8N^2-58N-36)X^{N+2} - (N^5+6N^4+14N^3+16N^2+9N+2)X^{N+1} + 16X^4 + 66X^3 + 36X^2 + 2X] / (X-1)^6 \quad (\text{A.4})$$

$$\langle R_{13}^2 \rangle_S = 2 \langle R_{31} R_{13} \rangle_S = 2 \langle R_{13} R_{31} \rangle_S = \frac{1}{3} \langle R^2 \rangle_S + \frac{2}{3} \langle R \rangle_S \quad (\text{A.5})$$

$$\langle R_{12} R_{21} \rangle_S = \langle R_{32} R_{23} \rangle_S = -\frac{1}{2} \langle R^2 \rangle_S + \frac{1}{2} (N+1) \langle R \rangle_S \quad (\text{A.6})$$

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Коллективный двойной резонанс
в присутствии теплового поля

E17-89-618

Рассмотрена проблема коллективного двойного резонанса в присутствии теплового поля. Получено стационарное решение матрицы плотности атомной системы для случая сильных внешних полей. Обсуждены спектральные и статистические свойства поля флуоресценции. Показано коллективное сужение спектральных компонент.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Collective Double Resonance in the
Presence of a Thermal Field

E17-89-618

The problem of collective double optical resonance in the presence of a thermal field reservoir is considered. The steady-state solution to the atomic density matrix is obtained for the case of strong external fields. The spectral and statistical properties of the fluorescence field are discussed. The collective narrowing of spectrum components is shown.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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