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A MATHEMATICAL MODEL OF HEAT TRANSMISSION IN ESSENTIALLY NONLINEAR CONJUGATE MEDIUMS

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1. INTRODUCTION

The study of regularities of spreading electromagnetic emission in mediums with nonlinear properties attracts nowadays a great number of investigators. So, for instance, the spreading of electromagnetic waves in mediums whose dielectric penetrability depends on the electric vector has been considered in refs.[1-5]. In ref.[6] the system of the Maxwell nonlinear equations for conjugate domains (cubes with a common centre, coaxis cylinders, concentric spheres) has been solved. In particular, this system of equations was solved for the following dependence of dielectric penetrability on the field

$$\epsilon_i(\omega/\vec{E}_i) = \epsilon_{oi}(\omega) - |\alpha_i(\omega))|\vec{E}^2, \tag{1}$$

where ω is the frequency of a wave, α_i is a parameter characterizing nonlinear properties of medium ($\alpha_i > 0$ corresponds to the case of a ray self-focusing; $\alpha_i < 0$, to defocusing), index "1" refers to the inner cube, cylinder or sphere; and index "2", to the corresponding outer figures. The classes of solutions satisfying conjugate conditions on the border line, for which a squared electric vector is a constant quantity, have been got: $\vec{E}_i = \epsilon_{oi}/|\alpha_i|$, $\alpha_i \neq 0$. In[6] (and in a more general form in[7]) possible restrictions on the problem parameters (first of all on the frequency ω) concerning the obtained solutions were also discussed.

On the basis of the solutions [6,7] it is possible to get expressions for the densities of heat flows given off in the volumes of domains considered. Really, for the densities of heat flows, as it is known from [26], it can be written

$$q_i = \frac{\omega}{8\pi}\epsilon_i''|\vec{E}_i|^2, \tag{2}$$

where ϵ_i'' is an imaginary part of dielectric penetrability.

Supposing that the electric induction vector is connected with the electric tension vector according to

$$d\vec{D}_i = \epsilon_i(\vec{E}_i)\,d\vec{D}_i \tag{3}$$

(see p.531 in [26]), one can show that the mentioned classes of solutions with allowance for (3) satisfy approximately (and in some cases precisely) the gauge conditions $\nabla\cdot\vec{D}_i = 0$, if the inequalities

$$R_i \left| \frac{\alpha_i \vec{E}_i \nabla\cdot\vec{E}_i}{\epsilon_{oi}} \right| \ll 1 \qquad (4)$$

are fulfilled, where R_i are characteristic dimensions in the systems considered. The perturbed solutions $\vec{E}_i = \vec{E}_i^o + \vec{E}_i'$, $\vec{H}_i = \vec{H}_i^o + \vec{H}_i'$ [6,7] in [27] have been used in the problem of electromagnetic wave dispersion on a spherical particle with nonlinear properties. It is worth mentioning that for $\vec{E}_i'$ and $\vec{H}_i'$ it led to a linear problem with dielectric penetrability $\tilde{\epsilon}_{oi} = -2\epsilon_{oi}$ and the absorption coefficient $\tilde{\chi}_i = -(8\pi\sigma_i/\omega)$.

On the basis of expressions (1)-(3) and taking account of constant values $\vec{E}_i^2$ let us find that the densities of heat flows are equal to

$$q_i = \frac{\omega \epsilon_{oi}''((\epsilon_{oi}')^2 + (\epsilon_{oi}'')^2)^{1/2}}{12\pi|\alpha_i|}, \qquad (5)$$

where $\epsilon_{oi}' = \mathrm{Re}(\epsilon_{oi})$. In particular, if the inequality $\epsilon_{oi}' \gg \epsilon_{oi}''$ holds, formula (5) transforms into

$$q_i = \frac{\omega \epsilon_{oi}'' \epsilon_{oi}'}{12\pi|\alpha_i|}. \qquad (6)$$

Further, for brevity we shall call the densities of heat streams $q_i(T_i)$ the heat sources.

The intensive investigations of heat processes taking place in mediums, whose properties (heat conductivity coefficient and some other) and a heat source depend on the temperature, are carried out nowadays. So, for instance, in refs. [8,9] the regularities of warming up and vaporization of transparent dielectrics in general conditions and laser emission field have been investigated. A great number of references (see [11-13]) devoted to the questions of getting infinite temperatures in limited domains during the final time and the questions of heat localization are summed up in a fundamental monograph [10]. A nonstationary heat emission equation with heat emission coefficient, density and heat source depending on the temperature (in the form of powers, as a rule) is used as a basis of this investigation.

The peculiarities of heat transmission in conjugate mediums on the basis of the corresponding heat conductivity equations with heat sources of the type (6), essentially depending on temperature, are studied in this paper. It is supposed that on the division surface of two mediums, in the general case, there can exist a supplementary heat source. In particular, a division surface can be a surface of phase transition so that its position could change in the course of time. The examples of those systems can be crystalline systems, systems "plasma (gas) - solid cover", liquid-crystalline systems with a travelling boundary surface, a transition surface of one liquid-crystalline phase to another.

2. GENERAL STATEMENT OF THE PROBLEM IN A QUASISTATIONARY APPROXIMATION. A POWER DEPENDENCE OF THE HEAT CONDUCTIVITY COEFFICIENT AND HEAT SOURCE ON THE TEMPERATURE

A quasistationary heat transmission in the considered conjugate systems (cubes with a common centre, co-axis cylinders, concentric spheres) can be described on the basis of heat conductivity equation (7) according to the existing edge conditions

$$\vec{\nabla}\cdot(\chi_i' \vec{\nabla} T_i) = -q_i/\chi_{oi}, \quad i=1,2, \qquad (7)$$

$$T_2|_{S_2} = T_o, \quad T_1|_{S_1} = T_2|_{S_1}, \qquad (8)$$

$$-\chi_1 \vec{\nabla} T_1|_{S_1} = -\chi_2 \vec{\nabla} T_2|_{S_2} + \vec{G}(S_1, T_S), \qquad (9)$$

where $\chi_i' = \chi_i \chi_{oi}$, χ_i is the heat conductivity coefficient, i are mediums, χ_{oi} is the meaning of χ_i at some fixed temperature, T_i is the temperature, S_1 is the surface limiting the inner domain, S_2 is the surface limiting the outer domain, T_s is the surface temperature S_1, $T_s = T_1(S_1)$. Condition (9) is the condition of heat balance on the boundary surface of the mediums. By $\vec{G}$ the surface density of heat flow is denoted which, in particular, can be conditioned by a phase transition on the surface S_1 which in its turn can be travelling: $S_1 = S_1(t)$, t - time.

2.1. Let us consider the spreading of heat in the system "cube in the cube" supposing that it is homogeneous in y and z. In this case, the edge problem (7)-(9) will be written as follows (a calculation diagram is given in figure 1):

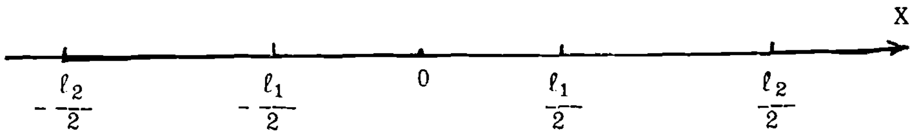

Fig.1. A calculation diagram.

$$\frac{d}{dx}\left(\chi_i'(T_i)\frac{dT_i}{dx}\right) + \frac{q_i(T_i)}{\chi_{oi}} = 0, \quad i = 1, 2,$$

$$T_2\left(\frac{\ell_2}{2}\right) = T_2\left(-\frac{\ell_2}{2}\right) = T_o, \quad T_1\left(-\frac{\ell_1}{2}\right) = T_1\left(\frac{\ell_1}{2}\right) \equiv T_s, \tag{10}$$

$$T_2\left(-\frac{\ell_1}{2}\right) = T_2\left(\frac{\ell_1}{2}\right) = T_s,$$

$$-\chi_1\frac{dT_1}{dx}\Big|_{\pm\ell/2} = -\chi_2\frac{dT_2}{dx}\Big|_{\pm\ell/2} + G\left(T_s, \pm\frac{\ell_1}{2}\right), \tag{11}$$

where ℓ_1 is the edge of the inner cube; ℓ_2, of the outer cube.

By changing $d\phi_i = \chi_i'(T_i)\,dT_i$, equation (10) is transformed into equation:

$$\frac{d^2\phi_i}{dx^2} + \frac{q_i(T_i(\phi_i))}{\chi_{oi}} = 0. \tag{12}$$

Equation (12) is integrated in quadratures /14/ so that coming back to T_i it can be written as

$$x - x_0 = \pm\int_{T_{oo}}^{T_i} \frac{\chi_i'(T_i)\,dT_i}{\left(c_i - \frac{2}{\chi_{oi}}\int_{T_{oo}}^{T_i} q_i(T_i)\,\chi_i'(T_i)\,dT_i\right)^{1/2}}, \tag{13}$$

where T_{oo} is some fixed temperature, c_i are constant values defined while solving the boundary problem, the sign "plus" or "minus" before the integral is chosen so (if there is such a necessity) that the function $T_i(x - x_o)$ is even.

Let us place equation (13) in the boundary conditions and get the following system for c_1, c_2, T_s:

$$\ell_1 = 2p\int_{T_{oo}}^{T_s} \frac{\chi_i'(T_i)\,dT_i}{\left(c_i - \frac{2}{\chi_{oi}}\int_{T_{oo}}^{T_i} q_i(T_i)\,\chi_i'(T_i)\,dT_i\right)^{1/2}}, \quad i = 1, 2, \tag{14}$$

$$\ell_2 = 2p\int_{T_{oo}}^{T_o} \frac{\chi_2'(T_2)\,dT_2}{\left(c_2 - \frac{2}{\chi_{02}}\int_{T_{oo}}^{T_2} q_2(T_2)\,\chi_2'(T_2)\,dT_2\right)^{1/2}}, \tag{15}$$

$$-\chi_{01}\,p\left(c_1 - \frac{2}{\chi_{01}}\int_{T_{oo}}^{T_s} q_1(T_1)\,\chi_1'(T_1)\,dT_1\right)^{1/2} =$$

$$= -\chi_{02}\,p\left(c_2 - \frac{2}{\chi_{02}}\int_{T_{oo}}^{T_s} q_2(T_2)\,\chi_2'(T_2)\,dT_2\right)^{1/2} + G\left(T_s, \pm\frac{\ell_1}{2}\right), \tag{16}$$

where p denotes the signs "plus" or "minus" chosen properly. It follows from (16) that the value G must be even with respect to its second argument, $G(T_s, \frac{\ell_1}{2}) = G(T_s, -\frac{\ell_1}{2})$. From expression (15) we can find c_2, then from (14), written for i=2, we can find a constant T_s. We shall conditionally call this calculation diagram for defining constants diagram $c_2 T_s c_1$. The heat balance condition (16) is the condition on the problem parameters, for example, on the outer cube dimension ℓ_2.

From the condition $-\chi_1\frac{dT_1}{dx}\big|_{\pm\frac{\ell_1}{2}} = 0$ we can get these dimensions ℓ_1, for which an adiabatic condition on the boundary surface will be fulfilled. As it follows from (16) this condition looks like

$$c_1(\ell_1, \ell_2) = \frac{2}{\chi_{01}} \int_{T_{00}}^{T_s(\ell_2,\ell_1)} q_1(T_1)\, \chi_1'(T_1)\, dT_1 . \qquad (17)$$

From (17) we can get one, several, a great number of or no values or ℓ_1, according to the adiabatic surfaces. In this case, the value ℓ_2 is gound from the following condition:

$$\chi_{02}^2 (c_2(\ell_2) - \frac{2}{\chi_{02}} \int_{T_{00}}^{T_s(\ell_2,\ell_1)} q_2(T_2)\, \chi_2'(T_2)\, dT_2) = G^2(\ell_2, \ell_1) . \qquad (18)$$

Thus, for special sets $\{\ell_1^{(n)}, \ell_2^{(n)}\}$ a heat insulation condition for the inner domain will hold (here n is the number of the adiabatic surface).

Let us consider the movement of the boundary ℓ_1 near $\ell_1^{(n)}$ supposing that it does not change essentially the geometry of the considered system ($\Delta\ell_1/\ell_1^{(n)} \ll 1$, $\Delta\ell_1 = |\ell_1 - \ell_1^{(n)}|$). By the law of conservation of energy one can write

$$\frac{dA}{dt} = \frac{G}{\rho_1 \ell_1},$$

where ρ_1 is density, dA is a specific work performed in shifting the border. At the same time, the work of the polytropic process is equal to $dA = \frac{C_{p1}}{\gamma_1} dT_s$, where C_{p1} is the thermal heat capacity under constant pressure, γ_1 is a polytropy degree. Accordingly, the change of the temperature T_s with time is defined by the formula

$$t(T_s) = \left| \int_{T_s^{(n)}}^{T_s} \frac{\rho_1 C_{p1} \ell_1(T_s)\, dT_s}{\gamma_1 G(T_s, \ell_1(T_s))} \right|,$$

where $T_s^{(n)} = T_s(\ell_1^{(n)})$. If the upper limit of the integral is equal to infinity or some critical value of temperature T_s^*, then $t_f = t(T_s^*)$ may be considered as the time of sharpening (/10/) for this process.

2.2. Let us suppose, following /10-13/, that $\chi_i' = \eta_i T_i^{\sigma_i}$, $q_i = q_{oi} T_i^{\beta_i}$. Indeed, a power dependence of a heat conductivity coefficient is characteristic of gases, plasma, crystalline substances at rather high temperatures. Power dependence of a heat source can be considered in some range of temperatures.

In this case, from (12) we get

$$\frac{d^2\phi_i}{dx^2} + a_i \phi_i^{\beta_i/(\sigma_i+1)} = 0, \qquad (19)$$

where a constant

$$a_i = \frac{q_{oi}}{\chi_{oi}} \left(\frac{\sigma_i + 1}{\eta_i}\right)^{\beta_i/(\sigma_i+1)} \qquad \text{if } \sigma_i > -1,$$

and in other cases it will be defined later. The integral (13) in this case is a differential binomial expressed in elementary functions if

$$\frac{\sigma_i + 1}{\beta_i + \sigma_i + 1} \quad \text{or} \quad \frac{\sigma_i - \beta_i + 1}{2(\beta_i + \sigma_i + 1)}$$

are whole numbers.

Here are the examples of these solutions.

2.2.1. Let $\frac{\sigma_i + 1}{\beta_i + \sigma_i + 1}$ be a whole number. It is interesting to consider the heat source for which $\frac{dq_i}{dT_i} > 0$. This is possible if the equation

$$\beta_i = |\sigma_i + 1| \frac{2k}{2k+1}, \qquad \sigma_i + 1 < 0,\ k = 1, 2, \ldots$$

is fulfilled. At $k = 1$, we have the following solution:

$$|x_i'| = F(z_i), \qquad (20)$$

where

$$x_i' = \frac{36 a_i^3 x}{c_i^{5/2}}, \quad F(z_i) = \sqrt{z_i}\left(\frac{z_i^2}{5} - \frac{2}{3} z_i + 1\right),$$

$$z_i = 1 + \frac{6a_i T_i^{(\sigma_i+1)/3}}{|\sigma_i + 1|^{1/3}\, c_i}, \quad a_i = \frac{q_{oi}}{\chi_{oi}}\left(\frac{|\sigma_i+1|}{\eta_i}\right)^{\frac{2}{3}|\sigma_i+1|}, \quad c_i \neq 0.$$

We find the constants c_1, c_2, T_s from the boundary conditions (according to the diagram $c_2 T_s c_1$) which becomes

$$\ell'_2 = 2F(z_2(T_0)), \quad \ell''_1 = 2F(z_2(T_s)), \quad \ell'_1 = 2F(z_1(T_s)),$$

where

$$\ell'_2 = 2x'_2\left(\frac{\ell_2}{2}\right), \quad \ell'_1 = 2x'_1\left(\frac{\ell_1}{2}\right), \quad \ell''_1 = 2x'_2\left(\frac{\ell_1}{2}\right).$$

As a result, we get the dependences $T_s = T_s(\ell_2, \ell_1)$, $c_2 = c_2(\ell_2)$, $c_1 = c_1(\ell_2, \ell_1)$. We find the edge ℓ_2 from heat balance conditions taking $c_i \neq 0$ as

$$\chi_{01}\left(c_1 + \frac{6a_1 T_s^{(\sigma_1+1)/3}}{|\sigma_1+1|^{1/3}}\right)^{1/2} = \chi_{02}\left(c_2 + \frac{6a_2 T_s^{(\sigma_2+1)/3}}{|\sigma_2+1|^{1/3}}\right)^{1/2} - G(T_s, \ell_1). \quad (21)$$

The same solutions can be obtained for other values of k, being greater than one (Chebyshev's [15] second substitution is used in integrating), and in writing the functions $F(z_1)$ higher degrees of z_i appear.

It follows from (21) that the adiabatic conditions can be fulfilled only if

$$c_1 = -\frac{6a_1 T_s^{(\sigma_1+1)/3}}{|\sigma_1+1|^{1/3}}.$$

Here, as it follows from (20) $\ell_1 = 0$ for i=1. It means that this condition holds only at the point. In particular, we can consider the case $T_s \to \infty$ for which $c_1 = 0$, $c_2 = G^2(\infty, 0)/\chi_{02}^2$. If $G(\infty, 0) = 0$, then $c_2 = 0$ and for ℓ_2 we get

$$\ell_2 = \frac{1}{5}\left(\frac{T_0^{\sigma_2+1}}{|\sigma_2+1|}\right)^{5/6} \sqrt{\frac{6}{a_2}}. \quad (22)$$

2.2.2. Let $\dfrac{\sigma_i - \beta_i + 1}{2(\beta_i + \sigma_i + 1)}$ be a whole number with $\sigma_i + 1 \neq \beta_i$.

For the value q_i to increase with temperature it is necessary to impose the condition

$$\beta_i = |\sigma_i + 1| \frac{2k-1}{2k+1}, \quad \sigma_i + 1 < 0, \quad k = 1, 2, \ldots .$$

In this case $a_i < 0$,

$$a_i = -\frac{q_{oi}|\sigma_i + 1|^{(2k-1)/(2k+1)}}{\chi_{oi}\, \eta_i}.$$

We give the solution for $k = 1$

$$|x_i| = \frac{4}{3} c_i \left[\frac{z_i}{12|a_i|\rho_i^2} + \frac{z_i}{24 a_i^2 \rho_i} + \frac{1}{144|a_i|^{5/2}} \ln\left|\frac{\sqrt{3|a_i|} + z_i}{\sqrt{3|a_i|} - z_i}\right|\right], \quad (23)$$

where

$$\rho_i = z_i^2 - 3|a_i|, \quad z_i = (|\sigma_i + 1|^{2/3} c_i + 3|a_i| T_i^{2/3(\sigma_i+1)})^{1/2} T_i^{-1/3(\sigma_i+1)}.$$

We find the constants c_1, c_2, T_s according to the digram c_2, c_1, T_s (as it has been done before). The heat balance condition becomes

$$\chi_{01}\left(c_1 + \frac{3|a_1| T_s^{2/3(\sigma_1+1)}}{|\sigma_1+1|^{2/3}}\right)^{1/2} = \chi_{02}\left(c + \frac{3|a_2| T_s^{2/3(\sigma_2+1)}}{|\sigma_2+1|^{2/3}}\right)^{1/2} - G(T_s, \ell_1). \quad (24)$$

The boundary surface will be adiabatic if

$$c_1 = -\frac{3|a_1| T_s^{2/3(\sigma_1+1)}}{|\sigma_1+1|^{2/3}}.$$

As follows from (23), this is possible only at the point $\ell_1=0$; in the case of $T_s\to\infty$ the final value of ℓ_2 is found only if $c_2\neq 0$. As $c_2=(G(\infty,0)/\chi_{02})^2$, one should require $G(\infty,0)\neq 0$.

2.2.3. Particularly, consider the case when the condition $\sigma_i+1=\beta_i$ is fulfilled. As we are interested in the condition $\beta_i>0$, then $\sigma+1>0$. The dependences $T_i(x)$ are

$$T_1=T_s\left(\frac{\cos(\sqrt{a_1}x)}{\cos(\sqrt{a_1}\ell_1/2)}\right)^{1/(\sigma_1+1)}, \quad T_2=T_0\left(\frac{\cos(\sqrt{a_2}x)}{\cos(\sqrt{a_2}\ell_2/2)}\right)^{1/(\sigma_2+1)},$$

$$T_s=T_0\left(\frac{\cos(\sqrt{a_2}\ell_1/2)}{\cos(\sqrt{a_2}\ell_2/2)}\right)^{1/(\sigma_2+1)}. \qquad (25)$$

As T_i, $T_s\geq 0$ in considering arbitrary dimensions of ℓ_i and values of a_i it is necessary to put some restrictions on the power degrees:

$$\frac{1}{\sigma_i+1}=2k_i \quad \text{or} \quad \frac{1}{\sigma_i+1}=\frac{2k_i}{2k_i+1}, \quad k_i=1,2,\dots .$$

On the boundary line we have the following condition which can be used for defining ℓ_2 or T_0

$$\frac{\chi_{01}T_s\sqrt{a_1}\sin(\sqrt{a_1}\ell_1/2)}{(\sigma_1+1)\cos(\sqrt{a_1}\ell_1/2)}=\frac{\chi_{02}T_0\sqrt{a_2}\sin(\sqrt{a_2}\ell_1/2)}{(\sigma_2+1)\cos(\sqrt{a_2}\ell_2/2)}+G(T_s,\ell_1/2). \qquad (26)$$

From (26) we find the dimensions of $\ell_1^{(n)}=2\pi n/\sqrt{a_1}$, n=0,1,2,... for which the heat flux from domain "1" to domain "2" is equal to zero. If G = 0, then from (26) we find $\ell_1^{(k)}=2\pi k/\sqrt{a_2}$, k = 0,1,2,... hence, it follows that in this case the condition $\frac{a_2}{a_1}=\frac{k^2}{n^2}$ must be hold. Any deflection of ℓ_1 from the adiabatic surface leads to the decrease of $\cos(a_1\ell_1/2)$ in the denominator of (25) thus increasing the temperature. In particular, if $\sqrt{a_1}\ell_1=\pi(2n+1)$, then the temperature $T_1\to\infty$. The value of the removed heat flow also tends to infinity. Therefore, the case where $\sigma_2>\sigma_1$, $\ell_1=\pi(2n+1)/\sqrt{a_1}$, $\ell_1=\pi(2k+1)/\sqrt{a_2}$, $T_s=0$ is interesting for us. In this case $T_1\to\infty$ and the localization of heat in the inner cube are realized simultaneously. The greater is $a_i\sim 1/|\alpha_i|$, the greater is the value of ℓ_1, as $\sqrt{\alpha_i}$.

2.3. In this section the systems consisting of two coaxis cylinders and two concentric spheres will be considered (see the calculation diagram in fig.2). Spherical geometry, in particular, is the most convenient in considering the processes occurring in liquid and liquid-crystalline heterogeneous two-component particles.

Supposing that the temperature T_i depends only on the radius, we simplify the boundary problem (7)-(9) for the system considered

$$\frac{1}{r^N}\frac{d}{dr}\left(r^N\chi_i'(T_i)\frac{dT_i}{dr}\right)+\frac{q_i(T_i)}{\chi_{oi}}=0, \qquad (27)$$

$$T_1(R_1)=T_2(R_1),\quad T_2(R_2)=T_0,\quad T_1(0)<\infty,$$
$$T_s\equiv T_1(R_1),\quad -\chi_1\frac{dT_1}{dr}\Big|_{R_1}=-\chi_2\frac{dT_2}{dr}\Big|_{R_1}+G(T_s,R_1), \qquad (28)$$

where N = 1 in the cylindric coordinate system, N = 2 in the spherical system; boundary surface temperature T_s is defined in the process of solving the problem (as it has been done before).

In considering the power dependences $\chi_i=\eta_i T_i^{\sigma_i}$, $q_i=q_{io}T_i^{\beta_i}$ we get the equation

$$r\frac{d^2\phi_i}{dr^2}+N\frac{d\phi_i}{dr}+a_i r\phi_i^{\beta_i/(\sigma_i+1)}=0 \qquad (29)$$

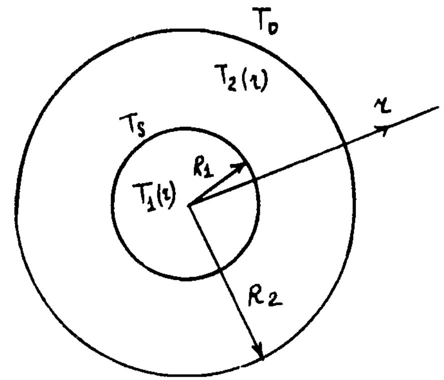

Fig.2. A calculation diagram.

instead of (27), where a_i is defined in the same way as in (19). In the spherical coordinate system at $\beta_i=k_i(\sigma_i+1)$, $k_i=0,1,2,\dots$, (29) is the Emden equation. If $\beta_i=\sigma_i+1$, then (29) is the Bassel equation, both at N = 1 and N = 2. Now we write down the solutions of the last case. In the cylindric coordinate system we get:

$$T_1 = T_s \left(\frac{J_0(\sqrt{a_1}\, r}{J_0(\sqrt{a_1} R_1)}\right)^{1/(\sigma_1+1)}, \tag{30}$$

$$T_2 = [\frac{\Phi_o(r)}{\Phi_o(R_1)}(T^{\sigma_2+1}\, Y_o(\sqrt{a_2}R_2) - T_o^{\sigma_2+1}\, Y_o(\sqrt{a_2}R_1)) +$$

$$+ T_o^{\sigma_2+1}\, Y_o(\sqrt{a_2}\, r)]^{1/(\sigma_2+1)}\, (Y_o(\sqrt{a_2}R_2))^{-1/(\sigma_2+1)}, \tag{31}$$

where

$$\Phi_n(r) = J_n(\sqrt{a_2}\, r)\, Y_o(\sqrt{a_2}R_2) - J_o(\sqrt{a_2}R_2)\, Y_n(\sqrt{a_2}\, r).$$

By $J_n(x)$, $Y_n(x)$ we denote n the order Bessel functions of the first and second rank, respectively. The temperature T_s is determined from the following transcendental equation that has been derived with the help of the heat balance condition

$$\frac{\chi_{01}\, \eta_1 \sqrt{a_1}\, T_s^{\sigma_1+1}\, J_1(\sqrt{a_1}R_1)}{(\sigma_1+1)\, J_0(\sqrt{a_1}\, R_1)} =$$

$$= \frac{\chi_{02}\eta_2\sqrt{a_2}}{\sigma_2+1}[\frac{\Phi_1(R_1)}{\Phi_0(R_1)}\, Y_0(\sqrt{a_2}\, R_2) - T_0^{\sigma_2+1}\, Y_0(\sqrt{a_2}R_1)) + \tag{32}$$

$$+ T_0^{\sigma_2+1}\, Y_1(\sqrt{a_2}R_1)]\, (Y_0(\sqrt{a_2}R_2))^{-1} + G(R_1, T_s).$$

In the spherical coordinate system the solutions are

$$T_1 = T_s \left(\frac{\sin(\sqrt{a_1}\, r)\, R_1}{\sin(\sqrt{a_1}R_1)\, r}\right)^{1/(\sigma_1+1)}, \tag{33}$$

$$T_2 = \left(\frac{A_1 \sin(\sqrt{a_2}r)}{r} + (T_0^{\sigma_2+1}\, R_2 - A_1)\, \frac{\cos(\sqrt{a_2}r}{r}\right)^{1/(\sigma_2+1)}, \tag{34}$$

where

$$A_1 = (T_0^{\sigma_2+1}\, R_2 \cos(\sqrt{a_2}R_1) - T_s^{\sigma_2+1}\, R_1)\cos(\sqrt{a_2}R_2)\sin(\sqrt{a_2}(R_2 - R_1)))^{-1}.$$

Here (just as in section 2.2.3) the restrictions on σ_i or the domain dimensions must be put. The temperature T_s is determined from the equation

$$\frac{\chi_{01}\, \eta_1\, T_s^{\sigma_1+1}}{\sigma_1+1}(R_1\sqrt{a_1}\, \rho_1 - 1) =$$

$$= \frac{\chi_{02}\eta_2 \sin(\sqrt{a_2}R_1)}{R_1(\sigma_2+1)}(A_1(R_1\sqrt{a_2}\, \rho_2 - 1 + R_1\sqrt{a_2} + \rho_2) - \tag{35}$$

$$- T_0^{\sigma_2+1}\, R_2(\sqrt{a_2}\, R_1 - \rho_2)) - G(R_1, T_s)\, R_1,$$

where

$$\rho_1 = \mathrm{ctg}(\sqrt{a_i}\, R_1).$$

Radii $R_1^{(n)}$ for which the adiabatic condition $-\chi_1 \frac{dT_1}{dr}|_{R_1} = 0$ holds can be found from the equations

$$J_1(\sqrt{a_1}R_1^{(n)}) = 0 \quad \text{or} \quad \sqrt{a_1}R_1^{(n)}\, \mathrm{ctg}(\sqrt{a_1}\, R_1^{(n)}) = 0. \tag{36}$$

The former is used for the cylindric coordinate system and the latter for the spherical coordinate system. If one of the conditions from (36) is fulfilled, the temperature T_s can be found, respectively, from the following equations:

$$\frac{\Phi_1(R_1)}{\Phi_0(R_1)}(T_s^{\sigma_2+1}\, Y_0(\sqrt{a_2}R_2) - T_0^{\sigma_2+1}\, Y_0(\sqrt{a_2}R_1)) +$$

$$+ T_0^{\sigma_2+1}\, Y_1(\sqrt{a_2}R_1) + G(T_s, R_1)\, Y_0(\sqrt{a_2}R_2) = 0, \tag{37}$$

$$A_1(R_1\sqrt{a_2}\rho_2 - 1 + R_1\sqrt{a_2} + \rho_2) - T_0^{\sigma_2+1}\, R_2(\sqrt{a_2}\, R_1 - \rho_2) - G(T_s, R_1) = 0,$$

where R_1 is equal to any of the set of values $\{R_1^{(n)}\}$. From (36) it follows (due to $a_1 \sim 1/|a_1|$) that the greater is the value of the parameter a_1, the larger is the domain $R_1^{(n)}$. The movement of the boundary surface near $R_1^{(n)}$ is supposed to be quasiadiabatic; thus (as it has been done in section 2.1) it can be written for the time

$$t = \frac{1}{N-1}\left| \int_{T_s^{(n)}}^{T_s} \frac{\rho_1(T_s)\, C_{p1}(T_s)\, R_1(T_s)\, dT_s}{\gamma_1(T_s)\, G(T_s)} \right| . \tag{38}$$

From expressions (30) and (33) it follows that in the process of movement $R_1(t)$ or at a certain setting of R_1, the denominators of the formulae turn to zero and $T_1 \to \infty$. These values of $R_{1f}^{(m)}$ are found from the expressions

$$R_{1f}^{(m)} = \frac{\pi m}{\sqrt{a_1}}, \quad N = 2, \quad J_0(R_{1f}^{(m)} \sqrt{a_1}) = 0, \; N = 1 . \tag{39}$$

Boundary transition time from the adiabatic surface to the nearest surface R_{1f} is determined (as in 2.1) according to the energy conservation law

$$t_f = \left| \int_{R_1^{(n)}}^{R_{1f}^{(n)}} \frac{\rho_1(T_s(R_1))\, L_f(T_s(R_1))\, dR_1}{G(T_s(R_1), R_1)} \right| , \tag{40}$$

$$dA = (N+1) L_f \, dR_1 / R_1$$

and plays the role of the time of sharpening.

Let us also consider the case when $m_i = 5$, $N = 2$ for which the analytic solution of Emden's[14] equation can be got

$$T_i = \left(\frac{3 b_i^2 \zeta_i}{r^2 + 3\zeta_i^2} \right)^{1/2(\sigma_i + 1)} , \tag{41}$$

where

$$b_i = (\sigma_i + 1)\, T_0 \sqrt{a_i}\, R_i \eta_i^{-1},$$

$$\zeta_i = [3b_i^2 \pm (9b_i^4 - 12\, T_s^{4(\sigma_i + 1)})^{1/2}]\, (6 T_s^{4(\sigma_i+1)})^{-1} .$$

On the boundary line R_1 the quasiadiabatic condition will be fulfilled if

$$\frac{\chi_{01} \eta_1 b_1 \sqrt{3\zeta_1}}{(\sigma_1 + 1) R_1} (1 + 3\zeta_1^2)^{-3/2} \le W , \tag{42}$$

where W is a small value of energy required according to the physical statement of the problem. The quantity T_s is determined from the condition

$$\frac{\chi_{02} \eta_2 b_2 \sqrt{3\zeta_2}}{(\sigma_2 + 1) R_1} (1 + 3\zeta_2^2)^{-3/2} + G(T_s, R_1) = W . \tag{43}$$

In this case, when the movement is R_1 the condition (43) is fulfilled continuously up to a certain movement R_1^0 within one and the same error W. This condition can easily be extended over the above-considered classes of solutions (2.2.1)-(2.2.2). And for the stationary boundary lines, we get a family $\ell_1(W)$ or $R_1(W)$ of these quasiadiabatic boundary for which the heat flow

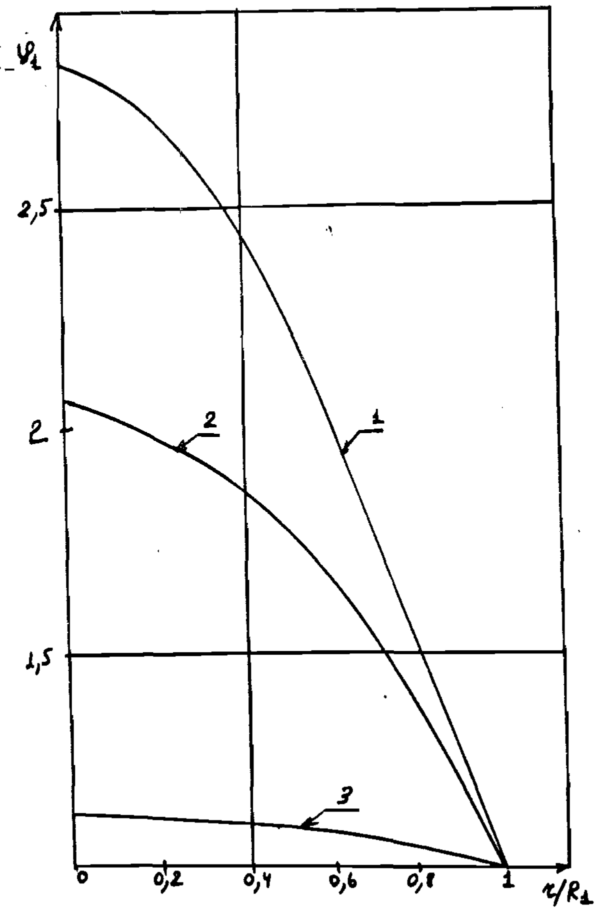

Fig.3. Dependences $\phi_1(r)$. The numeration of the curves corresponds to the following values of the parameters 1

1 - a=5, $\dfrac{\beta_1}{\sigma_1 + 1} = 1$;

2 - a=5, $\dfrac{\beta_1}{\sigma_1 + 1} = 0,5$;

3 - a=0,5, $\dfrac{\beta_1}{\sigma_1 + 1} = 4$.

$$-\chi_1 \frac{dT_1}{dx}\Big|_{\pm \ell_1/2} \lesssim W \quad \text{or} \quad -\chi_1 \frac{dT_1}{dr}\Big|_{R_1} \lesssim W .$$

Note that (as it has been shown in [10]) for $m_i > 5$ the solutions will also be strictly positive in R^3.

The diagrams of the solutions of $\phi_1(r)$ $(r \le R_1)$ obtained by integrating eq.(29) for different values of the parameters $\beta_1/(\sigma_1+1)$ and a_1 are given on Fig.3 as examples.

3. QUADRATIC DEPENDENCES OF THE q_i SOURCE ON THE TEMPERATURE

In some temperature range we approximate the dependence by a quadratic trinomial, $q_i(T_i) = Q_{oi} + Q_{1i}T_i + Q_{2i}T_i^2$.

For simplicity we suppose that $\chi_i = \chi_{oi}$ and this will help us to get a solution in the form of cnoidal waves. Approximating the heat conductivity coefficient χ_i by a quadratic trinomial, we can get a solution in the form of a linear combination of the Legendre´s elliptical integrals of the first, second and third rank.

3.1. Taking the dependence $q_i(T_i)$ as a quadratic trinomial (we´ll consider a more interesting case $Q_{2i} > 0$) from (13) we get

$$x = -\sqrt{\frac{3\chi_{oi}}{2Q_{2i}}} \int_{T_i}^{T_{1i}} \frac{dT_i}{((T_{1i} - T_i)(T_i - T_{2i})(T_i - T_{3i}))^{1/2}} , \qquad (44)$$

where T_{1i}, T_{2i}, T_{3i} are the roots of the cubic equation

$$T_i^2 + \frac{3}{2}\frac{Q_{1i}}{Q_{2i}}T_i^2 + \frac{3Q_{oi}}{Q_{2i}}T_i - \frac{3c_i^2\chi_{oi}}{2Q_{2i}} = 0 . \qquad (45)$$

c_i are the constants determined by the boundary conditions (11). For the roots (45) to be real it is necessary to fulfil the following conditions:

$$\frac{27Q_{oi}^3}{Q_{2i}} + \frac{9}{4}\frac{Q_{1i}^2Q_{oi}}{Q_{2i}^2} + \frac{3}{16}\frac{Q_{1i}^4Q_{oi}}{Q_{2i}^3} - \frac{23}{64}\frac{Q_{1i}^6}{Q_{2i}^4} + \frac{9}{4}c_i^2\chi_{oi}^2 + \frac{9Q_{1i}Q_{oi}}{Q_{2i}}c_i^2\chi_{oi} - \frac{3}{4}\frac{Q_{1i}c_i\chi_{oi}}{Q_{2i}^2} < 0 . \qquad (46)$$

When Q_{oi}, Q_{1i}, Q_{2i}, χ_{oi} are given, inequality (46) can be considered as some restriction on c_i. Further, we suppose that $T_{1i} > T_{2i} > T_{3i}$. Instead of (44) we can write:

$$x = p\,d_i F(\phi^{(i)}, k_i) , \qquad (47)$$

where

$$d_i = \left(\frac{6\chi_{oi}}{Q_{2i}(T_{1i} - T_{3i})}\right)^{1/2}, \quad k_i^2 = \frac{T_{1i} - T_{2i}}{T_{1i} - T_{3i}}, \quad \sin^2\phi^{(i)} = \frac{T_{1i} - T_i}{T_{1i} - T_{2i}} .$$

$F(\phi^{(i)}, k_i)$ is the Legendre elliptical integral of the first rank.

Transforming (47), we get an explicit expression for T_i in the cnoidal form

$$T_i = T_{1i} - (T_{1i} - T_{2i})\,\mathrm{sn}^2 \frac{x}{d_i} . \qquad (48)$$

Using (48) let us write the heat balance condition on the boundary line

$$\chi_{01}\frac{T_{11} - T_{21}}{d_1}\mathrm{cn}\left(\frac{\ell_1}{2d_1}\right)\mathrm{sn}\left(\frac{\ell_1}{2d_1}\right)\mathrm{dn}\left(\frac{\ell_1}{2d_1}\right) = \chi_{02}\frac{T_{12} - T_{22}}{d_2}\mathrm{cn}\left(\frac{\ell_1}{2d_2}\right)\mathrm{sn}\left(\frac{\ell_1}{2d_2}\right)\mathrm{dn}\left(\frac{\ell_1}{2d_2}\right) + \frac{G(T_s, \ell_1)}{2} , \qquad (49)$$

where snz, cnz, dnz are the Jacobi elliptical functions. From (49) it follows that the adiabatic conditions on the boundary surface are

$$\mathrm{cn}\frac{\ell_1}{2d_1} = 0 , \qquad \mathrm{sn}\frac{\ell_1}{2d_1} = 0. \qquad (50)$$

We have taken into account here that $\mathrm{dn}z > 0$ [16]. From (50) we find the dimensions

$$\ell_1^{(n)} = 2d\ \ K(k\ ), \quad n = 0, 1, 2, \ldots , \qquad (51)$$

where $K(k_1)$ is a full elliptical integral of the first rank, $K(k_1) = F(\pi/2, k_1)$. As in this case the boundary surface movement can be quasiadiabatic in the interval from one adiabatic surface to another. The temperature T_1 changes from T_{12} to T_{11}.

In the case when $k_i = 1$ and snz = thz, a soliton solution appears

$$T_i = T_{2i} + (T_{1i} - T_{2i})\,\mathrm{sech}^2 \frac{x}{d_i}. \qquad (52)$$

The adiabatic condition in this case holds at the point $\ell_1 = 0$, the heat flow removed from the surface S_1 increases with ℓ_1. At the same time, it follows from (52) that the temperature T_1 increases, in the general case, from T_{21} to T_{11}. As $Q_{2i} \sim 1/|\alpha_i|$, $d_i = d_{1i}\sqrt{|\alpha_1|}$ it follows from (52) that if it is necessary to get a high temperature in the domain "1" then the fulfillment of the inequality $|\alpha_1| >> \ell_1^2/4d_{1i}$ is more preferable.

3.2. Let find an approximate solution for the considered quadratic dependence $q_i(T_i)$ in the spherical coordinate system for the outer domain. To do this, we use the method developed in /17/. In ref. /17/ equations of the form

$$\ddot{q} + V(q, t) = 0 \qquad (53)$$

are considered and the conditions are found allowing us to get the functions $V(q, t)$ for which it is possible to get the first integral of the original equation (53).

The equation for the temperature $T_2(r)$, we are interested in, is

$$\theta'' + \frac{2}{y}\theta' + A_0 + A_1\theta + A_2\theta^2 = 0, \qquad (54)$$

where the following notation is introduced

$$\theta = \frac{T_2}{T_0}, \quad y = \frac{r}{R_2}, \quad q_2' = \frac{q_2 R_2^2}{\chi \;\; T}, \quad q_2' = A_0 + A_1\theta + A_2\theta^2 .$$

Using the standard transformation $\theta = u/y$ we get the following equation of the form (53):

$$u'' + A_0 y + A_1 u + \frac{A_2 u^2}{y} = 0. \qquad (55)$$

In particular, in /17/ the following equation has been considered

$$\ddot{q} + \omega^2(t)\, q + \alpha_0(t) + \alpha_2(t)\, q^2 = 0. \qquad (56)$$

(Hereafter we shall use the notation of the original paper).

It has also been shown that the first integral (56) can be obtained if we suppose

$$2\rho b = -\frac{1}{5}\alpha_2^{-2}\ddot{\alpha}_2 + \frac{6}{25}\alpha_2^{-3}\dot{\alpha}_2^2 + \frac{\omega^2}{\alpha_2},$$

$$\alpha_0 = (\rho b)^{\cdot\cdot} + \omega_0^2\rho b + c_0\alpha_2^{3/5} - \alpha_2(\rho b)^2 ,$$

$$\alpha_2 = \rho^5 ,$$

where c_0 is a constant.

In this case the first integral is

$$\frac{1}{2}[\rho\frac{d}{dt}(q + \rho b) - (q + \rho b)\,\rho]^2 + \frac{c_0}{\rho}(q + \rho b) + \frac{1}{3}\rho^{-3}(q + \rho b)^3 = \mathrm{const}.$$

We apply this method to eq.(55) and find out that disregarding the value $0{,}0816/A_2 y^3$ (and this can be done treating the outer sphere containing no point r = 0) and having conditions of the coefficients $A_0 = A_1^2/2A_2$, we have

$$2\rho b = -\frac{0{,}04}{A_2 y} + \frac{A_1 y}{2A_2}, \quad \rho = \left(\frac{A_2}{y}\right)^{1/5} .$$

Correspondingly, the first integral has the form

$$\frac{1}{2}\left[\left(\frac{A_2}{y}\right)^{1/5}\frac{dv}{dy} + \frac{v}{5}\frac{A_2^{1/5}}{y^{6/5}}\right]^2 + \frac{y^{3/5}\, v^3}{3A_2^{3/5}} = c_2 , \qquad (57)$$

where

$$v = u - \frac{0{,}02}{A_2 y} + \frac{A_1 y}{4A_2}.$$

Let's further replace $y^{3/5}v^3 = w^3$ and instead of (57) we get

$$\left(\frac{dw}{dy}\right)^2 = y^{4/5}\left(c_2 - \frac{2}{3}\frac{w^3}{A_2}\right). \qquad (58)$$

From (58) we find

$$\frac{5}{7}y^{7/5} = p\int\frac{dw}{\sqrt{c_2 - 2w/3A_2}}. \qquad (59)$$

The radicand has one real and two complex roots. We rewrite (59) with the help of the Legendre elliptical integral of the first rank:

$$y^{7/5} = -\frac{p}{ac_2} F(\phi, k), \tag{60}$$

where

$$a = \frac{5}{7}\sqrt{\frac{2}{\sqrt{3}A}}, \quad k = 0{,}5, \quad tg^2\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{3c_2^2}(c_2^2 - y).$$

Hence, for W we get

$$W = c_2^2\left(1 - \sqrt{3}\,\frac{1 - cn(ac_2 y^{7/5})}{1 + cn(ac_2 y^{7/5})}\right). \tag{61}$$

Correspondingly, the temperature in the outer sphere is equal to

$$T_2 = T_0\left[\frac{c_2^2 R_2^{6/5}}{r^{6/5}}\left(1 - \sqrt{3}\,\frac{1 - cn\left(ac_2\frac{r^{7/5}}{R_2^{7/5}}\right)}{1 + cn\left(ac_2\frac{r^{7/5}}{R_2^{7/5}}\right)}\right) + \frac{0{,}02 R_2^2}{A r^2} - \frac{A_1}{4A_2}\right]. \tag{62}$$

The constant c_2 is determined from the transcendental equation

$$c_2^2\left(1 - \sqrt{3}\,\frac{1 - cn(ac_2)}{1 + cn(ac_2)}\right) = 1 - \frac{0{,}02}{A_2} + \frac{A_1}{4A_2}.$$

From (62) it follows that if we have the condition

$$cn\left(ac_2\left(\frac{R_1}{R_2}\right)^{7/5}\right) = -1,$$

then the temperature T_2 can tend to infinity. The radius R_1 is determined by the formula

$$R_1 = R_2\left(\frac{2K(1+2n)}{ac_2}\right)^{5/7}, \quad n = 0, 1, \ldots$$

if $2K(1+2n)/ac_2 < 1$. The quantity K in this case is equal to 2.7681. Equating the right-hand side of expression (62) (when $r = R_1$) to zero, it is possible also to find R_1 such that $T_s = 0$. The radii of the adiabatic surface can be found from the expression

$$\frac{6c_2^2 R_2^{6/5}}{5R_1^{11/5}}\left(1 - \sqrt{3}\,\frac{1 - cn(d)}{1 + cn(d)}\right) + \frac{14\sqrt{3}\,c_2^2 sn(d)\, dn(d)}{5R_1^{4/5}R_2^{1/5}(1 + cn(d))^2} + \frac{0{,}04 R_2^2}{A_2 R_1^3} = 0.$$

where

$$d = ac_2\left(\frac{R_1}{R_2}\right)^{7/5}, \quad G(T_s, R_1) = 0,$$

The solutions considered in the second and third sections can be generalized by treating the dependences $q_i(T_i)$, $\chi_i(T_i)$ of different types in the inner and outer domains. Moreover, as has been shown, for instance, in ref. /18/, the dielectrics with anomalously high dielectric penetrability can be obtained by adding some metal ingradients. In these cases, as follows from formulae (5)-(6), the value of the heat source will be great (in these mediums ϵ''_{oi} is not equal to zero due to added ingradients). Considerable overheatings appearing in these cases can be described on the basis of the heat conductivity equations with a constant heat source (in formulas (10) or (27) q_i = const) for any dependence $\chi_i(T_i)$ thanks to the replacement $d\phi_i = \chi_i dT_i$.

4. ARBITRARY DEPENDENCES $\chi_i(T_i)$, $q_i(T_i)$

From expression (6) it follows that the greater is the warming-up in the cases considered, the greater is the growth of ϵ'_{oi} and ϵ''_{oi} with the temperature. Moreover, it is known that in the case when $d\epsilon'_{oi}/dT_i > 0$ the effect of the ray self-focusing /1,2/ appears. Therefore, it is interesting to consider the segnetoelectrics specified by a rapid growth of $\epsilon'_{oi}(T_i)$ according to the Curie-Weiss law $\epsilon'_{oi}(T_i) = B_i/(T_{vi} - T_i)$ /19/, where T_{vi} is the Curie-Weiss temperature. In a more general form we can write $\epsilon_{oi} = B_i/(T_{vi} - T_i)^{n_i}$. Liquid-crystalline systems, namely chiral smectic liquid crystalls of the phase C [20] can also possess segnetoelectric properties. The quantity $\epsilon''_{oi}(T_i)$ influences essentially the change of a heat source with the temperature. The change of $\epsilon''_{oi}(T_i)$ usually proceeds according to the exponential law (though in some ranges of temperatures ϵ''_i may be considered to be constant or changing with powers). So for dielectrics and wide-range semiconductors the following dependence can be written: $\epsilon''_{oi} = \epsilon_{ooi} T^{m_i} e^{-E_i/T_i}$ [21, 22]. Heat conductivity coefficien can change according to the power law for gases and plasma [23, 24]. Two types of dependences can be considered for crystalls: a power dependence at high temperatures and a dependence $\chi'_i = T^{\sigma_i} e^{-U_i/T_i}$ at relatively low temperatures [25]. A linear combination of these dependences may also occur. The quantity α_i can also change with temperature, for example, according to the law $D_i/(T_{vi} - T_i)^{\lambda_i}$ (for segnetoelectric materials). So the integrand in expression (13) will be a linear combination of the following integrals:

$$\int \theta_i^{\nu_i} (1-\theta_i)^{s_i} e^{-\Delta_i/\theta_i} d\theta_i , \tag{63}$$

where

$$\theta_i = \frac{T_i}{T_{vi}}, \quad \Delta_i = \frac{E_i}{T_{vi}} .$$

Using only two or three terms of the expansion of the integrand in powers of θ_i and $\bar{\theta}_i = 1 - \theta_i$ we get after the integration of (13) the Legendre elliptical integrals of the first-third rank. As an example we give a solution thus obtained for the constants $s_i = -0.5$, $\nu_i = 1$. This solution coicides in structure with the solution (48) in which it is necessary to suppose

$$d_i = (\chi_{oi})^{-1/2} (2q_{oi} |P_i| (T_{1i} - T_{3i}))^{1/2}, \quad P_i = 1 - \frac{\Delta_i}{2} + \frac{3}{8}\Delta_i^2 - \frac{5}{16}\Delta_i^3$$

being greater than $P_i > 0$. The temperatures T_{1i}, T_{2i}, T_{3i} are the roots of the equation

$$(1 - \frac{\Delta_i}{2} + \frac{3}{8}\Delta_i^2 - \frac{5}{16}\Delta_i^3)\theta_i^3 - (1\frac{5}{6} + \frac{c_i^2 \chi_{oi} T_{vi}}{2q_{oi}})\theta_i^2 - (\Delta - \frac{\Delta_i^3}{3})\theta_i + \frac{\Delta_i^3}{2} = 0,$$

$$T_{ji} = \theta_{ji} T_{vi} , \quad j = 1, 2, 3. \tag{64}$$

Correspondingly, all the conclusions of section 3.1 can be used here. In (64) and in P_i it can be formally supposed that $\Delta_i < 0$ considering, for example, metallic impurities. We also give the solution taking place at $P_i < 0$

$$T_i = T_{3i} + (T_{2i} - T_{3i}) \,\mathrm{sn}^2 \frac{x}{d} , \tag{65}$$

where T_{ji} are the roots of the same equation (64) and the parameter $k_i^2 = (T_{2i} - T_{3i})/(T_{1i} - T_{3i})$ so that soliton solutions in contrast with the previous cases for $T_{2i} = T_{1i}$ appear in this one. As for the adiabatic surfaces, they are calculated by formula (51) as in the previous case.

5. SOME ASPECTS OF NONSTATIONARY HEAT TRANSMISSION

5.1. Fundamental solutions. In investigating the possibilities of getting high temperatures in the restricted volumes, fundamental solutions equating the diffusion term in the transmission equation [12] to zero are very important. Due to the fact that the moving of the boundary $R_1(t)$ is of great importance in the obtained solutions, let us consider at N = 1 and N = 2 the following nonstationary boundary problem:

$$C_i(T_i)\rho_i(T_i, r)\frac{\partial T_i}{\partial t} = \frac{\chi_{oi}}{r^N}\left(\frac{\partial}{\partial r}\left(r^N \chi'_i(T_i)\frac{\partial T_i}{\partial r}\right)\right) + q_i(T_i, r), \tag{66}$$

$$T_1(r, 0) = T_{oi}(r), \quad T_1(R_1, t) = T_2(R_1, t), \quad T_2(R_2, t) = T_{oo}(t), \tag{67}$$

$$-\chi_1 \frac{\partial T_1}{\partial r}\Big|_{R_1} = -\chi_2 \frac{\partial T_2}{\partial r}\Big|_{R_1} + G(T_s, R_1, t), \tag{68}$$

where ρ_i is the density and c_i is the thermal heat capacity.

For fundamental solutions the conditions

$$r^N \chi'_i(T_i)\frac{\partial T_i}{\partial r} = \kappa_i(t) \tag{69}$$

must be fulfilled. Integrating (69) we have

$$\phi_i = \kappa_i(t)\,\psi(r, N) + \kappa_{1i}\ , \tag{70}$$

where

$$\psi(r, N) = \begin{cases} \ln r, & N = 1, \\ -\dfrac{1}{r}, & N = 2\ . \end{cases}$$

Substituting (70) into (66) we find the dependence

$$t = \int V_i(\kappa_i)\,d\kappa_i\ , \tag{71}$$

where

$$V_i(\kappa_i) = \frac{c_i(\phi_i)\,\rho_i(\phi_i, r)\,\psi(r,N)}{\chi_i'(\phi_i)\,q_i(\phi_i, r)}\,. \tag{72}$$

It is supposed here that from eq.(72) one can get in an explicit form only the dependence on $\kappa_i(t)$,and the dependence on r disappears in the process of multiplication of the functions $\rho_i\psi q_i^{-1}$.

From the initial and boundary conditions (67)-(68) we get

$$\begin{aligned} &\phi_2(T_{oo}) = \kappa_2(t)\,\psi(R_2, N) + \kappa_{12}\ , \\ &\phi_i(T_{oi}) = \kappa_i(0)\,\psi(r, N) + \kappa_{1i}\,, \\ &\chi_{01}\kappa_1(t) = \chi_{02}\kappa_2(t) - G(t)\,R_1^N\,, \\ &T_1(\kappa_1(t)\,\psi(R_1, N) + \kappa_{11}) = T_2(\kappa_2(t)\,\psi(R_2, N) + \kappa_{22}). \end{aligned} \tag{73}$$

From the system of equations (73) one can get some conditions on the problem parameters at fixed R_1 or the conditions on the problem parameters and the law of boundary movement $R_1(t)$. So at fixed $\kappa_1(t) \sim \kappa_2(t)$ should hold.

5.2. Let q_i be equal to $q_{oi}\zeta_i T_i^{\beta_i} Q_i(r)$, and ρ_i be equal to $\rho_{oi}\rho_i T_i^{\delta_i}\rho_i(r)$, where ζ_i, ρ_i are constants having the dimensions $\mathrm{grad}^{-\beta_i}$ and $\mathrm{grad}^{-\delta_i}$, respectively. The dependence of

the heat sources, determined by one of the formulas (4)-(6), on the radius can be conditioned by the dependence of the refractive index $n_i(r)$. Let's write the fundamental solutions for each domain in this case /12/:

$$T_i = T_{oi}\,(\psi_1(r))^{1/(\sigma_i+1)}\left(1 + \frac{t}{t_{fi}}\right)^{1/(\delta_i - \beta_i + 1)}\ , \tag{74}$$

where

$$\psi_1(r) = \begin{cases} \dfrac{R_2}{r}, & N = 2, \\ \ln\left(\dfrac{R_2}{r}\right), & N = 1, \end{cases}$$

$$t_{fi} = \frac{c_i\rho_{oi}\rho_i\,T_{oi}^{\delta_i - \beta_i + 1}}{(\delta_i + 1 - \beta_i)\,q_{oi}\,\zeta_i}\,.$$

For $\beta_i > \sigma_i + 1$ the quantity $t_{fi} < 0$ and $|t_{fi}|$ has the meaning of the sharpening time. If the density $\rho_i(T_i)$ decreases with temperature not faster than $1/T_i$, then $\beta_i > 0$. As it follows from formula (6) the growth of the heat source $q_i(T_i)$ can be connected, for example, with the growth of the dielectric penetrability ϵ_i' with the temperature. In this case, there is a certain correlation between the self-focusing effect and the existence of the sharpening time. For (/4) to be correct it is necessary to fulfil the condition

$$\rho_i'(r)\,(\psi_1(r))^{(\delta_i + 1 - \beta)/(\sigma_i + 1)} = Q_i(r)\ .$$

From the conditions $T_1(R_1, t) = T_2(R_1, t)$ and (68) we get rather strict restrictions on the problem parameters in case of $R_1 =$ = const, that is $\delta_1 - \beta_1 = \delta_2 - \beta_2$, $\sigma_1 = \sigma_2$, $T_{01} = T_{02}$, $t_{f1} = t_{f2}$, $\chi_{01}\eta_1 = \chi_{02}\eta_2$, $G = 0$. If $R_1 = R_1(t)$, then we get the following expressions for determining this dependence and the dependence $G(t)$ as well

$$R_1 = R_2 r,\ N = 2 \quad \text{or} \quad R_1 = R_2\exp\left(-\frac{1}{r}\right),\ N = 1, \tag{75}$$

where

$$r = \left[\frac{T_{02}\left(1 + \dfrac{t}{t_{f2}}\right)^{1/(\delta_2 - \beta_2 + 1)}}{T_{01}\left(1 + \dfrac{t}{t_{f1}}\right)^{1/(\delta_1 - \beta_1 + 1)}} \right]^{\sigma_{12}}, \quad \sigma_{12} = \frac{(\sigma_1 + 1)(\sigma_2 + 2)}{\sigma_1 - \sigma_2}, \tag{76}$$

$$\frac{\chi_{01}\eta_1 T_{01}^{\sigma_1+1}}{\sigma_1 + 1}\left(1 + \frac{t}{t_{fi}}\right)^{\nu_1} = \frac{\chi_{02}\eta_2 T_{02}^{\sigma_2+1}}{\sigma_2 + 1}\left(1 + \frac{t}{t_{f2}}\right)^{\nu_2} + \frac{G}{|\psi_1'(R_1)|},$$

where

$$\nu_i = \frac{\sigma_i + 1}{\delta_i - \beta_i + 1}, \quad \psi_1'(R_1) = \frac{d\psi_1}{dr}\Big|_{R_1}.$$

From (75) it follows that at $\sigma_{12} > 0$ the inner domain contrasts at a point: $R_1 \to 0$ as $t \to |t_{f1}|$, and on the contrary, it expands as $R_1 \to \infty (N = 2)$ or $R_1 \to R_2$ $(N = 1)$ at $t \to |t_{f2}|$. For $\sigma_{12} < 0$ opposite effects take place. As sharpening times in this case can be different the inequality $|t_{f1}| < |t_{f2}|$ can be fulfilled. Here, in the inner domain an infinite temperature is reached, while in the outer domain the temperature is limited. The mentioned inequality can be obtained, for example, if $|\alpha_1| < |\alpha_2|$. The analysis carried out shows also that at $|t_{f1}| < |t_{f2}|$ and at $\sigma_{12} > 0$ the sharpening conditions LS are realized (however, at the point $R_1 = 0$ the removed heat stream tends to infinity as $T_1 \to \infty$, i.e. at the moment of time $t \to |t_{f1}|$), and at $\sigma_{12} < 0$ the sharpening conditions *HS are* realized ([10]).

The boundary problem (66)-(68) can be satisfied also by the fundamental solutions of the following form:

$$T_i = T_{oi}\left(1 + \frac{t\psi_1(r)}{t_{fi}}\right)^{1/(\sigma_i + 1)}, \quad \psi_1(r)\,\rho_i'(r) = Q_i(r). \tag{77}$$

The condition $\beta_i = \delta_i - \sigma_i$ must also hold, i.e. in this case the mediums for which $\sigma_i < 0$ are to be considered whereas the conditions with sharpening appear at $\sigma_i < -1$. On the surfaces the radii of which are determined from the equation $[\psi_1(r_{oi})]^{-1} = t/|t_{fi}|$ an infinite temperature is reached. At a fixed R_1 condition $t_{f1} = t_{f2}$ must take place. In the opposite case, supposing that $\psi_1(R_1) = z/t$, $G = z_1|\psi_1'(R_1)|t$, we find the constants z, z_1 from the boundary conditions which take the form:

$$y = \Gamma(1 + py)^{\sigma} - 1, \quad y = \frac{z}{t_{f1}}, \quad p = \frac{t_{f1}}{t_{f2}}, \quad \sigma = \frac{\sigma_1 + 1}{\sigma_2 + 1} > 0, \tag{78}$$

$$z_1 = \frac{\chi_{01}\eta_1 T_{01}^{\sigma_1+1}}{(\sigma_1 + 1)t_{f1}} - \frac{\chi_{02}\eta_2 T_{02}^{\sigma_2+1}}{(\sigma_2 + 1)t_{f2}}. \tag{79}$$

From the solutions (78) only negative y have a physical meaning because $\psi_1(R_1) > 0$. For $y < -1$ the radius R_1 decreases and for $y > -1$ it increases. The surface r_{01} remain inside R_1 in this case.

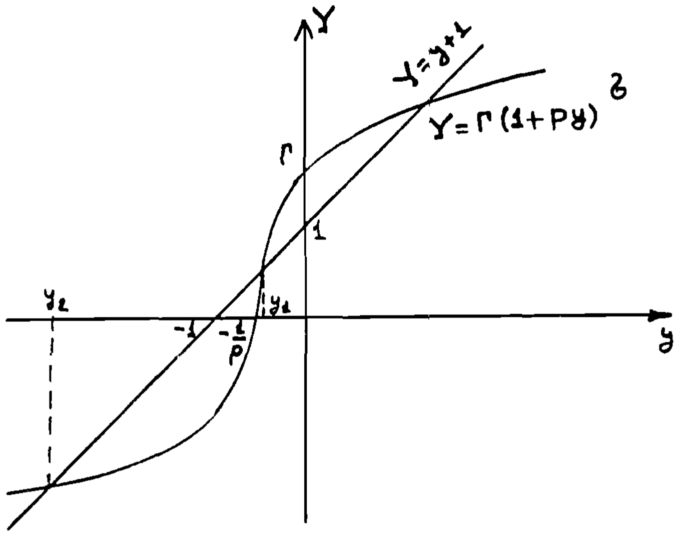

Fig.4. A diagram of a graphic determinative of the roots of equation (78), $\Gamma > 1$, $p > 1$, $\sigma < 1$.

5.3. It is interesting to consider homothermal solutions of (66) with the corresponding original conditions for arbitrary dependences on temperature. These solutions depend only on time and do not depend on hotothermal parameters of a system. Hence, these solutions allow us to consider different, not equal to each other times of sharpening $|t_{f1}|$ and $|t_{f2}|$. So, if $|\alpha_1| \ll |\alpha_2|$, then $|t_{f1}| \ll |t_{f2}|$ is fulfilled, thus leading to the appearance of high temperatures in the inner domain during much shorter time than in the outer one.

Let us first write eq.(66) in the dimensionless form supposing that $C_i(T_i) = C_{oi} C_i'(T_i)$, $\rho_i(T_i) = \rho_{oi} \rho_i'(T_i)$, $T_i = T_{oi} T_i'$, $q_i = q_{oi} q_i'(T_i)$, $r = r' R_1$, $t = t' t_0$ (t_0 is some time characteristic of the process),

$$C_i' \rho_i \frac{\partial T_i'}{\partial t'} = \frac{t_0}{t_{Di}} \frac{1}{(r')^N} \frac{\partial}{\partial r'}((r')^N \chi_i' \frac{\partial T_i'}{\partial r'}) + \frac{t_0}{t_{qi}} q_i', \qquad (80)$$

where a characteristic diffusion time $t_{Di} = C_{oi} \rho_{oi} R_1^2 / \chi_{oi}$ and a characteristic action of heat sources $t_{qi} = C_{oi} \rho_{oi} T_{oi} / q_{oi}$. If the condition $\max\{t_{q1}, t_{q2}\} \ll \min\{t_{D1}, t_{D2}\}$ is fulfilled, then it is possible to consider homothermal solutions in the inner and outer domains. Times of sharpening, if they do exist, are found by the formula

$$t_{fi} = \int_{T_{oi}}^{T_i^*} \frac{C_i(T_i) \rho_i(T_i) dT_i}{q_i(T_i)}, \qquad (81)$$

where T_i^* is equal to infinity or to some critical temperature for the physical process considered. Let us consider the following two examples of homothermal solutions supposing that the heat source q_i depends on the temperature through the parameters of formula (6).

5.3.1. Let us consider the substances with segnetoelectric properties supposing that $\epsilon_{oi}' = B_i/(T_{vi} - T_i)$. We neglect the dependences of other parameters on T_i. Then we get

$$T_i = T_{vi} - (T_{vi} - T_{oi})(1 - t/t_{fi})^{1/2}, \qquad (82)$$

where

$$t_{fi} = \frac{6\pi |\alpha_i| \rho_i C_i (T_{vi} - T_{oi})^2}{\epsilon_{oi}'' B_i \omega}.$$

From expression (82) it follows that as $t \to t_{fi}$ some critical temperature is reached, the Curie-Weiss temperature.

In this case $d\epsilon_{oi}'/dT_i > 0$, so that the medium is selffocusing. If we consider the temperatures higher than the Curie-Weiss temperature for which $d\epsilon_{oi}'/dT_i < 0$ (a defocusing medium), $\epsilon_{oi}' = B_i/(T_i - T_{vi})$, then no conditions with sharpenning occur. Indeed, the solution in this case is

$$T_i = T_{vi} + (T_{oi} - T_{vi})(\frac{t}{t_{fi}} + 1)^{1/2}. \qquad (83)$$

From (83) a monotonous increasing in temperature with time follows (because $t_{fi} < 0$), and $T_i \to \infty$ as $t \to \infty$. In particular, when the conditions $T_{01} = T_{02}$, $T_{v2} < T_{02} < T_{v1}$, $t_{f1} \ll \min\{t_{D1}, t_{D2}\}$ are fulfilled, it is possible to realize, within a homothermal model, the conditions with a sharpening in the inner domain of the heterogeneous system considered.

5.3.2. Let us find a homothermal solution of the problem supposing that the exponential dependence $\epsilon_{oi}'' = d_i'' e^{-E_i/T_i}$ is the most significant one in changing the heat source with temperature. Let us suppose also that $\rho_i = A_i/T_i$, $\epsilon_{oi} = d_i' T_i$. Then, we get the following homothermal solution:

$$t = \frac{b_i}{E_i}(e^{E_i/T_{oi}} - e^{E_i/T_i}), \qquad (84)$$

where

$$b_i = \frac{12\pi A_i C_i |\alpha_i|}{d_i'' d_i' \omega}.$$

From formula (84) it follows that as

$$t \to t_{fi} = \frac{b_i}{E_i}(e^{E_i/T_{oi}} - 1),$$

the temperature T_i increases without restrictions.

Summing up, we may say that the analysis carried out points out nontrivial effects accompanying heat transmission in nonlinear conjugate mediums, that is the appearance of adiabatic surfaces, the emergence of soliton solutions, the occurrence of the conditions with sharpening in self-focusing mediums. All the considered effects are essentially defined by: the values of α_i nonlinear parameters, the character of $\epsilon_i(T_i)$ dependences, the character of the boundary line movement.

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