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# SOFT MODE IN THE CLASS OF EXACTLY SOLUBLE MODELS OF PHASE TRANSITION

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## 1. Introduction

The interest in the theoretical aspects of the structural phase transitions does not decrease although our knowledge in this field has grown recently [1]. Being a significant branch of the statistical physics, the exactly soluble models (see e.g. Ref.[2]) has become an important tool in studies of the properties of phase transitions. Some time ago, Schneider et al.[3] proposed an exactly soluble model of structural phase transition with an anharmonic interaction of infinite range:

$$H = \sum_{l} \frac{P_l^2}{2M} + \frac{1}{4}\sum_{ll_1} \Phi_{ll_1}\left(Q_l - Q_{l_1}\right)^2 - \frac{A}{2}\sum_{l} Q_l^2 + \frac{B}{4N}\left(\sum_{l} Q_l^2\right)^2. \qquad (1.1)$$

The exact solution for all temperatures and the detailed discussion of the thermodynamical properties of the model (1.1) both in the classical and quantum limits, have been given by Plakida and Tonchev [4,5] within the approximating Hamiltonian method. Some modifications of this model have also been discussed by various authors [6-9].

Sarbach and Schneider [10] showed that the classical partition function of the whole class of models

$$H = \sum_{l} \frac{P_l^2}{2M} + \frac{1}{4}\sum_{ll_1} \varphi_{ll_1}\left(Q_l - Q_{l_1}\right)^2 + N\, V\left(\frac{1}{N}\sum_{l} Q_l^2\right), \qquad (1.2)$$

where N denotes the number of sites in d-dimensional lattice ( $\lambda = l_1, \dots l_d$ )) and V(x) is the well-defined potential (see below):

$$V(x) \geq V_0 \;, \quad V(x) \xrightarrow[x \to \infty]{} \infty \;, \quad V'(x) > 0 \quad (x > x_0), \tag{1.2a}$$

may be found in the thermodynamical limit $N \to \infty$ . Very recently, van Hemmen and Zagrebnov [11] also discussed some properties of model (1.2). It was shown [3-11] that the phase transition in the model (1.1) and also in the more general case, Eq.(1.2), is associated with the presence of the so-called soft-mode which is of the acoustic type in the ordered phase ($T < T_c$). It resembles a Goldstone-mode behaviour, which in order, is associated with the continuous symmetry breaking. In the previous paper [12], it has been shown that there is no any continuous symmetry in the model (1.1) and the phase transition corresponds to the broken discrete symmetry. It was also found that at T = 0, the branch of excitations reveals a singular behaviour at the centre of the Brillouine zone

$$\omega_q^2 = \frac{1}{M}\left(\varphi_q - \varphi_q\right) \;, \quad q \neq 0 \;, \tag{1.3a}$$

$$\omega_0^2 = \frac{2A}{M} \;, \tag{1.3b}$$

instead of an expected acoustic-type spectrum , $\omega_0^2 = 0$ .

There are two following problems associated with the properties of model (1.2). Firstly, why, the singular behaviour of the soft mode, $\omega_0^2(T)$, /at least at $T = 0$ / has not been found in the earlier treatments. /It is easy to show that the result (1.3) is proper for the model (1.2) - see Sec.3 /. The second problem is more interesting. It is generally believed that in any reasonable model of phase transition, the critical indices are classical for $d > d_c$ /$d_c$ - critical dimension/. Unambigiously the value of critical dimension for model (1.2) is $d_c = 4$, but according to previous results [3-11] the critical index $\gamma'$ $(T < T_c)$ is not defined for any $d > 2$, instead of to be equal to one, ( $\gamma = \gamma' = 1$ ) for $d > 4$.

Therefore, we have decided to study the behaviour of the susceptibility function $\chi$ ($\sim \Omega_0^{-2}$ - see below ) for the whole class of exactly soluble models (1.2). The aim of this paper is to give definite answer on the question what is the proper behaviour of the soft mode as a function of temperature. All our considerations are given in the classical limit for $d \geq 3$ ( $d = 3,4,5 \ldots$ ), where the discussed model seems to belong to the class of universality of spherical model.

The paper is organized as follows:

In Sec. II the meaning of the gapless mode within the models (1.2) will be explained; the behaviour of the susceptibility function at $T = 0$ and $T > 0$ is studied in Sec. III and Sec.IV, respectively; the connection with the spherical model is discussed in Sec.V and conclusions are given in the last Sec.VI.

## II. The soft-mode in the exactly soluble models of phase transition

In this Section we show how the free energy for the models (1.2) may be found in the limit of large N. In fact, the exact solution of the model (1.2) was given earlier [10-11] but as we want the paper to be self-contained, we shall do this once more but in another, a simpler way. It is found that a common feature for finite systems is the excitation whose frequency decreases with temperature and becomes of an order of $N^{-1}$ below some $T_N$. This phenomenon is interpreted as a signal of phase transition with a gapless mode in the ordered phase in the infinite system. /In Sec.IV it will be shown to what extent this interpretation is right/.

The partition function $\mathcal{Z}$ for the model (1.2) factorizes /within a classical approximation/

$$\mathcal{Z} = \mathcal{Z}_P \mathcal{Z}_Q \tag{2.1}$$

where the configurational part $\mathcal{Z}_Q$ may be found as follows:

$$\mathcal{Z}_Q = \int \prod_l dQ_l \exp\left[-\beta \mathcal{U}(\{Q_l\})\right] =$$

$$= \int_0^\infty dr\, e^{-\beta N V(r)} \int_{\frac{1}{N}\sum_l Q_l^2 = r} \prod_l dQ_l\, e^{-\beta \frac{1}{4}\sum_{ll_1} \varphi_{ll_1}(Q_l - Q_{l_1})^2} \tag{2.2}$$

$$= \int_0^\infty dr\, e^{-\beta N V(r)} \int \prod_l dQ_l\, \delta\left(Nr - \sum_l Q_l^2\right) e^{-\frac{1}{4}\beta \sum_{ll_1} \varphi_{ll_1}(Q_l - Q_{l_1})^2}$$

A usual representation for a $\delta$ - function

$$\delta(x) = \frac{1}{2\pi}\int dk\, e^{ikx}$$

allows one to find $Z_Q$ by using /twice/ a saddle point-type method /see also Ref.[10]/.

As it is a rather widely used, standard method / see, e.g. Ref.[2]/, let us only give a final formula for the free energy function

$$F_N = -\frac{kT}{N}\ln Z = \frac{kT}{N}\sum_q \ln\left(\frac{\hbar\omega_q}{kT}\right) + \left[V(r^*) - r^* V'(r^*)\right], \tag{2.3}$$

where the first term in formula (2.3) is a free energy of a system of harmonic oscillators with effective frequencies

$$\omega_q^2 = \frac{1}{M}\left[2V'(r^*) + \varphi_0 - \varphi_q\right] \tag{2.4}$$

and $r^*$ is defined by a self-consistency equation following from the condition on the saddle point

$$r^* = \frac{kT}{N}\sum_q\left[2V'(r^*) + \varphi_0 - \varphi_q\right]^{-1} \tag{2.5}$$

By definition

$$V(r) \xrightarrow[r\to\infty]{} \infty \,, \tag{2.6a}$$

$$V'(r) > 0 \,, \quad (r > r_0 \geq 0\,) \,. \tag{2.6b}$$

The necessary condition on V(r), for the phase transition in the model (1,2) is

$$V'(r_0) = 0 \quad , \quad r_0 > 0 \tag{2.6c}$$

/we also want V(r) to be at least two times differentiable function of r - see below/.

In fact, we find from (2.5) that when conditions (2.6) are fulfilled, then for $d \geq 3$ and for large N, $V(r^*)$ behaves as follows:

$$V'(r^*) \longrightarrow V'(r^*_N) \sim \frac{1}{N} \;\; \text{as} \;\; T \to T_N \, (\sim T_c) \tag{2.7a}$$

and

$$V'(r^*) \sim \frac{1}{N} \qquad \text{for} \qquad T < T_N, \left(r^* = r_0 + O\left(\frac{1}{N}\right)\right). \tag{2.7b}$$

We shall shortly comment on the above result but prior to that let us make a widely-used approximation which simplifies our further calculations

$$\varphi_0 - \varphi_q := s q^2 \, , \tag{2.8a}$$

$$\int d^{(d)} q := S^{(d)} \int_0^{q_B} q^{d-1} dq \, , \quad \frac{(2\pi)^d}{v \cdot d} S^{(d)} q_B^d := 1 \, . \tag{2.8b}$$

This approximation of an isotropic spectrum and a spherical Brillouine zone does not influence the qualitative character of our results, i.e. it does not influence the character of temperature dependence of susceptibility function or the value of the critical indices changing the values of the coefficients and the constants, e.g., $T_c$. The behaviour of $\omega_0^2\left(\sim 2V'(\tau^*)\right)$, Eq.(2.4), given by the formula (2.7) is interpreted as a signal of the phase transition in the infinitely large system /see Ref.[8-11]/. It may be verified by taking a thermodynamical limit $N \to \infty$ in Eq. (2.5)

$$\tau(t) = t g^{(d)}(\Delta) , \qquad (2.9)$$

where /see also Eq.(2.8)/

$$g^{(d)}(\Delta) = \int_0^1 \frac{x^{d-1}}{\Delta + x^2}\, dx , \qquad \Delta = \frac{2V'(\tau)}{sq_B^2} , \qquad (2.9a)$$

$$t = \frac{kT}{sq_B^2}\, d . \qquad (2.10)$$

It is found that Eq.(2.9) is reasonable ( $d \geq 3$ ) for $T > T_c$

$$\tau(t_c) = t_c\, g^{(d)}(0) \equiv \tau_0 , \qquad (2.11)$$

and

$$\omega_0^2 = \frac{2V'(\tau)}{M} \sim (t - t_c)^{\gamma} , \qquad (2.12)$$

where

$$\gamma = 2 \qquad \text{for } d = 3 \qquad (2.12a)$$

and

$$\gamma = 1 \qquad \text{for } d = 5,6,\ldots \qquad (2.12b)$$

Below $T_c$, there is no any solution of Eq.(2.9), but it may be shown that in this range the gap in the spectrum vanishes: $r = r_o$, $\omega_o^2 = 0$. This is made by adding a term of interaction with an external field to the Hamiltonian and taking a thermodynamical limit prior to the limit of a vanishing external field /see Ref.[9] and Sec. IV/. The temperature-dependence of the soft-mode (2.4)

$$\omega_o^2 \begin{cases} \sim (t-t_c)^{\gamma} & , \quad t > t_c \\ = 0 & , \quad t < t_c \end{cases} , \tag{2.13}$$

allows one to ask if formula (2.13) describes a Goldstone-type excitation associated with a spontaneously broken continuous symmetry. The answer /negative /to this question and the consequences following from it will be discussed in the next Sections.

## III. The excitation spectrum at T = 0

In this Section we shall study the properties of the spectrum of small oscillations around the minima of the potential energy of the model (1.2) /see also Ref.[12]/. In the absence of the harmonic interaction term, the system has a spherical symmetry / N-dimensional space - see below/, and small oscillations around any point of the hypersphere

$$\tilde{X} \circ \tilde{X} = r_o \tag{3.1a}$$

where $\tilde{X}$ is N-component column vector

$$\tilde{X} = \frac{1}{\sqrt{N}} \left( \{Q_i\} \right) , \tag{3.1b}$$

are characterized by N-1 times degenerated zero-frequency mode and excitation of perpendicular to the surface of the hypersphere with the frequency $\Omega_0$

$$\Omega_0^2 = \frac{1}{M} V''(r_0)\, 4 r_0 \qquad (3.2)$$

For the model (1.1), $V(r) = -\frac{A}{2} r + \frac{B}{4} r^2$,

$$\Omega_0^2 = \frac{2A}{M}$$

/c.f. Ref.[12]/ .

The presence of the harmonic interaction term in the Hamiltonian (1.2) breaks this spherical symmetry. This can be seen by expressing the potential energy $U$ in the $\hat{X}$ representation

$$U = N\left[ V\left(\hat{X}^{+} \circ \hat{X}\right) + \frac{1}{2} \hat{X}^{+} \Phi \hat{X} \right] , \qquad (3.3)$$

where

$$\hat{X} = \begin{pmatrix} x_0 \\ \{x_q\} \end{pmatrix} , \qquad \hat{X}^{+} = \left( x_0, \{ x_{-q} \} \right) , \qquad (3.3a)$$

$$x_q = \frac{1}{\sqrt{N}} Q_q \qquad \text{and}$$

$\Phi$ is a diagonal N x N matrix with elements $\varphi_0 - \varphi_q$.
We shall assume that these elements are non-negative (see also Eq.(2.8)/:

$$\varphi_0 - \varphi_q > 0 \qquad (q \neq 0) \qquad (3.4)$$

what means that the phase transition /in the infinitely large system/ takes place without the change of the translational symmetry. Then, the potential energy, Eq. (3.3), has a degenerated minimum

$$U(\hat{x}^*) = N V(\tau_o) \quad , \tag{3.5}$$

$$\hat{x}^* = \begin{pmatrix} x_o^* \\ 0 \\ 0 \\ \vdots \end{pmatrix} \quad , \quad {x_o^*}^2 = \tau_o \; . \tag{3.6}$$

The spectrum of small oscillations around one of the two minima (3.6) is given by the following formula

$$\Omega_q^2 = \frac{1}{M}\left(\varphi_o - \varphi_q\right) \quad , \tag{3.7a}$$

$$\Omega_o^2 = \frac{1}{M} V''(\tau_o)\, 4\tau_o \; . \tag{3.7b}$$

Let us shortly summarize the above discussion. There is a discrete, but not continuous, symmetry spontaneously broken by the phase transition in the class of models (1.2); therefore, there should not appear any of the Goldstone-type excitation. On the other hand, a singular $q$-dependence of a soft-mode in these models is expected, at least at T = 0 /see formula (3.7) for a finite, mechanical system /. This is in contradiction with the naive interpretation of the results obtained within the approximating Hamiltonian method or other methods [4-11] / see also Eqs.(2.7) and (2.13) /.

In the next Section, the proper temperature behaviour of the soft-mode frequency $\Omega_o^2(T)$ will be found.

IV. The susceptibility function

The inverse susceptibility $\chi^{-1}$

$$\chi = \left.\frac{\partial \langle Q\rangle(h)}{\partial h}\right|_{h=0} , \tag{4.1}$$

is by definition proportional to the square soft-mode frequency $\Omega_0^2$ . We shall show that $\Omega_0^2$ does not vanish within the whole ordered phase $T < T_c$ , so, $\omega_0^2$ /see Eq. (2.13)/ should not be interpreted as a soft-mode frequency /below $T_c$/ although

$$\Omega_0^2 = \omega_0^2 \quad , \quad T > T_c . \tag{4.2}$$

To study the properties of the susceptibility function we have to add to the Hamiltonian (1.2) a term of interaction with the external, homogeneous field h

$$H(h) = T + U - h\sum_{l} Q_l . \tag{4.3}$$

The free energy is given by the following formula:

$$F_N = \frac{kT}{N}\sum_{q} \ln\left(\frac{\hbar\omega_q}{kT}\right) + \left[V(r^*) - r^* V'(r^*)\right] - \frac{1}{2}\frac{h^2}{2V'(r^*)} , \tag{4.4}$$

where

$$\omega_q^2(h,T) = \frac{1}{M}\left[2V'(r^*) + \varphi_0 - \varphi_q\right] , \tag{4.5}$$

and $r^*$ is given by a self-consistency condition

$$r^* = r^*(h,T) = \left[\frac{h}{2V'(r^*)}\right]^2 + \frac{kT}{N}\sum_{q}(M\omega_q)^{-2} . \tag{4.6}$$

Most of the interesting properties of the model (1.2) may be studied by using Eqs.(4.4 - 4.6) which are obtained within the saddle-point type method with the Hamiltonian (4.3). The phase transition takes place in the infinitely large systems ($N \to \infty$), for $d \geq 3$, / see also Sec.II /. Then, the order parameter

$$s = \lim_{h \to 0} s_h = \lim_{h \to 0} \left[ \frac{h}{2V'(\tau^*)} \right] , \tag{4.7}$$

is different from zero for $T < T_c$, / Eqs.(2.11) and (4.6)/

$$s \sim (t_c - t)^{\frac{1}{2}} . \tag{4.8}$$

The susceptibility $\chi$ given by the formula (4.1) is

$$\chi = \frac{\partial}{\partial h} s_h \Big|_{h=0} \tag{4.9}$$

In the range $T < T_c$, $h \to 0$

$$\tau^* \simeq \tau_0 \left[ 1 + a_1 h^{b_1} \left( 1 + a_2 h^{b_2} \right) \right] , \tag{4.9a}$$

and

$$V'(\tau^*) \simeq V''(\tau_0)\, \tau_0\, a_1 h^{b_1} \left( 1 + a_2 h^{b_2} \right) , \tag{4.9b}$$

where the parameters $a_1$, $a_2$, $b_1$, $b_2$ are found from the condition (4.6).

The parameter $b_1$ is obviously equal to one, $b_1 = 1$, so

$$\chi = \left[ 2V''(\tau_0) a_1 \tau_0 \right]^{-1} \frac{\partial}{\partial h} \left[ \left( 1 + a_2 h^{b_2} \right)^{-1} \right] \Bigg|_{h=0} . \tag{4.10}$$

Let us separately consider two cases

a) T = 0

Then, one obtains from Eqs.(4.6) and (4.9)

$$b_2 = 1, \tag{4.11}$$

$$\chi = -\frac{a_2}{2V''(r_0)a_1 r_0} = \left[4V''(r_0)r_0\right]^{-1} \tag{4.12}$$

Thus, we find the coincidence with the result (3.7)

$$\chi = \frac{\partial}{\partial h}\left[\frac{h}{\omega_0^2(h)}\right]\Bigg|_{h=0} = \left(M\Omega_0^2\right)^{-1} \tag{4.12a}$$

b/ $T_c > T > 0$

The integral in Eq.(4.6) / see Eqs.(2.9)/ is easily calculated and one finds $b_2$ to be dependent on the dimension d

$$b_2 = \frac{1}{2}, \quad (d = 3) \tag{4.13a}$$

$$b_2 = 1, \quad (d = 5, 6, \ldots) \tag{4.13b}$$

Therefore, the susceptibility is not defined in this region for d = 3:

$$\chi^{-1} = 0 \tag{4.14}$$

and it is well-defined for $d \geq 5$:

$$\chi^{-1} \sim (t_c - t) \quad (d = 5, 6, \ldots). \tag{4.15}$$

So, we may conclude that the inverse susceptibility function $\chi^{-1} \sim \Omega_0^2$ for the class of models (1.2) reveals a singular behaviour at T = 0 for d = 3 and is a continuous function of T for $d \geq 5$ / see Fig.1 /. In the disordered phase

$$\chi \sim (t - t_c)^{-\gamma} \tag{4.16}$$

where

$\gamma = 2\ ,\ (\ d = 3\ )\ ,$ (4.17a)

$\gamma = 1\ ,\ (\ d \geq 5\ )\ ,$ (4.17b)

as follows from Eq.(4.6) .

In this case, the susceptibility at the critical dimension, d = 4, might be also found. After some simple algebra, one finds that the critical indices are classical but the logarithmic corrections appear. Namely

a/ $T > T_c$

$$\chi \sim \frac{\ln(T-T_c)}{T-T_c} \quad ,$$

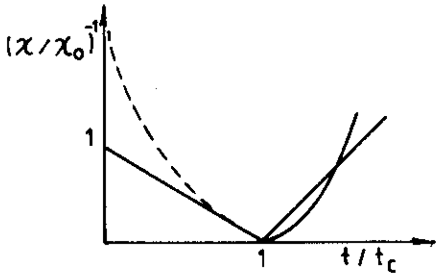
b/ $0 < T < T_c$

$$\chi^{-1} = \lim_{h \to 0} \left( \frac{\ln h}{T_c - T} \right)^{-1} = 0$$

as follows from Eq.(4.6) and / modified / formula (4.9) .

V. The correspondence with the spherical model

It is believed that the models defined by the Hamiltonian (1.2) belong to the class of universality of spherical model Refs. 3,4,5,10,11 /although $\chi$ ($T < T_c$ ) has not been defined there for any $d > 2$ /. As the finite value of the susceptibility function at T = 0, $\chi(0) = [4\, V''(\tau_0)\, \tau_0\, ]^{-1}$ is the common feature of models (1.2) /classical limit, $d \geq 3$ / independently of the behaviour $\chi$ for $T > 0$ /see the Figure/; so, we may ask if this phenomenon is observed in the spherical model. The answer is: " to some extent - yes".




Temperature dependence of the susceptibility function

| | |
|---|---|
| ——— | $d = 3$ |
| ——— | $d \geq 5$ |
| - - - - | spherical model ( $d \geq 5$) |

Firstly, let us note that by adding to the Hamiltonian (1.2) a condition of "sphericity"

$$\frac{1}{N} \sum_{l} Q_l^2 = u \tag{5.1}$$

one obtains, <u>in the classical approximation</u>, the usual spherical model / for any well-defined $V(r)$, also for $V(r) = Ar$, $A > 0$ / - see definition of $Z_Q$ (2.2).
In this case the condition of self-consistency (4.6), has the following form / in the limit $N \to \infty$ /:

$$u = s_h^2 + t g^{(d)}(\delta) \quad , \tag{5.2}$$

where

$$\delta = \frac{1}{s q_0^2} \frac{h}{s_h} = c \frac{h}{s_h} \quad , \tag{5.2a}$$

/compare with Eq.(2.9)/ and     is the function of h and T.

The susceptibility function is

$$\chi = \left(\frac{\partial}{\partial h} s_h\right)_{h=0} = - \frac{ct\, g(\delta)}{2 s_h^2 - ct\, g'(\delta) \cdot \frac{h}{s_h}} \quad (5.3)$$

In the ordered phase $T < T_c$, $s = \lim s_h \neq 0$ and, as

$$|g'(\delta)| \xrightarrow[\delta \to 0]{} \infty \ , \quad (d = 3) \ , \quad (5.4a)$$

$$|g'(\delta = 0)| < \infty \ , \quad (d = 5, 6 \ldots) \ , \quad (5.4b)$$

in the range of temperature $0 < T < T_c$

$$\chi = \infty \ , \quad (d = 3) \ , \quad (5.5a)$$

$$\chi < \infty \ , \quad (d \geq 5) \ . \quad (5.5b)$$

But, one finds that at $T = 0$

$$\chi = 0 \quad (!) \ . \quad (5.6)$$

Therefore, also in this case, the susceptibility function reveals the behaviour characteristic of the class of models (1.2). This means that $\chi$ $(T = 0)$ does not depend on d, and $\chi(T)$ is the discontinuous function of T at $T = 0$ for $d = 3$ but the continuous one for $d \geq 5$ $(\chi(T \sim 0) \sim T)$

VI. Final remarks

According to the results of other authors [3-11] the susceptibility function of models (1.2) is not defined in the ordered phase. In this paper it is shown that neither a pseudoharmonic, gapless mode $(T < T_c)$ [4,5,7,9,11] nor the susceptibility calculated in Refs.[3,10] does not correspond to the properly defined susceptibility function. As the soft mode is associated with the response of the system on the external perturbation, the square soft-mode frequency $\Omega_o^2$ which is proportional to the inverse susceptibility $\chi^{-1} = \left[ -\frac{\partial}{\partial h} \langle Q(h) \rangle \right]^{-1}$ was found. In fact $\chi$ is not defined below $T_c$, but only for $d \leq d_c = 4$, whereas it is a smooth function of temperature for $d > d_c$ and the critical index $\gamma'$ takes its classical value, $\gamma' = \gamma = 1$. In such a case it may be stated that the models (1.2) belong, in the classical limit, to the class of universality of the spherical model.

Let us point out that the singular behaviour of susceptibility at T = 0 for d = 3 / and also $d = d_c = 4$ / as a characteristic feature common for all known models belonging to this universality class / including spherical model / is closely related to the non-ergodicity of the model. It turns out that, in contradiction to the results of Ref.[3], the dynamics of model (1.1) and also (1.2) is quite nontrivial Namely, the adiabatic susceptibility,

$$\chi_{ad} := \lim_{\omega \to 0} \chi(\omega + i\varepsilon) \tag{6.1}$$

is a smooth function of temperature also in the ordered phase $T < T_c$,

$$\chi_{ad} = \frac{1}{M} \left[ \frac{4 V''(\tau_o) \langle Q \rangle^2}{M} \right]^{-1} \tag{6.2}$$

for any d( $\geq$ 3). Therefore, the adiabatic (6.1) and isothermic (4.9), susceptibilities are different for d = 3 / and d = 4 /

what actually means that the system is non-ergodic [13] within the ordered phase, $0 < T < T_c$. As it simply follows from Kubo considerations [13], at T = 0,

$$\chi_{ad} = \chi_{is} \tag{6.3}$$

in this case, where the potential energy is doubly degenerated (3.6). This is the reason of appearing of singularity in the plot of susceptibility /see the Figure/

$$\chi_{is}(T=0) = \left[4V''(r_0)\, r_0\right]^{-1} . \tag{6.4}$$

The extended discussion of the dynamics of model (1.2) is beyond the scope of this paper.

Let also shortly comment a new result for the spherical model, which to our data, has not been previously reported / see also a review paper [14] /.

In the spherical model, which may be considered as a limiting case of the model (1.2), instead of the formula (6.4) it was found that

$$\chi(T=0)=0 . \tag{6.5}$$

It is a somewhat surprising result, at the first sight. But let us remember that at T = 0, the system is in the position of the minimum potential energy and $\chi$ is a measure of the response to the homogeneous external perturbation. In the spherical model

$$\hat{X}^* = \begin{pmatrix} x_0^* \\ 0 \\ \vdots \end{pmatrix} , \qquad x_0^{*2} = u , \tag{6.6}$$

and there is no any response at T = 0, Eqs.(5.1), to the external perturbation $-hQ_0$, so $\chi = 0$, in agreement with Eq.(6.5).

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