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DYNAMIC IN DICKE MODEL

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The theory of superradiance is known to be based of the Dicke model whose Hamiltonian for a simplest point-like one-mode system in a perfect resonator is of the form /1-3/:

$$H = \omega\, a^{+}a + \omega_0 R_3 + g\, \{ R^{+} a^{m} + (a^{+})^{m} R^{-} \} , \qquad (1)$$

where

$$R_3 = \sum_{f=1}^{N} \sigma_f^{z} / 2 \qquad (2)$$

is the operator of level populations of a system of two-level atoms with the transition frequency ω_0 ; the operators

$$R^{+} = \sum_{f=1}^{N} (\sigma_f^{x} + i\, \sigma_f^{y}) / 2 , \qquad (2a)$$

$$R^{-} = \sum_{f=1}^{N} (\sigma_f^{x} - i\, \sigma_f^{y}) / 2 \qquad (2б)$$

describe , respectively , a transition to an upper (lower) level with absorption (emission) of m photons; for an exact resonance,

$$\omega_0 = m\,\omega \quad . \qquad (3)$$

a^{+} and a - are , respectively , the operators of creation and annihilation of photons with frequency ω ($\hbar = 1$); $\sigma_f^{(i)}$ $i = x,y,z$ are the Pauli operators; g - is the atom-field coupling constant inversely proportional to $V^{1/2}$, where V is the resonator volumne /1-3/.

That Hamiltonian describes interaction of two-level atoms in a resonator without loss . Using the commutation relations

$$[\, a , a^{+}] = 1, \qquad [\, a^{m}, \hat{n}\,] = m\, a^{m} \qquad (\hat{n} = a^{+}a\,) ,$$

$$[\, R_3 , R^{\pm}\,] = \pm R^{\pm}, \qquad [\, R^{+} , R^{-}\,] = 2\, R_3 \qquad (4)$$

and writing the Heisenberg equations

$$i\, \frac{d}{dt}\, R_3 = [\, R_3, H\,] = g\, \{ R^{+}a^{m} - (a^{+})^{m}R^{-} \} \qquad (5a)$$

$$i\, \frac{d}{dt}\, \hat{n} = [\, \hat{n} , H\,] = -\, g\, m\, \{ R^{+} a^{m} - (a^{+})^{m} R^{-} \} \qquad (5б)$$

we may be verify that the following constant of motion

$$\hat{n} + m\, R_3 = \mathrm{const}\ (\equiv \hat{M}_1). \qquad (6)$$

does exist if conditon (3) is satisfied (here $\hat{M}_1$ is a diagonal time-independent operator).

Usually , the problem of superradiance is formulated as follows : let a system at the initial moment of time t_0 be in a state $|0,+\rangle = N/2\ |0\rangle \otimes |+\rangle$, in which

i) the field is in a vacuum state $|0\rangle$;

ii) all atoms are on the upper level (the system is completely inverted , i.e. $R_3|+\rangle = N/2\ |+\rangle$).

It is necessary to determine the time evolution of the photon number $\hat{n}(t)$ and the level populations $R_3(t)$ related by (6) .

This problem can be solved within various approaches based on either kinetic equations or the Green function method . Consistent derivation of a kinetic equation for the problem with Hamiltonian (1) can be found in ref.4 for m=1 . It consists in eliminating the phonon variable from the Heisenberg equation

$$i \frac{d}{dt} \langle O \rangle_t = \langle [O, H] \rangle_t, \qquad (7)$$

where O is an arbitrary operator of the atomic subsystem and $\langle ... \rangle = Sp ... \rho_0$, with the use a special trick proposed in ref.5 for the polaron problem .

That trick is based on the assumption of possible seperation of variables into fast and slow variables and of adiabatic switching-on of the interaction between atomic and bosonic subsystems . As a result , we arrive at an equation of the form

$$i\, d/dt \langle O\rangle_t - i \langle [\omega_0 R_3, O]\rangle_t = g^2 \int_{t_0}^{t} d\tau \{ e^{-i\omega(t-\tau)} \langle [R^+(t), O(t)]\, R^-(\tau)\rangle + e^{i\omega(t-\tau)} \langle R^+(\tau)\, [O(t), R^-(t)]\rangle \} . \qquad (8)$$

This equation is examined , as a rule , within the Markovian approximation , when $R^{\pm}(\tau) = R^{\pm}(t)\exp\{\pm\omega_0(t-\tau)\}$, with a subsequent decupling some higher correlators , for instance

$$\langle R^+R^-R_3\rangle = \langle R_3R^+R^-\rangle \simeq \langle R^+R^-\rangle\langle R_3\rangle . \qquad (9)$$

In this case, hierarchy (9) reduces to the Rehler-Eberly equation /6-7/ :

$$\frac{d}{dt} \langle R_3\rangle_t = 2\Gamma\langle R_3\rangle_t^2 - 4\Gamma\langle R_3\rangle_t - 2\Gamma Q \qquad (10)$$

where the coefficient Γ depends on g , ω , ω_0 , and

$$\langle R^+R^-\rangle_t + \langle R_3\rangle_t^2 - 2\langle R_3\rangle_t = Q = const . \qquad (11)$$

The solution to eq. (10) is given by

$$\langle R_3\rangle_t = 1 + \sqrt{Q+1}\, th\{2\Gamma(t_D - t)\sqrt{Q+1}\}, \qquad (12)$$

$$t_D = \frac{Arth\{(\langle R_3\rangle_0 - 1)/\sqrt{Q+1}\}}{2\Gamma\sqrt{Q+1}} . \qquad (13)$$

For the intensity

$$I(t) = \frac{d}{dt}\, \omega \langle \hat{n}\rangle_t, \qquad (14)$$

that is an experimentally measerable characteristic of superradiance , we obtain , from (12) with (6) , the expresion

$$I(t) = 2\Gamma\omega\, (Q+1)\, sech^2\{2\Gamma(t_D - t)\sqrt{Q+1}\} , \qquad (15a)$$

which describes the dependence of a typical superradiant process on the delay time t_D determining the intensity maximum

$$I(t_D) = 1/2\ \Gamma\omega\, (N+2)^2, \quad (\langle R_3\rangle_0 = N/2, \quad \langle R^+R^-\rangle_0 = 2N) . \qquad (15б)$$

The dynamic behaviour described by equation (10) and relations (12) , (14) is in good agreement with experimental data [1-3,7,8].

At the same time it is to be noted the following : First , with the use of (4) one may prove the existence of the exact constant of motion

$$R^+R^- + R_3^2 - R_3 = \hat{M}_2, \qquad (16)$$

where M_2 is a time-independent diagonal operator . In terms of averages (16) does not coincide with (11), i.e. Q is not an exact constant of motion for a system with Hamiltonian (1). Consequently, decoupling in the hierarchic equation produces violation of the initial conservation laws.

Second, approximate equation (10) has a stationary solution

$$\langle R_3 \rangle_{t \gg t_D} = 1 - \sqrt{Q+1}\,, \qquad Q = N^2/4 + 1\,, \tag{17}$$

corresponding to the complete de-excitation of the system, and owing to (6)

$$\langle \hat{n} \rangle_{t \gg t_D} = N/2, \tag{18}$$

i.e. the resonance mode of the field is macroscopically occupied and that state is steady. On the other hand, Hamiltonian (1) describes both the processes of emission and absorption of resonance photons with transition of atoms to the upper state. That a stage of a dynamic process of that sort is absent in the solution to eq. (10) is probably due to the hypothesis of adiabatic switching-on the interaction and the Markovian approximation. Indeed, the presence of constant of motion (6) signifies that processes in an atomic system proceed with the same rate as in a photon system, but the Markovian approximation may result in irreversibility [9]. From a physical point of view, the irreversibility is a result of performing experiments by a scheme without resonator, which just account for a good agreement of the theoretical result (14) with the observed shape of the momentum[2,3,8]. In case of a perfect resonator or a resonator with a high quality, the process of emission and absorption should be periodic, and on the basis of (1) the corresponding solution is to be derived. Note in this connection that periodic generation of superradiance pulses is observed even in resonators with low quality [10,11]. Note also that in recent years interest in the problem of superradiance in a resonator has significantly increased [12]. Third, hierarchy is easily derived in the case of photon multiplicity $m = 1$. When $m = 2$, the analysis gets much complicated and derivation of equations requires a lot of extra assumptions[13,14]. It is to be emphasized that the problem of superradiance even in two-photon processes is not yet solved completely.

In view of this, it is of certain interest to find the solution of the problem of superradiance that i) would not directly require various approximations in describing the dynamic process which arise in deriving kinetic equations; ii) would be valid for studying the case with an arbitrary photon multiplicity m.

With this purpose in mind, let us consider eq.(5a) and pass to the second order derivative

$$\frac{d^2}{dt^2} R_3 = -\left[\, i\frac{d}{dt} R_3,\ H \,\right]. \tag{19}$$

We shall examine the general case, without resonance. To represent (19) as an equation of the type

$$\frac{d^2}{dt^2} R_3 = f(R_3) \tag{20}$$

where $f(R_3)$ is only a function of the variable R_3 and constants of motion, we should consider the term describing the interaction of photon field with an atomic system. From (1) and (6) we get

$$H_I = g\,[(a^+)^m R^- + R^+ a^m] = (H - \omega \hat{M}_1 - \Delta\omega\, R_3), \tag{21}$$

with $\Delta\omega = \omega_0 - m\omega$.

Taking account of (21) and the commutation relation (4)

$$[\, a^m, (a^+)^m] = \prod_{k=0}^{m-1} (\hat{n} + m - k) - \prod_{k=1}^{m} (\hat{n} - m + k) \tag{22}$$

as well as constants of motion (6) and (16), we find from (19):

$$
\frac{d^2}{dt^2} R_3 = \Delta\omega\,(\hat{H}-\omega\,\hat{M}_1) - (\Delta\omega)^2 R_3 + 2g^2\Big\{(\hat{M}_2-R_3-R_3^2)\prod_{k=1}^{m}(\hat{M}_1-mR_3+k) - (\hat{M}_2+R_3-R_3^2)\prod_{k=0}^{m-1}(\hat{M}_1-mR_3-m-k)\Big\} = P_{m-1}(R_3) \qquad (23a)
$$

or in another form

$$
\frac{d^2}{dt^2} R_3 = g\,\Big\{\Delta\omega\,(\hat{H}-\omega\,\hat{M}_1) - (\Delta\omega)^2 R_3 - \sum_{k=0}^{m-1}\Big\{[(\hat{M}_2-2R_3^2)(1+(-1)^{m+k}) + 2R_3(1-(-1)^{m+k})](\hat{M}_1-mR_3)^{k+1} + m(\hat{M}_2-2R_3^2+2R_3)(M-mR\ )\Big\}\times
$$

$$
\times\sum_{\{l_0\cdot\ldots\cdot l_{m-k-1}\}} l_0\cdot l_1\cdot\ldots\cdot l_{m-k-1}\ . \qquad (23б)
$$

Here $l_0=1$, $l_{i>0}=i,i+1,\ldots,i+k$, $\bigwedge_{i<j} l_i < l_j$, $P_{m+1}(R_3)$ is a polynomial of degree $m+1$ in the variable R_3.

Thus , instead of the hierarchic integro-differential equation (8) , the dynamic in the Dicke model (1) can be describe by the exact constant of motion (6) without any assumptions on the initial state of the system and the way of switching-on the interaction between atoms and the field .

Consider the commutator

$$
[R_3,\ i\tfrac{d}{dt}R_3] = g(\ R^+a^m + (a^+)\ R^m\ ) = H - \hat{M}_1(\equiv \hat{M}_3), \qquad (24)
$$

where $\ddot{M}_3$ is a time-independent operator proportional to the atom-field coupling constant g . This parameter is small compared to another typical parameter of the Hamiltonian ω /1,2/ ,its value decreasing with growing the photon multiplicity of the process . If the right-hand side of (24) is neglected ($\hat{M}_3 \simeq 0$), then multiplying the rhs and lhs of (23) by $^d/_{dt}R_3$ we may pass to a first-order equation with separable variables of the form

$$
\frac{d}{dt} R_3 = 2\sqrt{\int P_{m+1}(R_3)\ dR_3 + C\ }\ , \qquad (25)
$$

where C is a new time-independent operator . In the general case the solution of an equation like (25) will describe a periodic process,consequently periodic will also be the number of photons n. Assuming g to be small , solutions to the exact equation (28) may be found by the perturbation theory in the atom-field coupling constant , the solution to eq. (25) being a zeroth approximation . At the same time , a qualitative character of the dynamic behaviour of the system can be determined if in (23) we perform averaging over the initial state of the system $\rho_0 = |0;\alpha\rangle\langle 0;\alpha|$.

For simplicity we take advantage of a mean-field approximation when $\langle R_3^k\rangle_t \simeq \langle R_3\rangle_t^k$. Then , instead of (23) we obtain

$$
\ddot{X} = P_{m+1}(X)\ , \qquad X \equiv \langle R_3\rangle_t, \qquad (26)
$$

and the corresponding first-order equation is

$$
\dot{X} = 2\sqrt{\int P_{m+1}(X)\ dX + C}\ . \qquad (27)
$$

Consider particular cases :

1. One-photon process (m = 1) .

Equation (26) for m = 1 assumes the form

$$
\ddot{X} = g^2\{6X^2 - -(2M_1 + æ + 1)X - M_2 + 2æ(X_0 + \lambda)\}\ , \qquad (28)
$$

where $M_1 \equiv \langle\hat{M}_1\rangle$, $æ = 1/2(\Delta\omega/g)^2$, X_0 is a number equal to the level populations of upper and lower states at $t = 0$, λ is a coefficient for the interaction of a field with an atomic system :

$$
\lambda = \begin{cases} 0 & \text{if}\quad (n_0=0 \vee X_0=0) \\ \neq 0\ , & \text{otherwise}\ . \end{cases}
$$

The corresponding first-order equation is

$$
(\dot{X})^2 = g^2\,[4X^3 - 2(2M_1 + æ + 1)X^2 - 2(M_2 - 2æ(X+\lambda))X + C\ ] \qquad (29)
$$

where

$$C = g^{-2}(\dot{x})^2_{t=0} - 4x_0^3 + 2(2M_1 + \varkappa + 1)x_0^2 + 2(M_2 - 2\varkappa(x_0 + \lambda))x_0, \qquad (30)$$

x_0 and $(\dot{x})_{t=0}$ are initial values . Equation (29) is integrated in quadratures ; for this aim it should be represented in the normal Weierstrasse form :

$$\eta^2 = 4\xi^3 - g_2\xi - g_3, \qquad (31)$$

where g_2 , g_3 are invariants . Solution to (31) has the general form

$$\xi(t) = G(t + C) , \qquad (32)$$

where G is the elliptic Weierstrasse function , and constant C is defined by the initial conditions . Let ω_1 and ω_2 be half-periods of the elliptic Weierstrasse function G (ω_1 is real and ω_2 is purely imaginary) then $\omega_3 = \omega_1 + \omega_2$ is also half-periodic Weierstrasse function .Next , let e_i, i=1,2,3 is an irreducible set of zeros for the derivative of the Weierstrasse function G ', than $G(\omega_i) = e_i$, i = 1,2,3 . When the discriminant define as $\Delta = g_2^3 - 27 g_3^2$ is real and $\Delta > 0$ all zeros e_i are real , and

$$G(t+\omega_i) = e_i + \frac{(e_i - e_j)(e_i - e_k)}{G(t) - e_i} . \qquad (33)$$

In the special case that $\Delta > 0$ and $e_1 > e_2 > e_3$, the elliptic Weierstrasse function may be written in terms of the Jacobi elliptic functions , for instance ,

$$G(t) = e_3 + \frac{e_1 - e_3}{sn^2(t\sqrt{e_1 - e_3}, k)} . \qquad (34)$$

From comparison of (33) and (34) we get

$$G(t+\omega_i) = e_i + \frac{(e_i - e_j)(e_i - e_k)\ sn^2(t\sqrt{e_1 - e_3}, k)}{(e_1 - e_3) - (e_i - e_3)\ sn^2(t\sqrt{e_1 - e_3}, k)} . \qquad (35)$$

Let initial conditions be such that for t = 0 the function of level populations assumes one of the values x_1, x_2, x_3 that are zeros of equation (29) :

$$\begin{cases} x_1 = x_0 + n_0 + \frac{\varkappa}{2} + \frac{N - 2\varkappa(x_0 + \lambda)}{4x + 2n + \varkappa} \\ x_2 = x_0 \\ x_3 = -x_0 - \frac{N - 2\varkappa(x_0 + \lambda)}{4x_0 + 2n_0 + \varkappa} \end{cases} \qquad (36)$$

and $x_1 > x_2 > x_3$. The the solution to eq. (29) takes on the form

$$x(t) = x_i + \frac{(x_i - x_j)(x_i - x_k)sn^2(gt\sqrt{x_1 - x_3}, k')}{(x_1 - x_3) - (x_i - x_3)sn^2(gt\sqrt{x_1 - x_3}, k')} , \qquad (37)$$

where $k'^2 = (x_2 - x_3)/(x_1 - x_3)$.

Consider the following important cases :

1) $\omega_i = \omega_1$

$$x(t) = x_1 + (x_1 - x_2)sc^2(gt\sqrt{x_1 - x_3}, k') , \qquad (38)$$

2) $\omega_i = \omega_2$

$$x(t) = x_2 - \frac{(x_2 - x_3)sn^2(gt\sqrt{x_1 - x_3}, k')}{1 - \lambda' cn^2(gt\sqrt{x_1 - x_3}, k')} , \qquad (39)$$

где $\lambda' = (x_2 - x_3)/(x_1 - x_2)$.

3) $\omega_i = \omega_3$

$$x(t) = x_3 + (x_2 - x_3)sn^2(gt\sqrt{x_1 - x_3}, k') . \qquad (40)$$

That solution of eq. (29) coincides with the results by other authors/15/ . It describe a double-periodic process (see Fig.1) . As it is easy to see , the duration of stay of the atomic system in an upper (excited) state is larger than in a lower state . This is due to the transition from processes of spontaneous radiation to the collective spontaneous radiation , and the characteristic parameter t_0 , the so called collectivization time , decreases with increasing number of inverted atoms , which is in agreement

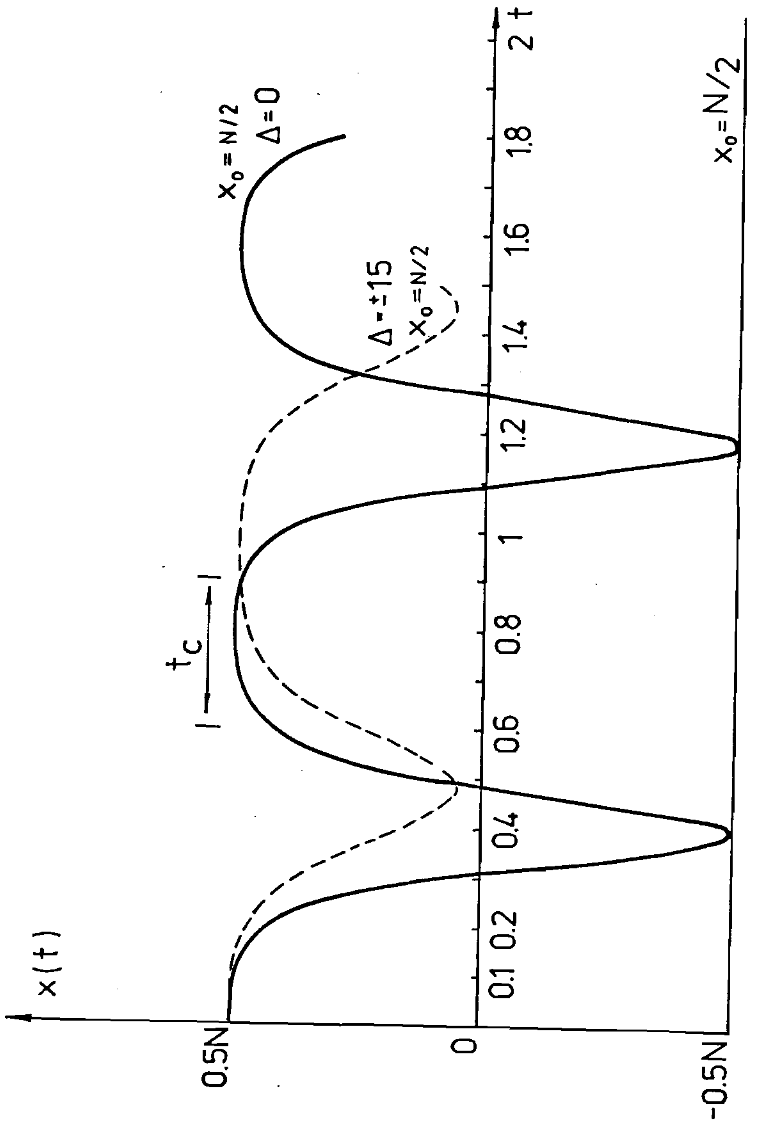

Fig. 1

with the results of semiphenomenological theories [1] (Fig. 1) .

The behaviour of the corresponding intensity

$$I(t) = -\omega \dot{x} \qquad (41)$$

defined by relations (29) and (30) is plotted in Fig. (2). Periods of emission are seen to alternate with periods of absorption (a dashed line) .

It is of interest to compare the results of two different approaches leading to formulae (12), (14) , and (29) respectively. First of all note that x(t) in (12) depends only on one constant of motion , (6) , arising for a decoupled system of equations , whereas (29) includes dependencies on both the "true" constants of motion (6) and (16) corresponding to the initial model problem (1) .

Besides , (29) describes a double-periodic process of the energy transfer between photon and atom subsystems in a perfect resonator , which , as indicated above , seems natural from a physical point of view .

A superradiance pulse defined by (15) is symmetric in the delay time t_D (Fig. 3) , whereas each of the pulses of emission corresponding to (29) has a typical asymmetry (Fig.2) . It is to be noted that the experimentally observed pulses are , as a rule , asymmetric , and to describe this asymmetry , additional physical mechanisms are employed in standard theories [13,14] .

In the solution we have found to (29) the time of delay that determines the position of the intensity maximum decreases with increasing number of atoms or initial level populations (Fig.4).

2) Two-photon process (m = 2)

For m = 2 equation (26) assumes the form

$$\ddot{x} = 2g^2\{16x^3-6(2M_1+1)x^2+2(M_1^2+M_1-2M_2+1+\tfrac{\delta}{2})x+(2M_1+1)M_2-\delta(x_0+æ) \qquad (42)$$

The corresponding first-order equation is

$$\dot{x} = 4g^2\{4x^4-2(2M_1+1)x^3+(M_1^2+M_1-2M_2+1+\tfrac{\delta}{2})x^2+[(2M_1+1)M_2-\delta(x_0+æ)]x+C$$

For the case that $\dot{x}$; 0 , at t=0 we obtain from (42)

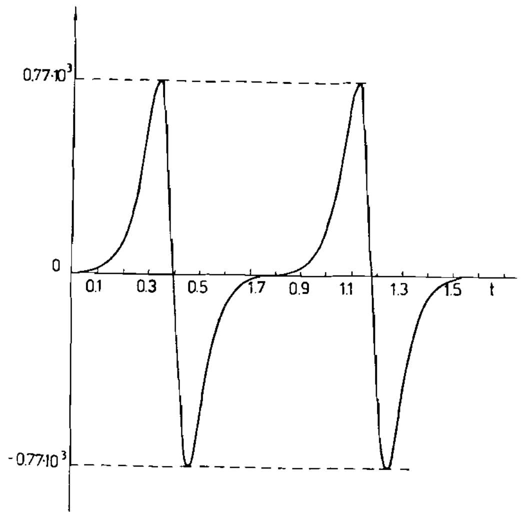

Fig. 2

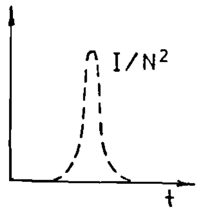

Fig. 3

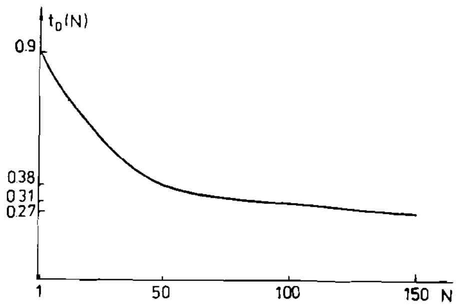

Fig. 4

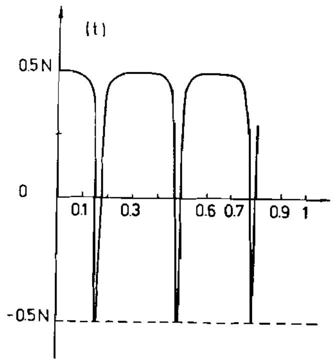

Fig. 5

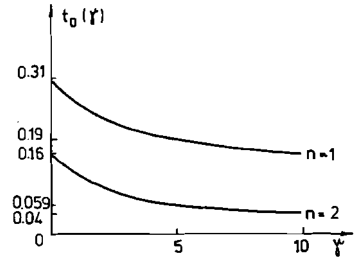

Fig. 6

$$C=-4g^2\{4x^4-2(2M_1+1)x^3+(M_1^2+M_1-2M_2+1+\frac{\delta}{2})x^2+[(2M_1+1)M_2-\delta(x_0+æ)]x+C$$

Equation (43) is integrated in quadratures , and its solution may be represented by the Weierstrasse function . However , this solution is much more complicated than the solution to eq. (29) , and therefore we restrict ourselves only to a numerical solution (Fig. 5 and 6) . It is then seen immediately that the higher the magnitude of photon multiplicity , the longer the time of stay an atom in an excited state and the lower the time of delay t_D .

Thus , the presented approach allows the description of periodic processes of emission of pulses of the electromagnetic field in a perfect resonator alternating with periods of absorption of the field (origin of an inverted state) for an arbitrary photon multiplicity of the transition . Superradiance pulses possess a characteristic asymmetry with respect to the delay time (position of the intensity maximum) .

We emphasize that the experimentally observed asymmetry of pulses is usually attributed different additional physical assumptions[3] .

As can seen from the time dependence of level populations , a system of two-level atoms is in an upper state for a longer time than in a lower state , because the process of de-excitation starts as a rather slow spontaneous decay of individual excited states and only gradually, through self-induction of correlations, the process becomes cooperative . So , the time of stay in the upper state , t_c , can be identified with the time of collectivization[3] .

The frequency tuning out from a resonance leads , on the one hand , to an increase in the time of collectivization and , on the other hand , to a decrease in the intensity of radiation caused only by a partial reverser of the initial inverted state in the process of generation (Fig. 1) .

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Динамика в модели Дикке

Исходя из модели Дикке получено точное дифференциальное операторное уравнение, зависящее только от интегралов движения и оператора полуразности населенности. После усреднения по начальному состоянию системы и использования приближения типа среднего поля переходим от операторного уравнения к уравнению для с-чисел. Рассмотрены частные случаи одно- и двухфотонного процесса, для которых уравнение интегрируется в квадратурах и решение получается в виде эллиптических функций Вейерштрасса. Проведенное рассмотрение позволяет описать периодический процесс испусканий импульсов электромагнитного поля в идеальном резонаторе, чередующихся с периодами поглощения поля для произвольной фотонной мультипликативности перехода. Сверхизлучательные импульсы имеют характерную асимметрию относительно времени задержки. Показано, что система двухуровневых атомов пребывает в верхнем состоянии дольше, чем в нижнем. Включение частотной отстройки от резонанса ведет к увеличению интенсивности излучения. Полученные результаты согласуются с экспериментом.

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Dynamic in Dicke Model

We derive an exact differential operator equation for the operator level populations7 Upon averaging over the initial state of the system and applying a mean-field approximation, we reduce the operator equation to an equation for c-numbers. Particular cases of one- and two-photon processes are considered for which the equations are integrated in quadratures and the solutions are the Weierstrasse elliptic functions. The approach allows us to describe a periodic emission of an electromagnetic field in the perfect resonator that alternates with absorption of the field for arbitrary multiplication of the transition. Superradiative intensity have a specific asymmetry with respect to the time delay. It is shown that the system of two-level atoms is in the upper state for a longer time than in the lower state. Switching-on of detuning increases the time of collectivization. The results are in agreement with experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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