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LOCALIZATION OF SOLITONS ON SMALL INHOMOGENEITIES IN JOSEPHSON JUNCTIONS

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1. Introduction

The effects of the soliton localization on microinhomogeneities (microresistances, or MR) in long (one - dimensional) Josephson junction (LJJ) were predicted and the ways of their experimental observation were considered in papers [1-5]. For instance, an easiest way of observing a localized soliton by specific dependence of critical current $\gamma_c(h)$ on the external magnetic field, **h**, was suggested in ref.[4].Due to the soliton and antisoliton localization on MR there arises a cross - shaped dependence of γ_c on **h** in a weak field, the so-called "soliton cross" [4] that has recently been observed experimentally [6].

However, it was only a qualitative agreement with the theoretical model [4]. The ratio of the current for a junction with a localized soliton, $\gamma_c^s(0)$, to the maximum critical current $\gamma_c^m(0)$ is much larger than predicted one. This discrepancy is not in fact surprising since: 1) MR in the sample used in ref.[6] differs essentially from the ideal one considered in ref.[4]*) (the thickness of an insulating layer in the real MR are $\geqslant 2\lambda_L$ where λ_L is the London penetration depth); 2) the distribution of the bias current $I_B(x)$ in the real LJJ is usually inhomogeneous whereas in ref.[4] a homogeneous distribution along the whole junction was considered. It is suggested in the present paper that the internal structure of the inhomogeneity and the inhomogeneous distribution of $I_B(x)$ are to be taken into account in calculating $\gamma_c(h)$.

In the next section we remind the basic assumption of the model used in ref.[4] and formulate a new ("realistic") model. In sect.3 a qualitative discussion of possible new effects and simple estimation of $\gamma_c^m(0)$ are suggested. Sect.4 contains the most typical results of numerical calculations from which it follows that the predictions of the new model are in good agreement with available data. In conclusion, an extensive experimental verification of the proposed model is discussed.

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*) Preparation of artificial MR with a priori given properties has first been proposed in refs.[1,2]. All subsequent theoretical studies have treated only those inhomogeneities for which all the basic effects are determined by the vanishing of the linear density of the critical Josephson current $j(x)$ in the inhomogeneity.

2. The realistic model

The theory [1-4] has been developed for a model that can be called the "$\alpha\beta\gamma$ - model" for LJJ. It is determined by the equation

$$\ddot{\varphi} - \varphi'' + \left[1 - \sum_i \mu_i \delta(x-x_i) \right] \operatorname{Sin}\varphi + \alpha \dot{\varphi} - \beta \dot{\varphi}'' + \gamma = 0 \qquad (1)$$

where $\dot{\varphi} = \varphi_t$, $\varphi' = \varphi_x$, $2\pi\, \varphi(x,t) = \Phi(x,t) / \Phi_0$ *); $\Phi(x,t)$ is the distribution of the magnetic flux along the junction; α, β, γ and μ_i are some constants. The parameters α and β describe the effects of damping due to penetration of the normal current through the junction and along the surface; γ is the bias current flowing through the junction (eq.(1) corresponds to the "overlap geometry" of the junction in which one can achieve γ independent of x). Inhomogeneities (thickenings) of the junction dielectric layer are approximately described by the δ - functions, $\mu_i\delta(x-x_i)$. It is assumed that in the vicinity of each point x_i, $| x-x_i | < \delta_i$, the Josephson current vanishes. For a sufficiently small δ_i, this corresponds to $\mu_i = 2\delta_i$ since

$$\int_{x_i-\delta_i}^{x_i+\delta_i} dx \left[1 - \mu_i \delta(x-x_i) \right] \operatorname{Sin}\varphi(x,t) = (2\delta_i - \mu_i) \operatorname{Sin}\varphi(x_i,t).$$

Microinhomogeneities can also be treated for which $\mu_i<0$ (microshorts). However, it is very difficult to prepare LJJ with controlled microshorts. Therefore, we shall further assume that LJJ may have only some random microshorts with $| \mu_i | \ll 1$. For the LJJ we study microresistances for which $\mu_i \sim 1$; thus, microshorts can be neglected in the first approximation. In comparing the predictions of this model with the observations one should remember that some of the assumptions used in it can be violated. In the present paper we consider the effects arising due to a finite length of inhomogeneity. Moreover, in the places of the layer thickenings not only the Josephson current becomes zero but also the inductance

*) Here Φ_0 is the magnetic flux quantum , x is the distance in the units of the Josephson length, λ ; t is the time in the units of ω_J^{-1}; ω_J is the Josephson frequency, the magnetic field is normalized to $h_0 = \Phi_0 / 4\pi \lambda_L \lambda_J$.

and capacitance of the junction change. We also take into account the effects of inhomogeneous distribution of the bias current I_B.

To construct a model accurately taking account of the above effects we return to the initial equation for the phase distribution in LJJ [2] (see also refs.[7,8])

$$C\varphi_{tt} - \left(L^{-1}\varphi_x\right)_x + \frac{2e}{\hbar} I \,\mathrm{Sin}\varphi + (\sigma_1+\sigma_2 \mathrm{Cos}\varphi)\varphi_t - \frac{2e}{\hbar} R^{-1} \varphi_{txx} + I_B = 0. \quad (2)$$

where for a while we come back to physical units (**L, C** and **R** are inductance, capacitance and surface resistance per LJJ length unit, **I(x)** is the critical Josephson current, $I_B(x)$ is the bias current). In dimensionless variables defined by the values of λ_J and ω_J in homogeneous intervals, eq.(2) can be rewritten as

$$a\,\ddot{\varphi} - \varphi'' + b\,\varphi' + j\,\mathrm{Sin}\varphi + \alpha\,(\,1+\varepsilon\,\mathrm{Cos}\varphi\,)\,\dot{\varphi} - \beta\,\dot{\varphi}'' + \gamma = 0\,, \qquad (3)$$

where

$$a = \frac{L(x)}{L_0}\frac{C(x)}{C_0}\;;\quad b = \left(\ln\frac{L(x)}{L_0}\right)'\;;\quad j = \frac{L(x)}{L_0}\frac{I(x)}{I_0}\;;$$
$$\alpha = \frac{L(x)}{L_0}\frac{\sigma_1}{\omega_0 C_0}\;;\quad \beta = \omega_0\frac{L(x)}{R(x)}\;;\quad \gamma = \frac{L(x)}{L_0}\frac{I(x)}{I_0}\;;\quad \varepsilon = \frac{\sigma_2}{\sigma_1} \qquad (4)$$

L_0, C_0, I_0, λ_0 and ω_0 are the values of $L(x), C(x), I(x), \lambda_J$ and ω_J in homogeneous LJJ intervals. If L, C, I_B, R and σ_1 are constant, $\varepsilon = 0$, and inhomogeneities in the distribution are local, then the $\alpha\beta\gamma\delta$ - model (1) follows from (3).

Consider inhomogeneity whose design is shown in fig.1.

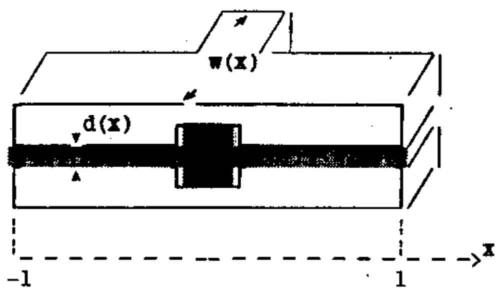


Fig.1. A design of LJJ with MR.

One can easily show that

$$\frac{L(x)}{L_0} = \frac{2\lambda_L+d(x)}{2\lambda_L+d_0}\ ;\quad \frac{C(x)}{C_0} = \frac{w(x)}{w_0}\,\frac{d_0}{d(x)}\ ;\quad \frac{R(x)}{R_0} = \frac{w_0}{w(x)}\ , \tag{5}$$

whence it can be seen how the design of the junction influences the dependence of the functions α, β, γ, a, b and j on x; moreover, $\dot{\gamma}(x)$ depends on the I_B distribution (see below). Inside the inhomogeneity the current $j(x)$ vanishes and in the homogeneous intervals $j(x)=1$. Conductivities σ_1 and σ_2 in the inhomogeneities vanish whereas outside them they are constant. We shall assume $d(x)$ and $w(x)$ to suffer a discontinuity

$$d(x) = \sum_i\left[d_0\bar{\theta}_i(x) + d_i\theta_i(x)\right];\quad w(x) = \sum_i\left[w_0\bar{\theta}_i(x) + w_i\theta_i(x)\right], \tag{6}$$

where by definition

$$\theta_i(x)=0,\ |\ x-x_i|>\delta_i;\ \theta_i(x)=1,\ |\ x-x_i|<\delta_i;\ \ \bar{\theta}_i= 1 - \theta_i .$$

Taking account of (4-6) we can easily find that

$$a(x) \cong \sum_i\left[\ \bar{\theta}_i(x) + \theta_i(x)\,\frac{d_0}{d_i}\,\left(1 + \frac{d_i}{2\lambda_L}\right)\ \right] \tag{7}$$

$$\alpha(x) \cong \sum_i \bar{\theta}_i(x)\ \alpha_0;\quad \beta(x) \cong \beta_0\left[\ 1 + \sum \theta_i(x)\,\frac{d_i}{2\lambda_L}\right]$$

$$b(x) \cong 2\sum_i \theta_i'(x)\ \left(\frac{d_i - d_0}{d_i+4\lambda_L} - \frac{w_i - w_0}{w_i + w_0}\right);\quad j(x) = \sum_i \bar{\theta}_i(x) \tag{8}$$

$$\gamma(x) \cong \frac{I_B(x)}{I_0}\sum_i\left[\ \bar{\theta}_i(x) + \theta_i(x)\,\frac{w_0}{w_i}\,\left(\ 1 + \frac{d_i}{2\lambda_L}\right)\ \right] \tag{9}$$

(here $\cong$ means neglect of the corrections $\sim d\ /\ 2\lambda_L$). For a final formulation of the model one should give $I_B(x)$. As is known (see for instance ref.[9]), by using a superconducting screen one can achieve the current $I_B(x)$ to be uniform along the junction. Otherwise, the current concentrates near the junction edges which is described by the simple expression (see ref.[9])

$$\frac{I_B(x)}{I_0} = \frac{\gamma_0}{\pi}\,\frac{2l}{\sqrt{l^2 - x^2}}\ , \tag{10}$$

where γ_0 is the mean bias current flowing through the junction. This completes the formulation of the realistic (or $\alpha\beta\gamma\theta$) model of the junction with artificial microresistances.

3. Qualitative discussion and estimates of the effects

If $a=1$, $b=0$, $\alpha=\alpha_0$, $\varepsilon=0$, $\beta=0$ and $\gamma=\text{const}$, then the new model reduces to the $\alpha\beta\gamma\delta$ – model but with "smeared" δ – functions. In ref.[4,5] the basic phenomena have been shown to be qualitatively independent of smearing and that the quantitative dependence is not essential if $\mu<1$ (it is assumed that all inhomogeneities are rather far from each other and from the edges of the junction). New effects can arise if $d \geq 2\lambda$ or $w_i \ll w_0$ as well as due to inhomogeneous distribution of the current I_B.

For $d_i/d_0 \ll 1$ the velocity $v \equiv \sqrt{a(x)} \sim \sqrt{d_i/d_0}$ is very large inside inhomogeneity. Moreover, as

$$\theta_i'(x) = \delta(x-x_i+\delta_i) - \delta(x-x_i-\delta_i)$$

then for $d_0 \ll d_i \sim 2\lambda_L$ the very strong barriers arise at the edges of inhomogeneity. Since the terms containing θ_i' are proportional to φ', one should use "smeared" δ- functions (or smoothed θ – functions). In this case some results may depend on a specific dimension of smoothing. It is very important that these effects of sharp edges of inhomogeneity can be removed. It is easily seen that taking

$$w_i \cong w_0 (1 + d_i/2\lambda_L)$$

$b(x)$ becomes zero. On the contrary, at $w_0 \ll w_i$ the effect of a sharp edge increases, and the value of $\gamma(x)$ also strongly increases at $| x-x_i|<\delta_i$.

All these effects can essentially influence the motion of solitons in the junctions with inhomogeneities, the current voltage curves and the spectrum and the nature of static states of the junction. In this paper we consider only static states and their bifurcations with changing the total bias current γ and external magnetic field h. We shall show that a simple version of the $\alpha\beta\gamma\theta$-model allows one to make the agreement between theoretical and experimental curves rather good.

To analyze static states suffice it to study the problem

$$\varphi'' - b(x)\, \varphi' - j(x)\, \text{Sin}\varphi - \gamma(x) = 0, \qquad (11)$$

where b, j and γ are defined by eq.(8-10). The model (9-10) for the bias current may turn out to be insufficient for describing real samples of LJJ in which $w_i \ll w_0$ and $d_i \gg d_0$ since a very sharp change in the LJJ structure in the region of inhomogeneity may cause a strong change in the distribution $I_B(x)$. In this paper, we do not attempt to take these corrections into account. We also assume $w_1 = w_0$ and consider only critical current $\gamma_c(h)$.

In particular, consider $r(0) = \gamma_c^m(0) / \gamma_c^s(0)$. To explain the discrepancy between the theoretical and observed $r(0)$ we propose to take into account two effects: 1) the inhomogeneity of distribution of the current $I_B(x)$ which is described by (10); 2) the change of inductance inside inhomogeneity which is caused by considerable change in the thickness d ($d_0 \ll d_1 \sim 2\lambda_L$) and is described by (8). Both the effects diminish the ratio $r(0)$, which provides a qualitative agreement of the predictions of the theoretical model with experimental data. For an initial orientation in the estimates for the effects, let us discuss them qualitatively and give approximate formulae for $\gamma_c(0)$. Remember that for the long homogeneous junction $\gamma_c^m(0)=1$. With the inhomogeneities present in the junction, $\gamma_c^m(0)$ decreases.

One can get a good estimate of $\gamma_c^m(0)$ in the $\alpha\beta\gamma\delta$- model. Using the piecewise - linear approximation [2] we can easily show that for $l \gg 1$ (inhomogeneity is far from the edge)

$$\gamma_c^m(0) \cong \left(1 + \frac{\mu}{\sqrt{4\pi + 2\mu^2}} \right)^{-1}. \qquad (12)$$

Qualitatively, with increasing γ a frozen "breather" is localized on inhomogeneity (see fig.1 in ref.[4]). Both halves of this "breather" are attracted to each other and to inhomogeneity whereas the bias current γ attracts them to each other. The balance of forces is broken at $\gamma \geqslant \gamma_c^m(0)$. In the realistic model we failed to derive such a simple formula, but for rough estimates it is sufficient to use eq.(12).

For the inhomogeneous distribution of I_B the maximal value of γ_0 is considered as a critical current (see (10)); we retain for this quantity the notation $\gamma_c^{m,s}(0)$ *).

Using the simplest approximation of the standard soliton perturbation theory (for $l \gg 1$), one can get estimates for $\gamma^s(0)$ ta-

*) Remember that $\gamma^{m,s}(h)$ is the ratio of the total critical current flowing through the junction in the m- or s- state to the maximal current flowing through the corresponding homogeneous junction.

king account of changing in L, inhomogeneity of the bias current and finite length of the inhomogeneity. We shall restrict ourselves to one inhomogeneity placed at the center of the junction and assume that $\mu_1 = 2\delta_1$, $L_1 \neq L_0$ and $w_1 = w_0$. Instead of formula (8) of ref.[4], for the homogeneous distribution we can get

$$2\pi\ \gamma_c^s(0) = \frac{8\mu_1}{3\sqrt{3}} \left[\frac{\mathrm{sh}\mu_1 / \mu_1}{\left(1 + \frac{2}{3}\ \mathrm{sh}^2(\mu_1/2)\right)^2} \right] \left(2 - \frac{L_1}{L_0} \right) . \qquad (13)$$

The first factor in the right-hand side provides the value of the critical current on the δ - inhomogeneity, and the second one for a finite length of the inhomogeneity (its value at μ_1=1 is 0.85). In the same approximation, for the inhomogeneous distribution of the current one can find

$$\frac{\gamma_{cB}^s(0)}{\gamma_c^s(0)} \underset{l\gg1}{\cong} \frac{\pi}{4} \sqrt{2\pi}\ f(\mu_1), \qquad (14)$$

where $f(0)=1$, $f'(0)=0$ and $f(\mu_1)$ weakly depends on μ_1 at small values of μ_1 ($f(0.5) \cong 1.02$).

According to (13) one can use the δ-model of real inhomogeneity if $\mu_1 = 2\delta_1$ is replaced by $\mu_1[\ldots](\ldots) = \mu_1^{eff}$. Changing μ_1 by μ_1^{eff} for rough estimates of $\gamma_c^m(0)$ one can use also formula (12). Note that for asymmetric configuration of the junction or boundary conditions the value of $\gamma_c(h)$ is maximal at some $h\neq0$. In these cases, formula (12) provides a maximal value of the current, and formulae (13) and (14) are to be altered.

Let us compare the results of paper [6] with our estimates.

For LJJ investigated in ref.[6]: $2l\approx15$, $\mu_1 = 2\delta_1 \approx \frac{1}{7}$, $d_1/2\lambda_L \approx 3$, $\gamma_c^m(0)\sim0.68$ and $\gamma_c^s(0)\sim0.4$. By formulae (12-14) we get $\gamma_c^m(0)=0.93$ and $\gamma_c^s(0)=0.12$. Thus, a quantitative discrepancy between the theory and experiment is essentially less than for the $\alpha\beta\gamma\delta$ - model. It is to be noted that the most probable source of divergence is a large value of $d_1/2\lambda_L$. Determining μ_1^{eff} by $\gamma_c^s(0)$ from the formula

$$2\pi\ \gamma_c^s(0) = \frac{8\mu_1^{eff}}{3\sqrt{3}}$$

and substituting this into (12) we get $\gamma_c^m(0) \approx 0.72$, which agrees well with experimental data.The other data obtained by the authors of ref.[6] (for samples with smaller values of $d_1/2\lambda_L$) are in rough agreement with formulae (12-14). We can conclude that the experiment confirms the basic prediction of paper [4] - the existence of the soliton branch $\gamma_c(h)$ and soliton cross.

4. Numerical analysis of the realistic model

As in ref.[4] we are to solve the problem (11) numerically. The critical value of γ at given h is determined from the condition of vanishing the eigenvalue ω^2 of the problem (see [1-5])

$$\psi'' - b(x)\,\psi' - j(x)\,\cos\varphi\,\psi + \omega^2\psi = 0\,. \qquad (15)$$

In this case, discontinuities in the functions $b(x)$ and $j(x)$ should be smoothed and the current distribution (10) regularized. Thus, in the distribution (10) we substitute

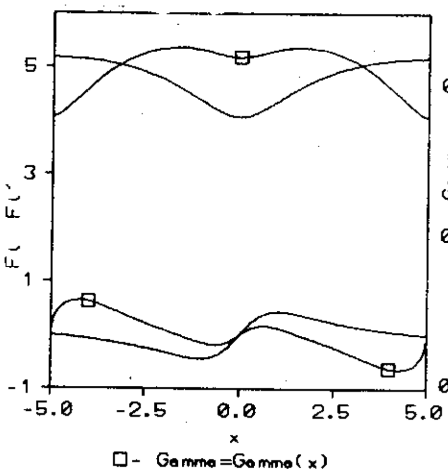
$$\sqrt{l^2-x^2} \to \sqrt{l^2+\delta_\gamma^2-x^2}$$

and smooth discontinuity in $b(x)$ by changing the function $\theta(x)$ by $\mathrm{th}(x/\delta_b)$. If the edge of the junction is sharp and $d(x)$ is changed abruptly (i.e., at the distances $\leqslant 2\lambda_L$), then a physically natural regularization is apparently $\delta_\gamma \sim \delta_b \sim \lambda_L$. Regularization with larger values of δ_γ, δ_b corresponds to smoothing of the junction structure while its preparation.

For a detailed numerical algorithm of solving the problem see paper [11]. We consider only the general idea of the method. Equations (11) and (15) together with the boundary conditions and the condition of normalization of the function ψ will be treated as a unique system that can be closed by setting two of three parameters γ, h and ω^2; the third parameter is found as a solution of the problem. The simplest is the case when the quantities γ and h are fixed; in this case, the system splits into the above - mentioned two subsystems for $\varphi(x)$ and ($\psi(x),\omega^2$). In the rest two cases (either γ and ω^2 or h and ω^2 are fixed) the system should be treated as a nonlinear problem for eigenvalues with spectral parameter h or γ, respectively. A continuous analog of the Newton method [12] with an iteration step taken with respect to the minimum discrepancy is applied to solve this problem.

Here are some results of calculations. The effect of inhomogeneity $\gamma(x)$ can be understood from fig.2 representing $\varphi(x)$ and $\varphi'(x)$ at $h = 0$ for γ_0 close to $\gamma_c^m(0)$. Calculations were made for a very large value of δ_b. With decreasing δ_b the peaks at the junction edges increase but the picture does not change qualitatively. Fig.3 shows the calculated curves for two LJJ with MR. The parameters of the former coincide with those of the junction studied in ref.[6], and $\delta_\gamma \sim \delta_b \sim 2\lambda_L$ is taken. Finally fig.4 represents the curves $\gamma_c(h)$ for a nonsymmetric position of MR. It is seen that the vertex curve $\gamma_c^m(h)$ is shifted a little whereas the soliton cross is shifted and strongly deformed. Comparison with experimental data, formulae (12-14) and the relevant curves from ref.[4] is not difficult. More complete results and their discussion will be published in a journal.

Fig 2. The distribution of $\varphi(x)$ and $\varphi'(x)$ inside LJJ for $\gamma(x)$ and γ=**const** when γ_0 (or γ) is near $\gamma_c^m(0)$:
$2l=10$, $\mu_1= 2\delta_1= 1.2$, $h=0$, $\delta_b= 0.2$.



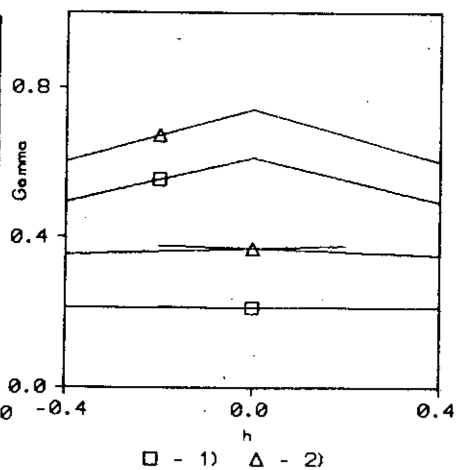


Fig.3. The **h** - dependence of the critical current for different states with the nonuniform distribution of $\gamma(x)$:
1) $2l=15$, $\mu_1= 2\delta_1=0.14$, $L_0/L_1 = 0.25$;
2) $2l=10$, $\mu_1= 2\delta_1=0.6$, $L_0/L_1 = 0.5$.

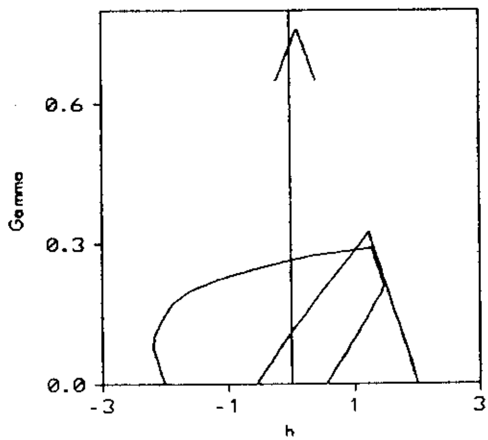


Fig.4. The **h** - dependence of the critical current for the asymmetric location of the inhomogeneity with the nonuniform distribution of $\gamma(\mathbf{x})$; inhomogeneity is displaced to $\mathbf{x}_1 = 2$:

$$2l=10, \ \mu_1 = 2\delta_1 = 1.2, \ L_0/L_1 = 1.$$

5. Conclusion

Localization of solitons on microinhomogeneities predicted in papers [1-4] was confirmed experimentally [6]. However, for describing real experiments the model used in refs.[1-4] should be modified, which has been done in the present paper. For a thorough verification of this model it is necessary to study experimentally the dependence of the critical currents $\gamma_c^{m,s}(\mathbf{h})$ on the length, width, thickness and position of inhomogeneity making particular efforts to control thickness of inhomogeneity and nature of its edges. It is also to be emphasized that it is not difficult to realize experimentally the conditions under which the model used in refs.[1-4] is applicable. It is necessary that $2\delta_1 \leqslant 1$, $d_1/2\lambda_L \ll 1$, $w_1 = w_0$ and the current distribution $I_B(\mathbf{x})$ is homogeneous. In more detail this model will be discussed and compared with experiment in a more extended paper.

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